Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.3-Inverse-hyperbolic-tangent/7.3.3-d+e-x-^m-a+b-arctanh-c-x^n-^p

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Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [49]. This is test number [193].

1.1   Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.

2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.

3. Maple 2021.1 (64 bit) on windows 10.

4. Maxima 5.44 on Linux. (via sagemath 9.3)

5. Fricas 1.3.7 on Linux (via sagemath 9.3)

6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)

7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.

8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 93.88 (46)	% 6.12 (3)
Mathematica	% 95.92 (47)	% 4.08 (2)
Maple	% 97.96 (48)	% 2.04 (1)
Maxima	% 59.18 (29)	% 40.82 (20)
Fricas	% 32.65 (16)	% 67.35 (33)
Sympy	% 20.41 (10)	% 79.59 (39)
Giac	% 34.69 (17)	% 65.31 (32)
Mupad	% 34.69 (17)	% 65.31 (32)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	93.88	0.00	0.00	6.12
Mathematica	73.47	0.00	22.45	4.08
Maple	53.06	28.57	16.33	2.04
Maxima	40.82	18.37	0.00	40.82
Fricas	14.29	14.29	4.08	67.35
Sympy	20.41	0.00	0.00	79.59
Giac	18.37	16.33	0.00	65.31
Mupad	4.08	30.61	0.00	65.31

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.

Antiderivative Grade distribution for each CAS

Numbers shown on bars are total percentage solved for each CAS
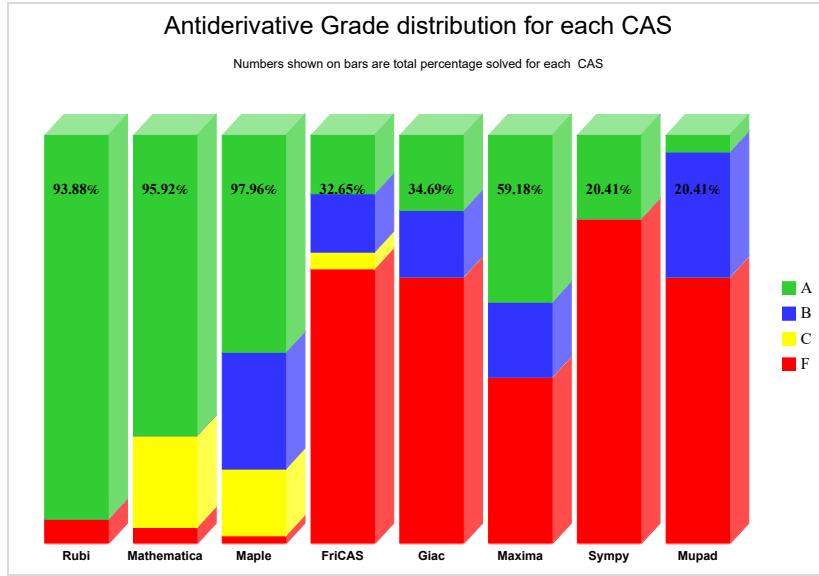
The figure below compares the CAS systems for each grade level.
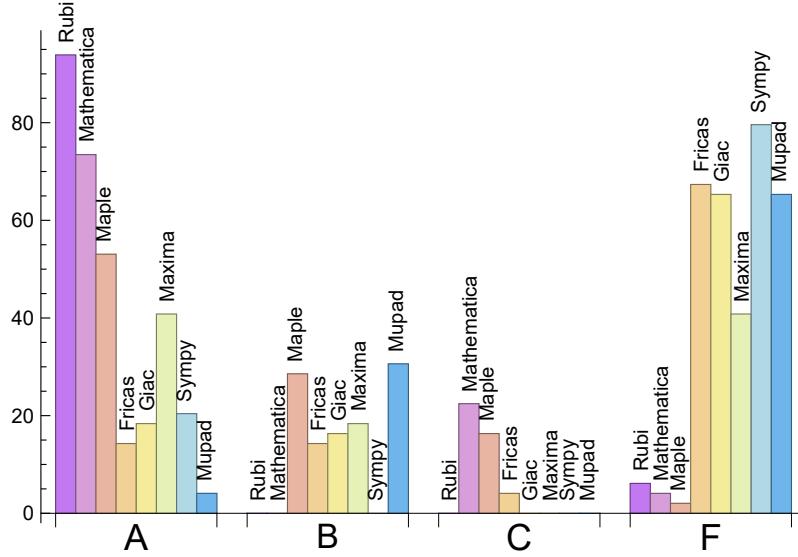
The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	3	100.00 %	0.00 %	0.00 %
Mathematica	2	100.00 %	0.00 %	0.00 %
Maple	1	100.00 %	0.00 %	0.00 %
Maxima	20	100.00 %	0.00 %	0.00 %
Fricas	33	96.97 %	3.03 %	0.00 %
Sympy	39	66.67 %	33.33 %	0.00 %
Giac	32	93.75 %	6.25 %	0.00 %
Mupad	32	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

1.3   Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.43	268.20	0.98	190.00	1.00
Mathematica	5.72	301.87	1.26	272.00	1.08
Maple	0.58	2060.08	4.59	355.00	1.63
Maxima	0.42	233.38	1.37	209.00	1.38
Fricas	4.54	1185.88	4.83	361.50	2.35
Sympy	7.57	1960.50	12.68	501.50	2.99
Giac	8.61	728.29	4.84	374.00	1.63
Mupad	1.85	592.82	2.70	309.00	2.07

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.

Normalized mean size of antiderivative

Lower is better
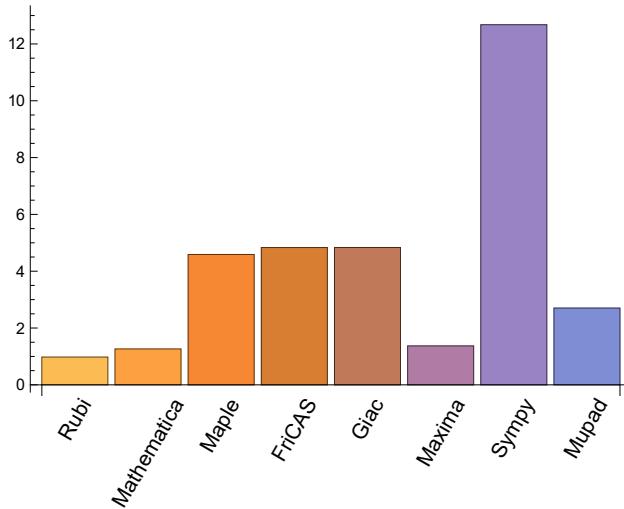
Mean time used (seconds)

Lower is better
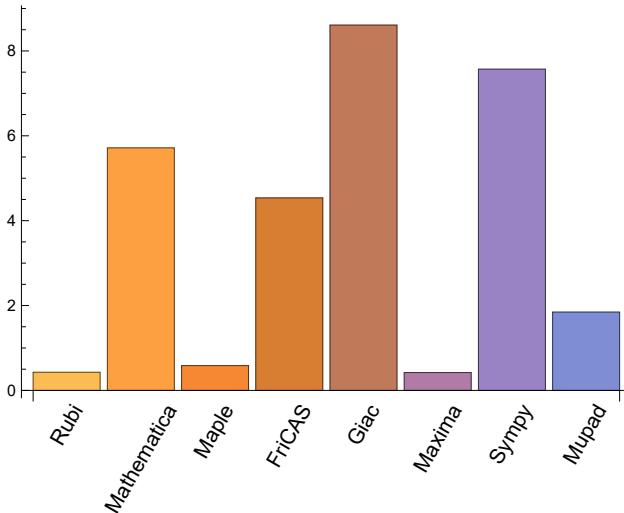
1.4 list of integrals that has no closed form antiderivative

$\{30, 31\}$

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {29}

Mathematica {5, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 21, 22, 29, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7   Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A <u>time limit</u> of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8   Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9   Important notes about some of the results

1.9.1   Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2   Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3   Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://`

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4   Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives sin(x)^2/2

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
   The following field present only in Rubi and Mathematica Tables
13. integer. 1 if result was verified or 0 if not verified.
   The following fields present only in Rubi Tables
14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,..}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49 }

B grade: { }

C grade: { }

F grade: { 26, 28, 34 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 15, 16, 17, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 47, 48, 49 }

B grade: { }

C grade: { 5, 12, 13, 14, 19, 21, 22, 26, 34, 44, 46 }

F grade: { 18, 20 }

2.1.3 Maple

A grade: { 4, 5, 6, 7, 8, 13, 14, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 35, 36, 44, 45, 46, 47, 48, 49 }

B grade: { 1, 2, 3, 9, 10, 11, 21, 37, 38, 39, 40, 41, 42, 43 }

C grade: { 12, 15, 16, 17, 18, 19, 20, 34 }

F grade: { 29 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 6, 7, 23, 24, 25, 27, 28, 30, 31, 32, 33, 35, 36, 37, 38, 39 }

B grade: { 8, 9, 10, 11, 22, 40, 41, 42, 43 }

C grade: { }

F grade: { 5, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 26, 29, 34, 44, 45, 46, 47, 48, 49 }

2.1.5 FriCAS

A grade: { 3, 4, 23, 24, 25, 30, 31 }

B grade: { 1, 2, 6, 7, 8, 27, 28 }

C grade: { 32, 33 }

F grade: { 5, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 26, 29, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 6, 7, 8, 23, 24, 25 }

B grade: { }

C grade: { }

F grade: { 5, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49 }

2.1.7 Giac

A grade: { 23, 24, 25, 27, 30, 31, 32, 33, 35 }

B grade: { 1, 2, 3, 4, 6, 7, 8, 28 }

C grade: { }

F grade: { 5, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 26, 29, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49 }

2.1.8 Mupad

A grade: { 30, 31 }

B grade: { 1, 2, 3, 4, 6, 7, 8, 23, 24, 25, 27, 28, 32, 33, 35 }

C grade: { }

F grade: { 5, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 26, 29, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	274	395	273	322	381	2351	272
normalized size	1	1.00	1.84	2.65	1.83	2.16	2.56	15.78	1.83
time (sec)	N/A	0.141	0.189	0.034	0.334	0.493	2.961	0.233	1.696

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	205	308	209	244	279	1375	197
normalized size	1	1.00	1.64	2.46	1.67	1.95	2.23	11.00	1.58
time (sec)	N/A	0.141	0.135	0.031	0.326	0.800	1.982	0.190	1.150

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	129	218	137	163	178	785	127
normalized size	1	1.00	1.34	2.27	1.43	1.70	1.85	8.18	1.32
time (sec)	N/A	0.122	0.099	0.032	0.312	0.658	1.264	0.390	0.941

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	96	92	83	98	92	292	67
normalized size	1	1.00	1.14	1.10	0.99	1.17	1.10	3.48	0.80
time (sec)	N/A	0.076	0.011	0.029	0.316	0.604	0.674	0.206	0.827

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	257	148	0	0	0	0	-1
normalized size	1	1.00	2.25	1.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.258	0.089	0.000	0.768	0.000	0.000	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	102	114	99	182	690	374	115
normalized size	1	1.00	1.10	1.23	1.06	1.96	7.42	4.02	1.24
time (sec)	N/A	0.067	0.123	0.036	0.317	0.944	3.618	0.181	3.596

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	133	154	190	454	3216	1197	427
normalized size	1	1.00	1.02	1.18	1.46	3.49	24.74	9.21	3.28
time (sec)	N/A	0.129	0.156	0.039	0.332	0.854	7.126	0.195	3.998

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	173	223	339	859	10946	3320	418
normalized size	1	1.00	0.99	1.27	1.94	4.91	62.55	18.97	2.39
time (sec)	N/A	0.188	0.271	0.040	0.336	1.705	12.914	0.238	2.295

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	506	1430	782	0	0	0	-1
normalized size	1	1.00	1.41	3.98	2.18	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.534	0.931	0.069	0.562	1.111	0.000	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	319	1050	524	0	0	0	-1
normalized size	1	1.00	1.24	4.09	2.04	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.409	0.655	0.066	0.565	1.736	0.000	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	174	462	313	0	0	0	-1
normalized size	1	1.00	1.09	2.89	1.96	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.330	0.440	0.058	0.557	0.581	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	759	1170	0	0	0	0	-1
normalized size	1	1.00	4.04	6.22	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	10.613	0.743	0.000	0.552	0.000	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	317	605	0	0	0	0	-1
normalized size	1	1.00	0.99	1.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.313	4.600	0.072	0.000	0.970	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	480	480	470	824	0	0	0	0	-1
normalized size	1	1.00	0.98	1.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.499	7.064	0.079	0.000	0.608	0.000	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	614	614	830	6104	0	0	0	0	-1
normalized size	1	1.00	1.35	9.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.183	2.111	11.293	0.000	1.217	0.000	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	591	4600	0	0	0	0	-1
normalized size	1	1.00	1.53	11.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.804	1.314	6.212	0.000	0.815	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	331	12404	0	0	0	0	-1
normalized size	1	1.00	1.36	50.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.602	0.746	1.246	0.000	0.740	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	0	2367	0	0	0	0	-1
normalized size	1	1.00	0.00	8.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.057	98.024	0.655	0.000	0.486	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	517	517	813	3497	0	0	0	0	-1
normalized size	1	1.00	1.57	6.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.525	13.851	0.888	0.000	0.601	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	953	953	0	53538	0	0	0	0	-1
normalized size	1	1.00	0.00	56.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.027	85.894	3.862	0.000	0.638	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	109	240	118	0	0	0	0	-1
normalized size	1	1.63	3.58	1.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.278	0.043	0.000	0.941	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	108	272	127	144	0	0	0	-1
normalized size	1	1.23	3.09	1.44	1.64	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.100	0.043	0.417	0.698	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	220	254	306	240	519	622	290	823
normalized size	1	1.21	1.40	1.68	1.32	2.85	3.42	1.59	4.52
time (sec)	N/A	0.223	0.277	0.036	0.424	0.947	18.880	66.415	2.045

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	170	223	171	401	2907	194	309
normalized size	1	1.00	1.08	1.41	1.08	2.54	18.40	1.23	1.96
time (sec)	N/A	0.210	0.171	0.031	0.410	0.684	15.808	1.038	1.385

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	94	104	91	95	249	294	143	242
normalized size	1	0.80	0.89	0.78	0.81	2.13	2.51	1.22	2.07
time (sec)	N/A	0.093	0.055	0.030	0.415	1.342	10.478	0.365	1.310

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	A	F	F	F(-1)	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	0	285	362	0	0	0	0	-1
normalized size	1	0.00	0.88	1.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.063	17.365	0.072	0.000	0.702	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	261	181	172	634	0	271	727
normalized size	1	1.00	1.57	1.09	1.04	3.82	0.00	1.63	4.38
time (sec)	N/A	0.279	0.364	0.038	0.413	8.201	0.000	0.226	2.598

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	A	A	B	F(-1)	B	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	0	379	310	311	1639	0	507	2016
normalized size	1	0.00	1.68	1.37	1.38	7.25	0.00	2.24	8.92
time (sec)	N/A	0.066	0.681	0.044	0.424	48.701	0.000	41.457	5.280

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1085	1216	684	0	0	0	0	0	-1
normalized size	1	1.12	0.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.438	3.002	0.421	0.000	0.458	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.119	44.302	0.352	0.000	0.808	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.369	42.011	0.612	0.000	0.914	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	332	299	500	295	9282	0	383	1081
normalized size	1	0.99	0.89	1.49	0.88	27.62	0.00	1.14	3.22
time (sec)	N/A	0.511	0.280	0.035	0.418	2.842	0.000	4.759	1.819

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	333	362	248	3928	0	304	621
normalized size	1	1.00	1.17	1.27	0.87	13.78	0.00	1.07	2.18
time (sec)	N/A	0.452	0.104	0.031	0.422	2.098	0.000	2.178	1.033

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	C	F	F	F(-1)	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	523	0	515	182	0	0	0	0	-1
normalized size	1	0.00	0.98	0.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.062	100.895	0.150	0.000	0.589	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	534	591	394	0	0	595	2638
normalized size	1	1.00	1.29	1.43	0.95	0.00	0.00	1.44	6.37
time (sec)	N/A	0.773	0.520	0.043	0.427	0.000	0.000	28.280	1.405

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	160	309	246	0	0	0	-1
normalized size	1	1.00	0.82	1.58	1.26	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.601	0.571	0.057	0.558	1.107	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	130	276	208	0	0	0	-1
normalized size	1	1.00	0.81	1.72	1.30	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.429	0.391	0.056	0.556	0.567	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	96	243	166	0	0	0	-1
normalized size	1	1.00	0.80	2.02	1.38	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.260	0.225	0.056	0.553	0.947	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	75	186	101	0	0	0	-1
normalized size	1	1.00	0.96	2.38	1.29	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.098	0.051	0.597	0.649	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	72	217	159	0	0	0	-1
normalized size	1	1.00	1.04	3.14	2.30	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.244	0.129	0.059	0.523	0.725	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	118	315	248	0	0	0	-1
normalized size	1	1.00	1.01	2.69	2.12	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.363	0.336	0.067	0.565	1.048	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	158	348	291	0	0	0	-1
normalized size	1	1.00	1.01	2.22	1.85	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.456	0.570	0.071	0.569	1.215	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	187	381	330	0	0	0	-1
normalized size	1	1.00	0.97	1.98	1.72	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.571	0.809	0.071	0.563	0.959	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	558	651	0	0	0	0	-1
normalized size	1	1.00	1.21	1.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.783	2.934	0.097	0.000	0.834	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	337	539	0	0	0	0	-1
normalized size	1	1.00	0.90	1.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.485	1.461	0.065	0.000	0.779	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	432	462	0	0	0	0	-1
normalized size	1	1.00	1.36	1.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.322	1.617	0.062	0.000	0.610	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	302	540	0	0	0	0	-1
normalized size	1	1.00	0.84	1.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.587	1.149	0.070	0.000	0.710	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	360	620	0	0	0	0	-1
normalized size	1	1.00	0.87	1.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.720	1.644	0.074	0.000	0.667	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	506	506	394	741	0	0	0	0	-1
normalized size	1	1.00	0.78	1.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.872	2.741	0.081	0.000	0.702	0.000	0.000	0.000

2.3   Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [29] had the largest ratio of [2.167]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	4	1.00	16	0.250
2	A	6	4	1.00	16	0.250
3	A	6	4	1.00	16	0.250
4	A	6	4	1.00	14	0.286
5	A	4	4	1.00	16	0.250
6	A	6	4	1.00	16	0.250
7	A	4	3	1.00	16	0.188
8	A	4	3	1.00	16	0.188
9	A	19	14	1.00	18	0.778
10	A	15	12	1.00	18	0.667
					Continued on next page	

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
11	A	12	9	1.00	16	0.562
12	A	1	1	1.00	18	0.056
13	A	12	6	1.00	18	0.333
14	A	18	10	1.00	18	0.556
15	A	29	15	1.00	18	0.833
16	A	20	13	1.00	18	0.722
17	A	14	10	1.00	16	0.625
18	A	1	1	1.00	18	0.056
19	A	9	7	1.00	18	0.389
20	A	21	11	1.00	18	0.611
21	A	4	4	1.63	17	0.235
22	A	4	4	1.23	15	0.267
23	A	19	10	1.21	18	0.556
24	A	12	10	1.00	18	0.556
25	A	10	7	0.80	16	0.438
26	F	0	0	N/A	0	N/A
27	A	10	7	1.00	18	0.389
28	F	0	0	N/A	0	N/A
29	A	104	39	1.12	18	2.167
30	A	0	0	0.00	0	0.000
31	A	0	0	0.00	0	0.000
32	A	25	14	0.99	18	0.778
33	A	23	13	1.00	16	0.812
34	F	0	0	N/A	0	N/A
35	A	20	12	1.00	18	0.667
						Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\dfrac{\text{number of rules}}{\text{integrand leaf size}}$
36	A	19	10	1.00	26	0.385
37	A	14	10	1.00	26	0.385
38	A	9	9	1.00	24	0.375
39	A	5	4	1.00	23	0.174
40	A	5	7	1.00	26	0.269
41	A	9	9	1.00	26	0.346
42	A	14	9	1.00	26	0.346
43	A	20	9	1.00	26	0.346
44	A	20	11	1.00	23	0.478
45	A	15	10	1.00	21	0.476
46	A	11	5	1.00	20	0.250
47	A	15	11	1.00	23	0.478
48	A	19	13	1.00	23	0.565
49	A	24	13	1.00	23	0.565

Chapter 3

Listing of integrals

3.1 $\int (d + ex)^4 \left(a + b \tanh^{-1}(cx) \right) dx$

Optimal. Leaf size=149

$$\frac{(d + ex)^5 \left(a + b \tanh^{-1}(cx) \right)}{5e} - \frac{b(cd - e)^5 \log(cx + 1)}{10c^5 e} + \frac{b(cd + e)^5 \log(1 - cx)}{10c^5 e} + \frac{be^2 x^2 \left(10c^2 d^2 + e^2 \right)}{10c^3} + \frac{bdex \left(2c^2 d^2 - \right)}{c^3}$$

[Out] b*d*e*(2*c^2*d^2+e^2)*x/c^3+1/10*b*e^2*(10*c^2*d^2+e^2)*x^2/c^3+1/3*b*d*e^3
*x^3/c+1/20*b*e^4*x^4/c+1/5*(e*x+d)^5*(a+b*arctanh(c*x))/e+1/10*b*(c*d+e)^5
*ln(-c*x+1)/c^5/e-1/10*b*(c*d-e)^5*ln(c*x+1)/c^5/e

Rubi [A]   time = 0.14, antiderivative size = 149, normalized size of antiderivative =
1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$
= 0.250, Rules used = {5926, 702, 633, 31}

$$\frac{(d + ex)^5 \left(a + b \tanh^{-1}(cx) \right)}{5e} + \frac{be^2 x^2 \left(10c^2 d^2 + e^2 \right)}{10c^3} + \frac{bdex \left(2c^2 d^2 + e^2 \right)}{c^3} - \frac{b(cd - e)^5 \log(cx + 1)}{10c^5 e} + \frac{b(cd + e)^5 \log(1 - }{10c^5 e}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^4*(a + b*ArcTanh[c*x]),x]

[Out] (b*d*e*(2*c^2*d^2 + e^2)*x)/c^3 + (b*e^2*(10*c^2*d^2 + e^2)*x^2)/(10*c^3) +
 (b*d*e^3*x^3)/(3*c) + (b*e^4*x^4)/(20*c) + ((d + e*x)^5*(a + b*ArcTanh[c*x
]))/(5*e) + (b*(c*d + e)^5*Log[1 - c*x])/(10*c^5*e) - (b*(c*d - e)^5*Log[1
+ c*x])/(10*c^5*e)

Rule 31

Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\int (d+ex)^4 \left(a+b\tanh^{-1}(cx)\right) dx = \frac{(d+ex)^5 \left(a+b\tanh^{-1}(cx)\right)}{5e} - \frac{(bc)\int \frac{(d+ex)^5}{1-c^2x^2}\, dx}{5e}$$

$$= \frac{(d+ex)^5 \left(a+b\tanh^{-1}(cx)\right)}{5e} - \frac{(bc)\int \left(-\frac{5de^2\left(2c^2d^2+e^2\right)}{c^4} - \frac{e^3\left(10c^2d^2+e^2\right)x}{c^4} - \frac{5de^4x^2}{c^2}\right)}{}$$

$$= \frac{bde\left(2c^2d^2+e^2\right)x}{c^3} + \frac{be^2\left(10c^2d^2+e^2\right)x^2}{10c^3} + \frac{bde^3x^3}{3c} + \frac{be^4x^4}{20c} + \frac{(d+ex)^5\left(a+\right.}{}$$

$$= \frac{bde\left(2c^2d^2+e^2\right)x}{c^3} + \frac{be^2\left(10c^2d^2+e^2\right)x^2}{10c^3} + \frac{bde^3x^3}{3c} + \frac{be^4x^4}{20c} + \frac{(d+ex)^5\left(a+\right.}{}$$

$$= \frac{bde\left(2c^2d^2+e^2\right)x}{c^3} + \frac{be^2\left(10c^2d^2+e^2\right)x^2}{10c^3} + \frac{bde^3x^3}{3c} + \frac{be^4x^4}{20c} + \frac{(d+ex)^5\left(a+\right.}{}$$

Mathematica $[\mathbf{A}]$ time = 0.19, size = 274, normalized size = 1.84

$$3c^4e^3x^4(20acd+be) + 20c^4de^2x^3(6acd+be) + 6c^2ex^2\left(20ac^3d^3+be\left(10c^2d^2+e^2\right)\right) + 60c^2dx\left(ac^3d^3+be\left(2c^2d^2+\right.\right.}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^4*(a + b*ArcTanh[c*x]),x]

[Out] (60*c^2*d*(a*c^3*d^3 + b*e*(2*c^2*d^2 + e^2))*x + 6*c^2*e*(20*a*c^3*d^3 + b
e(10*c^2*d^2 + e^2))*x^2 + 20*c^4*d*e^2*(6*a*c*d + b*e)*x^3 + 3*c^4*e^3*(
20*a*c*d + b*e)*x^4 + 12*a*c^5*e^4*x^5 + 12*b*c^5*x*(5*d^4 + 10*d^3*e*x + 1
0*d^2*e^2*x^2 + 5*d*e^3*x^3 + e^4*x^4)*ArcTanh[c*x] + 6*b*(5*c^4*d^4 + 10*c
^3*d^3*e + 10*c^2*d^2*e^2 + 5*c*d*e^3 + e^4)*Log[1 - c*x] + 6*b*(5*c^4*d^4
- 10*c^3*d^3*e + 10*c^2*d^2*e^2 - 5*c*d*e^3 + e^4)*Log[1 + c*x])/(60*c^5)

__fricas__ $[\mathbf{B}]$ time = 0.49, size = 322, normalized size = 2.16

$$12\,ac^5e^4x^5 + 3\left(20\,ac^5de^3 + bc^4e^4\right)x^4 + 20\left(6\,ac^5d^2e^2 + bc^4de^3\right)x^3 + 6\left(20\,ac^5d^3e + 10\,bc^4d^2e^2 + bc^2e^4\right)x^2 + 60\,\big(a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="fricas")

[Out] 1/60*(12*a*c^5*e^4*x^5 + 3*(20*a*c^5*d*e^3 + b*c^4*e^4)*x^4 + 20*(6*a*c^5*d
^2*e^2 + b*c^4*d*e^3)*x^3 + 6*(20*a*c^5*d^3*e + 10*b*c^4*d^2*e^2 + b*c^2*e^
4)*x^2 + 60*(a*c^5*d^4 + 2*b*c^4*d^3*e + b*c^2*d*e^3)*x + 6*(5*b*c^4*d^4 -
10*b*c^3*d^3*e + 10*b*c^2*d^2*e^2 - 5*b*c*d*e^3 + b*e^4)*log(c*x + 1) + 6*(
5*b*c^4*d^4 + 10*b*c^3*d^3*e + 10*b*c^2*d^2*e^2 + 5*b*c*d*e^3 + b*e^4)*log(
c*x - 1) + 6*(b*c^5*e^4*x^5 + 5*b*c^5*d*e^3*x^4 + 10*b*c^5*d^2*e^2*x^3 + 10
*b*c^5*d^3*e*x^2 + 5*b*c^5*d^4*x)*log(-(c*x + 1)/(c*x - 1)))/c^5

__giac__ $[\mathbf{B}]$ time = 0.23, size = 2351, normalized size = 15.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="giac")

[Out] -1/15*(15*(c*x + 1)^5*b*c^4*d^4*log(-(c*x + 1)/(c*x - 1) + 1)/(c*x - 1)^5 -
 75*(c*x + 1)^4*b*c^4*d^4*log(-(c*x + 1)/(c*x - 1) + 1)/(c*x - 1)^4 + 150*(
c*x + 1)^3*b*c^4*d^4*log(-(c*x + 1)/(c*x - 1) + 1)/(c*x - 1)^3 - 150*(c*x +
 1)^2*b*c^4*d^4*log(-(c*x + 1)/(c*x - 1) + 1)/(c*x - 1)^2 + 75*(c*x + 1)*b*
c^4*d^4*log(-(c*x + 1)/(c*x - 1) + 1)/(c*x - 1) - 15*b*c^4*d^4*log(-(c*x +
1)/(c*x - 1) + 1) - 15*(c*x + 1)^5*b*c^4*d^4*log(-(c*x + 1)/(c*x - 1))/(c*x
 - 1)^5 + 60*(c*x + 1)^4*b*c^4*d^4*log(-(c*x + 1)/(c*x - 1))/(c*x - 1)^4 -
90*(c*x + 1)^3*b*c^4*d^4*log(-(c*x + 1)/(c*x - 1))/(c*x - 1)^3 + 60*(c*x +
1)^2*b*c^4*d^4*log(-(c*x + 1)/(c*x - 1))/(c*x - 1)^2 - 15*(c*x + 1)*b*c^4*d
^4*log(-(c*x + 1)/(c*x - 1))/(c*x - 1) - 30*(c*x + 1)^4*a*c^4*d^4/(c*x - 1)
^4 + 120*(c*x + 1)^3*a*c^4*d^4/(c*x - 1)^3 - 180*(c*x + 1)^2*a*c^4*d^4/(c*x
 - 1)^2 + 120*(c*x + 1)*a*c^4*d^4/(c*x - 1) - 30*a*c^4*d^4 - 60*(c*x + 1)^4

```
*b*c^3*d^3*e*log(-(c*x + 1)/(c*x - 1))/(c*x - 1)^4 + 180*(c*x + 1)^3*b*c^3*
d^3*e*log(-(c*x + 1)/(c*x - 1))/(c*x - 1)^3 - 180*(c*x + 1)^2*b*c^3*d^3*e*l
og(-(c*x + 1)/(c*x - 1))/(c*x - 1)^2 + 60*(c*x + 1)*b*c^3*d^3*e*log(-(c*x +
 1)/(c*x - 1))/(c*x - 1) - 120*(c*x + 1)^4*a*c^3*d^3*e/(c*x - 1)^4 + 360*(c
*x + 1)^3*a*c^3*d^3*e/(c*x - 1)^3 - 360*(c*x + 1)^2*a*c^3*d^3*e/(c*x - 1)^2
 + 120*(c*x + 1)*a*c^3*d^3*e/(c*x - 1) - 60*(c*x + 1)^4*b*c^3*d^3*e/(c*x -
1)^4 + 240*(c*x + 1)^3*b*c^3*d^3*e/(c*x - 1)^3 - 360*(c*x + 1)^2*b*c^3*d^3*
e/(c*x - 1)^2 + 240*(c*x + 1)*b*c^3*d^3*e/(c*x - 1) - 60*b*c^3*d^3*e + 30*(
c*x + 1)^5*b*c^2*d^2*e^2*log(-(c*x + 1)/(c*x - 1) + 1)/(c*x - 1)^5 - 150*(c
*x + 1)^4*b*c^2*d^2*e^2*log(-(c*x + 1)/(c*x - 1) + 1)/(c*x - 1)^4 + 300*(c*
x + 1)^3*b*c^2*d^2*e^2*log(-(c*x + 1)/(c*x - 1) + 1)/(c*x - 1)^3 - 300*(c*x
 + 1)^2*b*c^2*d^2*e^2*log(-(c*x + 1)/(c*x - 1) + 1)/(c*x - 1)^2 + 150*(c*x
+ 1)*b*c^2*d^2*e^2*log(-(c*x + 1)/(c*x - 1) + 1)/(c*x - 1) - 30*b*c^2*d^2*e
^2*log(-(c*x + 1)/(c*x - 1) + 1) - 30*(c*x + 1)^5*b*c^2*d^2*e^2*log(-(c*x +
 1)/(c*x - 1))/(c*x - 1)^5 + 60*(c*x + 1)^4*b*c^2*d^2*e^2*log(-(c*x + 1)/(c
*x - 1))/(c*x - 1)^4 - 120*(c*x + 1)^3*b*c^2*d^2*e^2*log(-(c*x + 1)/(c*x -
1))/(c*x - 1)^3 + 180*(c*x + 1)^2*b*c^2*d^2*e^2*log(-(c*x + 1)/(c*x - 1))/(
c*x - 1)^2 - 90*(c*x + 1)*b*c^2*d^2*e^2*log(-(c*x + 1)/(c*x - 1))/(c*x - 1)
 - 180*(c*x + 1)^4*a*c^2*d^2*e^2/(c*x - 1)^4 + 360*(c*x + 1)^3*a*c^2*d^2*e^
2/(c*x - 1)^3 - 240*(c*x + 1)^2*a*c^2*d^2*e^2/(c*x - 1)^2 + 120*(c*x + 1)*a
*c^2*d^2*e^2/(c*x - 1) - 60*a*c^2*d^2*e^2 - 60*(c*x + 1)^4*b*c^2*d^2*e^2/(c
*x - 1)^4 + 180*(c*x + 1)^3*b*c^2*d^2*e^2/(c*x - 1)^3 - 180*(c*x + 1)^2*b*c
^2*d^2*e^2/(c*x - 1)^2 + 60*(c*x + 1)*b*c^2*d^2*e^2/(c*x - 1) - 60*(c*x + 1
)^4*b*c*d*e^3*log(-(c*x + 1)/(c*x - 1))/(c*x - 1)^4 + 60*(c*x + 1)^3*b*c*d*
e^3*log(-(c*x + 1)/(c*x - 1))/(c*x - 1)^3 - 60*(c*x + 1)^2*b*c*d*e^3*log(-(
c*x + 1)/(c*x - 1))/(c*x - 1)^2 + 60*(c*x + 1)*b*c*d*e^3*log(-(c*x + 1)/(c*
x - 1))/(c*x - 1) - 120*(c*x + 1)^4*a*c*d*e^3/(c*x - 1)^4 + 120*(c*x + 1)^3
*a*c*d*e^3/(c*x - 1)^3 - 120*(c*x + 1)^2*a*c*d*e^3/(c*x - 1)^2 + 120*(c*x +
 1)*a*c*d*e^3/(c*x - 1) - 60*(c*x + 1)^4*b*c*d*e^3/(c*x - 1)^4 + 180*(c*x +
 1)^3*b*c*d*e^3/(c*x - 1)^3 - 220*(c*x + 1)^2*b*c*d*e^3/(c*x - 1)^2 + 140*(
c*x + 1)*b*c*d*e^3/(c*x - 1) - 40*b*c*d*e^3 + 3*(c*x + 1)^5*b*e^4*log(-(c*x
 + 1)/(c*x - 1) + 1)/(c*x - 1)^5 - 15*(c*x + 1)^4*b*e^4*log(-(c*x + 1)/(c*x
 - 1) + 1)/(c*x - 1)^4 + 30*(c*x + 1)^3*b*e^4*log(-(c*x + 1)/(c*x - 1) + 1)
/(c*x - 1)^3 - 30*(c*x + 1)^2*b*e^4*log(-(c*x + 1)/(c*x - 1) + 1)/(c*x - 1)
^2 + 15*(c*x + 1)*b*e^4*log(-(c*x + 1)/(c*x - 1) + 1)/(c*x - 1) - 3*b*e^4*l
og(-(c*x + 1)/(c*x - 1) + 1) - 3*(c*x + 1)^5*b*e^4*log(-(c*x + 1)/(c*x - 1)
)/(c*x - 1)^5 - 30*(c*x + 1)^3*b*e^4*log(-(c*x + 1)/(c*x - 1))/(c*x - 1)^3
 - 15*(c*x + 1)*b*e^4*log(-(c*x + 1)/(c*x - 1))/(c*x - 1) - 30*(c*x + 1)^4*a
*e^4/(c*x - 1)^4 - 60*(c*x + 1)^2*a*e^4/(c*x - 1)^2 - 6*a*e^4 - 12*(c*x + 1
)^4*b*e^4/(c*x - 1)^4 + 24*(c*x + 1)^3*b*e^4/(c*x - 1)^3 - 24*(c*x + 1)^2*b
*e^4/(c*x - 1)^2 + 12*(c*x + 1)*b*e^4/(c*x - 1))*c/((c*x + 1)^5*c^6/(c*x -
1)^5 - 5*(c*x + 1)^4*c^6/(c*x - 1)^4 + 10*(c*x + 1)^3*c^6/(c*x - 1)^3 - 10*
(c*x + 1)^2*c^6/(c*x - 1)^2 + 5*(c*x + 1)*c^6/(c*x - 1) - c^6)
```

maple [B]   time = 0.03, size = 395, normalized size = 2.65

$$\frac{b\,e^3 dx}{c^3}+\frac{b\,e^4 \operatorname{arctanh}(cx)\,x^5}{5}+b\operatorname{arctanh}(cx)\,x\,d^4+\frac{b\operatorname{arctanh}(cx)\,d^5}{5e}-\frac{b\ln(cx+1)\,d^5}{10e}+\frac{b\,e^4 x^2}{10c^3}+2ae\,x^2 d^3+2a\,e^2 x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(a+b*arctanh(c*x)),x)
```

```
[Out] b/c^3*e^3*d*x-1/10*b/e*ln(c*x+1)*d^5+1/5*b*e^4*arctanh(c*x)*x^5+1/10/c^3*b*
e^4*x^2+2*a*e*x^2*d^3+2*a*e^2*x^3*d^2+a*e^3*x^4*d+b*arctanh(c*x)*x*d^4+1/10
/c^5*b*e^4*ln(c*x-1)+1/10/c^5*b*e^4*ln(c*x+1)+1/2/c*b*ln(c*x-1)*d^4+1/2/c*b
*ln(c*x+1)*d^4+1/10*b/e*ln(c*x-1)*d^5+1/5*b/e*arctanh(c*x)*d^5+1/5*a/e*d^5+
1/5*a*e^4*x^5+a*x*d^4+1/c*b*e^2*x^2*d^2+b*e^3*arctanh(c*x)*x^4*d+2*b*e^2*ar
ctanh(c*x)*x^3*d^2+2*b*e*arctanh(c*x)*x^2*d^3+1/c^2*b*e*ln(c*x-1)*d^3+1/c^3
*b*e^2*ln(c*x-1)*d^2+1/2/c^4*b*e^3*ln(c*x-1)*d-1/2/c^4*b*e^3*ln(c*x+1)*d-1/
c^2*b*e*ln(c*x+1)*d^3+1/c^3*b*e^2*ln(c*x+1)*d^2+2*b/c*e*d^3*x+1/3*b*d*e^3*x
^3/c+1/20*b*e^4*x^4/c
```

maxima [A]   time = 0.33, size = 273, normalized size = 1.83

$$\frac{1}{5}\,ae^4 x^5+ade^3 x^4+2\,ad^2 e^2 x^3+2\,ad^3 ex^2+\left(2\,x^2\operatorname{artanh}(cx)+c\left(\frac{2\,x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)\right)bd^3 e+\left(2\,x^3\operatorname{arta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e^4*x^5 + a*d*e^3*x^4 + 2*a*d^2*e^2*x^3 + 2*a*d^3*e*x^2 + (2*x^2*arct
anh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*b*d^3*e + (2*
x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*d^2*e^2 + 1/6*(6*x
^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x
 - 1)/c^5))*b*d*e^3 + 1/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 +
 2*log(c^2*x^2 - 1)/c^6))*b*e^4 + a*d^4*x + 1/2*(2*c*x*arctanh(c*x) + log(-
c^2*x^2 + 1))*b*d^4/c
```

mupad [B]   time = 1.70, size = 272, normalized size = 1.83

$$\frac{a\,e^4 x^5}{5}+a\,d^4 x+\frac{b\,d^4\ln\left(c^2 x^2-1\right)}{2\,c}+\frac{b\,e^4\ln\left(c^2 x^2-1\right)}{10\,c^5}+2\,a\,d^2 e^2 x^3+\frac{b\,e^4 x^4}{20\,c}+\frac{b\,e^4 x^2}{10\,c^3}+b\,d^4 x\operatorname{atanh}(c\,x)+2\,a\,d^3 e\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))*(d + e*x)^4,x)
```

```
[Out] (a*e^4*x^5)/5 + a*d^4*x + (b*d^4*log(c^2*x^2 - 1))/(2*c) + (b*e^4*log(c^2*x
^2 - 1))/(10*c^5) + 2*a*d^2*e^2*x^3 + (b*e^4*x^4)/(20*c) + (b*e^4*x^2)/(10*
```

```
c^3) + b*d^4*x*atanh(c*x) + 2*a*d^3*e*x^2 + a*d*e^3*x^4 + (b*e^4*x^5*atanh(
c*x))/5 + (2*b*d^3*e*x)/c + (b*d*e^3*x)/c^3 - (2*b*d^3*e*atanh(c*x))/c^2 -
(b*d*e^3*atanh(c*x))/c^4 + 2*b*d^3*e*x^2*atanh(c*x) + b*d*e^3*x^4*atanh(c*x
) + (b*d*e^3*x^3)/(3*c) + 2*b*d^2*e^2*x^3*atanh(c*x) + (b*d^2*e^2*log(c^2*x
^2 - 1))/c^3 + (b*d^2*e^2*x^2)/c
```

sympy [A]   time = 2.96, size = 381, normalized size = 2.56

$$
\begin{cases}
ad^4x + 2ad^3ex^2 + 2ad^2e^2x^3 + ade^3x^4 + \frac{ae^4x^5}{5} + bd^4x\operatorname{atanh}(cx) + 2bd^3ex^2\operatorname{atanh}(cx) + 2bd^2e^2x^3\operatorname{atanh}(cx) + ba \\[2ex]
a\left(d^4x + 2d^3ex^2 + 2d^2e^2x^3 + de^3x^4 + \frac{e^4x^5}{5}\right)
\end{cases}
$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**4*(a+b*atanh(c*x)),x)

[Out] Piecewise((a*d**4*x + 2*a*d**3*e*x**2 + 2*a*d**2*e**2*x**3 + a*d*e**3*x**4
+ a*e**4*x**5/5 + b*d**4*x*atanh(c*x) + 2*b*d**3*e*x**2*atanh(c*x) + 2*b*d*
*2*e**2*x**3*atanh(c*x) + b*d*e**3*x**4*atanh(c*x) + b*e**4*x**5*atanh(c*x)
/5 + b*d**4*log(x - 1/c)/c + b*d**4*atanh(c*x)/c + 2*b*d**3*e*x/c + b*d**2*
e**2*x**2/c + b*d*e**3*x**3/(3*c) + b*e**4*x**4/(20*c) - 2*b*d**3*e*atanh(c
*x)/c**2 + 2*b*d**2*e**2*log(x - 1/c)/c**3 + 2*b*d**2*e**2*atanh(c*x)/c**3
+ b*d*e**3*x/c**3 + b*e**4*x**2/(10*c**3) - b*d*e**3*atanh(c*x)/c**4 + b*e*
*4*log(x - 1/c)/(5*c**5) + b*e**4*atanh(c*x)/(5*c**5), Ne(c, 0)), (a*(d**4*
x + 2*d**3*e*x**2 + 2*d**2*e**2*x**3 + d*e**3*x**4 + e**4*x**5/5), True))

3.2 $\int (d + ex)^3 \left(a + b \tanh^{-1}(cx) \right) dx$

Optimal. Leaf size=125

$$\frac{(d + ex)^4 \left(a + b \tanh^{-1}(cx) \right)}{4e} - \frac{b(cd - e)^4 \log(cx + 1)}{8c^4 e} + \frac{b(cd + e)^4 \log(1 - cx)}{8c^4 e} + \frac{bex \left(6c^2 d^2 + e^2 \right)}{4c^3} + \frac{bde^2 x^2}{2c} + \frac{be^3 x^3}{12c}$$

```
[Out] 1/4*b*e*(6*c^2*d^2+e^2)*x/c^3+1/2*b*d*e^2*x^2/c+1/12*b*e^3*x^3/c+1/4*(e*x+d
)^4*(a+b*arctanh(c*x))/e+1/8*b*(c*d+e)^4*ln(-c*x+1)/c^4/e-1/8*b*(c*d-e)^4*l
n(c*x+1)/c^4/e
```

Rubi [A]   time = 0.14, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5926, 702, 633, 31}

$$\frac{(d + ex)^4 \left(a + b \tanh^{-1}(cx) \right)}{4e} + \frac{bex \left(6c^2 d^2 + e^2 \right)}{4c^3} - \frac{b(cd - e)^4 \log(cx + 1)}{8c^4 e} + \frac{b(cd + e)^4 \log(1 - cx)}{8c^4 e} + \frac{bde^2 x^2}{2c} + \frac{be^3 x^3}{12c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*e*(6*c^2*d^2 + e^2)*x)/(4*c^3) + (b*d*e^2*x^2)/(2*c) + (b*e^3*x^3)/(12*c
) + ((d + e*x)^4*(a + b*ArcTanh[c*x]))/(4*e) + (b*(c*d + e)^4*Log[1 - c*x])
/(8*c^4*e) - (b*(c*d - e)^4*Log[1 + c*x])/(8*c^4*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```


Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```


Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```


Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\int (d + ex)^3 \left(a + b \tanh^{-1}(cx)\right) dx = \frac{(d + ex)^4 \left(a + b \tanh^{-1}(cx)\right)}{4e} - \frac{(bc) \int \frac{(d+ex)^4}{1-c^2x^2} dx}{4e}$$

$$= \frac{(d + ex)^4 \left(a + b \tanh^{-1}(cx)\right)}{4e} - \frac{(bc) \int \left(-\frac{e^2\left(6c^2d^2+e^2\right)}{c^4} - \frac{4de^3x}{c^2} - \frac{e^4x^2}{c^2} + \frac{c^4d^4+6c^2d^2}{}\right)}{4e}$$

$$= \frac{be\left(6c^2d^2 + e^2\right)x}{4c^3} + \frac{bde^2x^2}{2c} + \frac{be^3x^3}{12c} + \frac{(d + ex)^4 \left(a + b \tanh^{-1}(cx)\right)}{4e} - \frac{b \int \frac{c^4d}{}}{}$$

$$= \frac{be\left(6c^2d^2 + e^2\right)x}{4c^3} + \frac{bde^2x^2}{2c} + \frac{be^3x^3}{12c} + \frac{(d + ex)^4 \left(a + b \tanh^{-1}(cx)\right)}{4e} + \frac{\left(b(cd -}{}$$

$$= \frac{be\left(6c^2d^2 + e^2\right)x}{4c^3} + \frac{bde^2x^2}{2c} + \frac{be^3x^3}{12c} + \frac{(d + ex)^4 \left(a + b \tanh^{-1}(cx)\right)}{4e} + \frac{b(cd +}{}$$

Mathematica [A]   time = 0.14, size = 205, normalized size = 1.64

$$2c^3e^2x^3(12acd + be) + 12c^3dex^2(3acd + be) + 6cx\left(4ac^3d^3 + be\left(6c^2d^2 + e^2\right)\right) + 6ac^4e^3x^4 + 6bc^4x \tanh^{-1}(cx)\left(4d^3\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (6*c*(4*a*c^3*d^3 + b*e*(6*c^2*d^2 + e^2))*x + 12*c^3*d*e*(3*a*c*d + b*e)*x
^2 + 2*c^3*e^2*(12*a*c*d + b*e)*x^3 + 6*a*c^4*e^3*x^4 + 6*b*c^4*x*(4*d^3 +
6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)*ArcTanh[c*x] + 3*b*(4*c^3*d^3 + 6*c^2*d^
2*e + 4*c*d*e^2 + e^3)*Log[1 - c*x] + 3*b*(4*c^3*d^3 - 6*c^2*d^2*e + 4*c*d*
e^2 - e^3)*Log[1 + c*x])/(24*c^4)
```

fricas [B]   time = 0.80, size = 244, normalized size = 1.95

$$6\,ac^4e^3x^4 + 2\left(12\,ac^4de^2 + bc^3e^3\right)x^3 + 12\left(3\,ac^4d^2e + bc^3de^2\right)x^2 + 6\left(4\,ac^4d^3 + 6\,bc^3d^2e + bce^3\right)x + 3\left(4\,bc^3d^3 - 6\,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/24*(6*a*c^4*e^3*x^4 + 2*(12*a*c^4*d*e^2 + b*c^3*e^3)*x^3 + 12*(3*a*c^4*d^
2*e + b*c^3*d*e^2)*x^2 + 6*(4*a*c^4*d^3 + 6*b*c^3*d^2*e + b*c*e^3)*x + 3*(4
*b*c^3*d^3 - 6*b*c^2*d^2*e + 4*b*c*d*e^2 - b*e^3)*log(c*x + 1) + 3*(4*b*c^3
*d^3 + 6*b*c^2*d^2*e + 4*b*c*d*e^2 + b*e^3)*log(c*x - 1) + 3*(b*c^4*e^3*x^4
 + 4*b*c^4*d*e^2*x^3 + 6*b*c^4*d^2*e*x^2 + 4*b*c^4*d^3*x)*log(-(c*x + 1)/(c
*x - 1)))/c^4
```

giac [**B**]   time = 0.19, size = 1375, normalized size = 11.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] -1/3*(3*(c*x + 1)^4*b*c^3*d^3*log(-(c*x + 1)/(c*x - 1) + 1)/(c*x - 1)^4 - 1
2*(c*x + 1)^3*b*c^3*d^3*log(-(c*x + 1)/(c*x - 1) + 1)/(c*x - 1)^3 + 18*(c*x
 + 1)^2*b*c^3*d^3*log(-(c*x + 1)/(c*x - 1) + 1)/(c*x - 1)^2 - 12*(c*x + 1)*
b*c^3*d^3*log(-(c*x + 1)/(c*x - 1) + 1)/(c*x - 1) + 3*b*c^3*d^3*log(-(c*x +
 1)/(c*x - 1) + 1) - 3*(c*x + 1)^4*b*c^3*d^3*log(-(c*x + 1)/(c*x - 1))/(c*x
 - 1)^4 + 9*(c*x + 1)^3*b*c^3*d^3*log(-(c*x + 1)/(c*x - 1))/(c*x - 1)^3 - 9
*(c*x + 1)^2*b*c^3*d^3*log(-(c*x + 1)/(c*x - 1))/(c*x - 1)^2 + 3*(c*x + 1)*
b*c^3*d^3*log(-(c*x + 1)/(c*x - 1))/(c*x - 1) - 6*(c*x + 1)^3*a*c^3*d^3/(c*
x - 1)^3 + 18*(c*x + 1)^2*a*c^3*d^3/(c*x - 1)^2 - 18*(c*x + 1)*a*c^3*d^3/(c
*x - 1) + 6*a*c^3*d^3 - 9*(c*x + 1)^3*b*c^2*d^2*e*log(-(c*x + 1)/(c*x - 1))
/(c*x - 1)^3 + 18*(c*x + 1)^2*b*c^2*d^2*e*log(-(c*x + 1)/(c*x - 1))/(c*x -
1)^2 - 9*(c*x + 1)*b*c^2*d^2*e*log(-(c*x + 1)/(c*x - 1))/(c*x - 1) - 18*(c*
x + 1)^3*a*c^2*d^2*e/(c*x - 1)^3 + 36*(c*x + 1)^2*a*c^2*d^2*e/(c*x - 1)^2 -
 18*(c*x + 1)*a*c^2*d^2*e/(c*x - 1) - 9*(c*x + 1)^3*b*c^2*d^2*e/(c*x - 1)^3
 + 27*(c*x + 1)^2*b*c^2*d^2*e/(c*x - 1)^2 - 27*(c*x + 1)*b*c^2*d^2*e/(c*x -
 1) + 9*b*c^2*d^2*e + 3*(c*x + 1)^4*b*c*d*e^2*log(-(c*x + 1)/(c*x - 1) + 1)
/(c*x - 1)^4 - 12*(c*x + 1)^3*b*c*d*e^2*log(-(c*x + 1)/(c*x - 1) + 1)/(c*x
 - 1)^3 + 18*(c*x + 1)^2*b*c*d*e^2*log(-(c*x + 1)/(c*x - 1) + 1)/(c*x - 1)^2
 - 12*(c*x + 1)*b*c*d*e^2*log(-(c*x + 1)/(c*x - 1) + 1)/(c*x - 1) + 3*b*c*d
*e^2*log(-(c*x + 1)/(c*x - 1) + 1) - 3*(c*x + 1)^4*b*c*d*e^2*log(-(c*x + 1)
/(c*x - 1))/(c*x - 1)^4 + 3*(c*x + 1)^3*b*c*d*e^2*log(-(c*x + 1)/(c*x - 1))
/(c*x - 1)^3 - 9*(c*x + 1)^2*b*c*d*e^2*log(-(c*x + 1)/(c*x - 1))/(c*x - 1)^
2 + 9*(c*x + 1)*b*c*d*e^2*log(-(c*x + 1)/(c*x - 1))/(c*x - 1) - 18*(c*x + 1
)^3*a*c*d*e^2/(c*x - 1)^3 + 18*(c*x + 1)^2*a*c*d*e^2/(c*x - 1)^2 - 6*(c*x +
 1)*a*c*d*e^2/(c*x - 1) + 6*a*c*d*e^2 - 6*(c*x + 1)^3*b*c*d*e^2/(c*x - 1)^3
 + 12*(c*x + 1)^2*b*c*d*e^2/(c*x - 1)^2 - 6*(c*x + 1)*b*c*d*e^2/(c*x - 1) -
 3*(c*x + 1)^3*b*e^3*log(-(c*x + 1)/(c*x - 1))/(c*x - 1)^3 - 3*(c*x + 1)*b*
e^3*log(-(c*x + 1)/(c*x - 1))/(c*x - 1) - 6*(c*x + 1)^3*a*e^3/(c*x - 1)^3 -
 6*(c*x + 1)*a*e^3/(c*x - 1) - 3*(c*x + 1)^3*b*e^3/(c*x - 1)^3 + 6*(c*x + 1
```

```
)^2*b*e^3/(c*x - 1)^2 - 5*(c*x + 1)*b*e^3/(c*x - 1) + 2*b*e^3)*c/((c*x + 1)
^4*c^5/(c*x - 1)^4 - 4*(c*x + 1)^3*c^5/(c*x - 1)^3 + 6*(c*x + 1)^2*c^5/(c*x
 - 1)^2 - 4*(c*x + 1)*c^5/(c*x - 1) + c^5)
```

maple [B]   time = 0.03, size = 308, normalized size = 2.46

$$\frac{a\,x^4 e^3}{4}+ad\,e^2 x^3+\frac{3a\,d^2 e\,x^2}{2}+ax\,d^3+\frac{a\,d^4}{4e}+\frac{b\,e^3\,\operatorname{arctanh}(cx)\,x^4}{4}+b\,e^2\,\operatorname{arctanh}(cx)\,x^3 d+\frac{3be\,\operatorname{arctanh}(cx)\,x^2 d^2}{2}+b\,\operatorname{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^3*(a+b*arctanh(c*x)),x)

[Out] 1/4*a*x^4*e^3+a*d*e^2*x^3+3/2*a*d^2*e*x^2+a*x*d^3+1/4*a/e*d^4+1/4*b*e^3*arc
tanh(c*x)*x^4+b*e^2*arctanh(c*x)*x^3*d+3/2*b*e*arctanh(c*x)*x^2*d^2+b*arcta
nh(c*x)*x*d^3+1/4*b/e*arctanh(c*x)*d^4+1/12*b*e^3*x^3/c+1/2*b*d*e^2*x^2/c+3
/2*b/c*e*x*d^2+1/4*b/c^3*x*e^3+1/8*b/e*ln(c*x-1)*d^4+1/2/c*b*ln(c*x-1)*d^3+
3/4/c^2*b*e*ln(c*x-1)*d^2+1/2/c^3*b*e^2*ln(c*x-1)*d+1/8/c^4*b*e^3*ln(c*x-1)
-1/8*b/e*ln(c*x+1)*d^4+1/2/c*b*ln(c*x+1)*d^3-3/4/c^2*b*e*ln(c*x+1)*d^2+1/2/
c^3*b*e^2*ln(c*x+1)*d-1/8/c^4*b*e^3*ln(c*x+1)

maxima [A]   time = 0.33, size = 209, normalized size = 1.67

$$\frac{1}{4}\,ae^3 x^4+ade^2 x^3+\frac{3}{2}\,ad^2 ex^2+\frac{3}{4}\left(2\,x^2\,\operatorname{artanh}(cx)+c\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)\right)bd^2 e+\frac{1}{2}\left(2\,x^3\,\operatorname{artanh}(cx)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="maxima")

[Out] 1/4*a*e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + 3/4*(2*x^2*arctanh(c*x) + c
*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*b*d^2*e + 1/2*(2*x^3*arct
anh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*d*e^2 + 1/24*(6*x^4*arctan
h(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5
))*b*e^3 + a*d^3*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b*d^3/c

mupad [B]   time = 1.15, size = 197, normalized size = 1.58

$$\frac{a\,e^3\,x^4}{4}+a\,d^3\,x+\frac{b\,d^3\,\ln\left(c^2\,x^2-1\right)}{2\,c}+\frac{b\,e^3\,x^3}{12\,c}+b\,d^3\,x\,\operatorname{atanh}(c\,x)+\frac{3\,a\,d^2\,e\,x^2}{2}+a\,d\,e^2\,x^3+\frac{b\,e^3\,x}{4\,c^3}-\frac{b\,e^3\,\operatorname{atanh}(c\,x)}{4\,c^4}+\frac{b\,e^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x))*(d + e*x)^3,x)

[Out] (a*e^3*x^4)/4 + a*d^3*x + (b*d^3*log(c^2*x^2 - 1))/(2*c) + (b*e^3*x^3)/(12*
c) + b*d^3*x*atanh(c*x) + (3*a*d^2*e*x^2)/2 + a*d*e^2*x^3 + (b*e^3*x)/(4*c^

```
3) - (b*e^3*atanh(c*x))/(4*c^4) + (b*e^3*x^4*atanh(c*x))/4 + (3*b*d^2*e*x)/
(2*c) - (3*b*d^2*e*atanh(c*x))/(2*c^2) + (3*b*d^2*e*x^2*atanh(c*x))/2 + b*d
*e^2*x^3*atanh(c*x) + (b*d*e^2*log(c^2*x^2 - 1))/(2*c^3) + (b*d*e^2*x^2)/(2
*c)
```

sympy [A]   time = 1.98, size = 279, normalized size = 2.23

$$
\begin{cases}
ad^3x + \frac{3ad^2ex^2}{2} + ade^2x^3 + \frac{ae^3x^4}{4} + bd^3x \operatorname{atanh}(cx) + \frac{3bd^2ex^2 \operatorname{atanh}(cx)}{2} + bde^2x^3 \operatorname{atanh}(cx) + \frac{be^3x^4 \operatorname{atanh}(cx)}{4} + \frac{bd^3 \log}{} \\
a\left(d^3x + \frac{3d^2ex^2}{2} + de^2x^3 + \frac{e^3x^4}{4}\right)
\end{cases}
$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**3*(a+b*atanh(c*x)),x)

[Out] Piecewise((a*d**3*x + 3*a*d**2*e*x**2/2 + a*d*e**2*x**3 + a*e**3*x**4/4 + b
*d**3*x*atanh(c*x) + 3*b*d**2*e*x**2*atanh(c*x)/2 + b*d*e**2*x**3*atanh(c*x
) + b*e**3*x**4*atanh(c*x)/4 + b*d**3*log(x - 1/c)/c + b*d**3*atanh(c*x)/c
+ 3*b*d**2*e*x/(2*c) + b*d*e**2*x**2/(2*c) + b*e**3*x**3/(12*c) - 3*b*d**2*
e*atanh(c*x)/(2*c**2) + b*d*e**2*log(x - 1/c)/c**3 + b*d*e**2*atanh(c*x)/c*
*3 + b*e**3*x/(4*c**3) - b*e**3*atanh(c*x)/(4*c**4), Ne(c, 0)), (a*(d**3*x
+ 3*d**2*e*x**2/2 + d*e**2*x**3 + e**3*x**4/4), True))

3.3 $\int (d + ex)^2 \left(a + b \tanh^{-1}(cx) \right) dx$

Optimal. Leaf size=96

$$\frac{(d + ex)^3 \left(a + b \tanh^{-1}(cx) \right)}{3e} - \frac{b(cd - e)^3 \log(cx + 1)}{6c^3 e} + \frac{b(cd + e)^3 \log(1 - cx)}{6c^3 e} + \frac{bdex}{c} + \frac{be^2 x^2}{6c}$$

[Out] b*d*e*x/c+1/6*b*e^2*x^2/c+1/3*(e*x+d)^3*(a+b*arctanh(c*x))/e+1/6*b*(c*d+e)^
3*ln(-c*x+1)/c^3/e-1/6*b*(c*d-e)^3*ln(c*x+1)/c^3/e

Rubi [A]   time $= 0.12$, antiderivative size $= 96$, normalized size of antiderivative $= 1.00$, number of steps used $= 6$, number of rules used $= 4$, integrand size $= 16$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used $= \{5926, 702, 633, 31\}$

$$\frac{(d + ex)^3 \left(a + b \tanh^{-1}(cx) \right)}{3e} - \frac{b(cd - e)^3 \log(cx + 1)}{6c^3 e} + \frac{b(cd + e)^3 \log(1 - cx)}{6c^3 e} + \frac{bdex}{c} + \frac{be^2 x^2}{6c}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^2*(a + b*ArcTanh[c*x]),x]

[Out] (b*d*e*x)/c + (b*e^2*x^2)/(6*c) + ((d + e*x)^3*(a + b*ArcTanh[c*x]))/(3*e)
+ (b*(c*d + e)^3*Log[1 - c*x])/(6*c^3*e) - (b*(c*d - e)^3*Log[1 + c*x])/(6*
c^3*e)

Rule 31

Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]

Rule 633

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]

Rule 702

Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\int (d + ex)^2 \left(a + b\tanh^{-1}(cx)\right) dx = \frac{(d + ex)^3 \left(a + b\tanh^{-1}(cx)\right)}{3e} - \frac{(bc)\int \frac{(d+ex)^3}{1-c^2x^2}\,dx}{3e}$$

$$= \frac{(d + ex)^3 \left(a + b\tanh^{-1}(cx)\right)}{3e} - \frac{(bc)\int \left(-\frac{3de^2}{c^2} - \frac{e^3x}{c^2} + \frac{c^2d^3+3de^2+e\left(3c^2d^2+e^2\right)x}{c^2\left(1-c^2x^2\right)}\right) d}{3e}$$

$$= \frac{bdex}{c} + \frac{be^2x^2}{6c} + \frac{(d + ex)^3 \left(a + b\tanh^{-1}(cx)\right)}{3e} - \frac{b\int \frac{c^2d^3+3de^2+e\left(3c^2d^2+e^2\right)x}{1-c^2x^2}\,dx}{3ce}$$

$$= \frac{bdex}{c} + \frac{be^2x^2}{6c} + \frac{(d + ex)^3 \left(a + b\tanh^{-1}(cx)\right)}{3e} + \frac{\left(b(cd - e)^3\right)\int \frac{1}{-c-c^2x}\,dx}{6ce} -$$

$$= \frac{bdex}{c} + \frac{be^2x^2}{6c} + \frac{(d + ex)^3 \left(a + b\tanh^{-1}(cx)\right)}{3e} + \frac{b(cd + e)^3 \log(1 - cx)}{6c^3e} - \frac{b(}$$

Mathematica [A]   time = 0.10, size = 129, normalized size = 1.34

$$\frac{1}{6}\left(\frac{ex^2(6acd + be)}{c} + \frac{6dx(acd + be)}{c} + 2ae^2x^3 + \frac{b\left(3c^2d^2 + 3cde + e^2\right)\log(1 - cx)}{c^3} + \frac{b\left(3c^2d^2 - 3cde + e^2\right)\log(cx}{c^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] ((6*d*(a*c*d + b*e)*x)/c + (e*(6*a*c*d + b*e)*x^2)/c + 2*a*e^2*x^3 + 2*b*x*
(3*d^2 + 3*d*e*x + e^2*x^2)*ArcTanh[c*x] + (b*(3*c^2*d^2 + 3*c*d*e + e^2)*L
og[1 - c*x])/c^3 + (b*(3*c^2*d^2 - 3*c*d*e + e^2)*Log[1 + c*x])/c^3)/6
```

fricas [A]   time = 0.66, size = 163, normalized size = 1.70

$$\frac{2\,ac^3e^2x^3 + \left(6\,ac^3de + bc^2e^2\right)x^2 + 6\left(ac^3d^2 + bc^2de\right)x + \left(3\,bc^2d^2 - 3\,bcde + be^2\right)\log(cx + 1) + \left(3\,bc^2d^2 + 3\,bcde\right.}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e^2*x^3 + (6*a*c^3*d*e + b*c^2*e^2)*x^2 + 6*(a*c^3*d^2 + b*c^2
*d*e)*x + (3*b*c^2*d^2 - 3*b*c*d*e + b*e^2)*log(c*x + 1) + (3*b*c^2*d^2 + 3
*b*c*d*e + b*e^2)*log(c*x - 1) + (b*c^3*e^2*x^3 + 3*b*c^3*d*e*x^2 + 3*b*c^3
*d^2*x)*log(-(c*x + 1)/(c*x - 1)))/c^3
```

giac $[\mathbf{B}]$ time = 0.39, size = 785, normalized size = 8.18

$$-\left(\frac{3\,(cx+1)^3 bc^2 d^2 \log\left(-\frac{cx+1}{cx-1}+1\right)}{(cx-1)^3} - \frac{9\,(cx+1)^2 bc^2 d^2 \log\left(-\frac{cx+1}{cx-1}+1\right)}{(cx-1)^2} + \frac{9\,(cx+1) bc^2 d^2 \log\left(-\frac{cx+1}{cx-1}+1\right)}{cx-1} - 3\,bc^2 d^2 \log\left(-\frac{cx+1}{cx-1}+1\right) - \frac{3\,(cx+1)^3 b\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] -1/3*(3*(c*x + 1)^3*b*c^2*d^2*log(-(c*x + 1)/(c*x - 1) + 1)/(c*x - 1)^3 - 9
*(c*x + 1)^2*b*c^2*d^2*log(-(c*x + 1)/(c*x - 1) + 1)/(c*x - 1)^2 + 9*(c*x +
 1)*b*c^2*d^2*log(-(c*x + 1)/(c*x - 1) + 1)/(c*x - 1) - 3*b*c^2*d^2*log(-(c
*x + 1)/(c*x - 1) + 1) - 3*(c*x + 1)^3*b*c^2*d^2*log(-(c*x + 1)/(c*x - 1))/
(c*x - 1)^3 + 6*(c*x + 1)^2*b*c^2*d^2*log(-(c*x + 1)/(c*x - 1))/(c*x - 1)^2
 - 3*(c*x + 1)*b*c^2*d^2*log(-(c*x + 1)/(c*x - 1))/(c*x - 1) - 6*(c*x + 1)^
2*a*c^2*d^2/(c*x - 1)^2 + 12*(c*x + 1)*a*c^2*d^2/(c*x - 1) - 6*a*c^2*d^2 -
6*(c*x + 1)^2*b*c*d*e*log(-(c*x + 1)/(c*x - 1))/(c*x - 1)^2 + 6*(c*x + 1)*b
*c*d*e*log(-(c*x + 1)/(c*x - 1))/(c*x - 1) - 12*(c*x + 1)^2*a*c*d*e/(c*x -
1)^2 + 12*(c*x + 1)*a*c*d*e/(c*x - 1) - 6*(c*x + 1)^2*b*c*d*e/(c*x - 1)^2 +
 12*(c*x + 1)*b*c*d*e/(c*x - 1) - 6*b*c*d*e + (c*x + 1)^3*b*e^2*log(-(c*x +
 1)/(c*x - 1) + 1)/(c*x - 1)^3 - 3*(c*x + 1)^2*b*e^2*log(-(c*x + 1)/(c*x -
1) + 1)/(c*x - 1)^2 + 3*(c*x + 1)*b*e^2*log(-(c*x + 1)/(c*x - 1) + 1)/(c*x
 - 1) - b*e^2*log(-(c*x + 1)/(c*x - 1) + 1) - (c*x + 1)^3*b*e^2*log(-(c*x +
 1)/(c*x - 1))/(c*x - 1)^3 - 3*(c*x + 1)*b*e^2*log(-(c*x + 1)/(c*x - 1))/(c*
x - 1) - 6*(c*x + 1)^2*a*e^2/(c*x - 1)^2 - 2*a*e^2 - 2*(c*x + 1)^2*b*e^2/(c
*x - 1)^2 + 2*(c*x + 1)*b*e^2/(c*x - 1))*c/((c*x + 1)^3*c^4/(c*x - 1)^3 - 3
*(c*x + 1)^2*c^4/(c*x - 1)^2 + 3*(c*x + 1)*c^4/(c*x - 1) - c^4)
```

maple $[\mathbf{B}]$ time = 0.03, size = 218, normalized size = 2.27

$$\frac{a\,x^3 e^2}{3} + ade\,x^2 + ax\,d^2 + \frac{a\,d^3}{3e} + \frac{b\,e^2 \operatorname{arctanh}(cx)\,x^3}{3} + be\operatorname{arctanh}(cx)\,x^2 d + b\operatorname{arctanh}(cx)\,x\,d^2 + \frac{b\operatorname{arctanh}(cx)\,d^3}{3e} + \frac{b\,e\cdots}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a+b*arctanh(c*x)),x)
```

[Out] 1/3*a*x^3*e^2+a*d*e*x^2+a*x*d^2+1/3*a/e*d^3+1/3*b*e^2*arctanh(c*x)*x^3+b*e*
arctanh(c*x)*x^2*d+b*arctanh(c*x)*x*d^2+1/3*b/e*arctanh(c*x)*d^3+1/6*b*e^2*
x^2/c+b*d*e*x/c+1/6*b/e*ln(c*x-1)*d^3+1/2*c*b*ln(c*x-1)*d^2+1/2*c^2*b*e*ln(
c*x-1)*d+1/6/c^3*b*e^2*ln(c*x-1)-1/6*b/e*ln(c*x+1)*d^3+1/2*c*b*ln(c*x+1)*d^
2-1/2/c^2*b*e*ln(c*x+1)*d+1/6/c^3*b*e^2*ln(c*x+1)

maxima [A]   time = 0.31, size = 137, normalized size = 1.43

$$\frac{1}{3}\,ae^2x^3+adex^2+\frac{1}{2}\left(2\,x^2\operatorname{artanh}(cx)+c\left(\frac{2\,x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)\right)bde+\frac{1}{6}\left(2\,x^3\operatorname{artanh}(cx)+c\left(\frac{x^2}{c^2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="maxima")

[Out] 1/3*a*e^2*x^3 + a*d*e*x^2 + 1/2*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x
+ 1)/c^3 + log(c*x - 1)/c^3))*b*d*e + 1/6*(2*x^3*arctanh(c*x) + c*(x^2/c^2
+ log(c^2*x^2 - 1)/c^4))*b*e^2 + a*d^2*x + 1/2*(2*c*x*arctanh(c*x) + log(-c
^2*x^2 + 1))*b*d^2/c

mupad [B]   time = 0.94, size = 127, normalized size = 1.32

$$\frac{a\,e^2\,x^3}{3}+a\,d^2\,x+\frac{b\,d^2\,\ln\left(c^2\,x^2-1\right)}{2\,c}+\frac{b\,e^2\,\ln\left(c^2\,x^2-1\right)}{6\,c^3}+\frac{b\,e^2\,x^2}{6\,c}+a\,d\,e\,x^2+b\,d^2\,x\operatorname{atanh}(c\,x)+\frac{b\,e^2\,x^3\operatorname{atanh}(c\,x)}{3}+b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x))*(d + e*x)^2,x)

[Out] (a*e^2*x^3)/3 + a*d^2*x + (b*d^2*log(c^2*x^2 - 1))/(2*c) + (b*e^2*log(c^2*x
^2 - 1))/(6*c^3) + (b*e^2*x^2)/(6*c) + a*d*e*x^2 + b*d^2*x*atanh(c*x) + (b*
e^2*x^3*atanh(c*x))/3 + (b*d*e*x)/c - (b*d*e*atanh(c*x))/c^2 + b*d*e*x^2*at
anh(c*x)

sympy [A]   time = 1.26, size = 178, normalized size = 1.85

$$\begin{cases} ad^2x+adex^2+\frac{ae^2x^3}{3}+bd^2x\operatorname{atanh}(cx)+bdex^2\operatorname{atanh}(cx)+\frac{be^2x^3\operatorname{atanh}(cx)}{3}+\frac{bd^2\log\left(x-\frac{1}{c}\right)}{c}+\frac{bd^2\operatorname{atanh}(cx)}{c}+\frac{bdex}{c}+ \\ a\left(d^2x+dex^2+\frac{e^2x^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**2*(a+b*atanh(c*x)),x)

```
[Out] Piecewise((a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + b*d**2*x*atanh(c*x) + b*
d*e*x**2*atanh(c*x) + b*e**2*x**3*atanh(c*x)/3 + b*d**2*log(x - 1/c)/c + b*
d**2*atanh(c*x)/c + b*d*e*x/c + b*e**2*x**2/(6*c) - b*d*e*atanh(c*x)/c**2 +
 b*e**2*log(x - 1/c)/(3*c**3) + b*e**2*atanh(c*x)/(3*c**3), Ne(c, 0)), (a*(
d**2*x + d*e*x**2 + e**2*x**3/3), True))
```

3.4 $\int (d + ex)\left(a + b\tanh^{-1}(cx)\right) dx$

Optimal. Leaf size=84

$$\frac{(d+ex)^2\left(a+b\tanh^{-1}(cx)\right)}{2e} - \frac{b(cd-e)^2\log(cx+1)}{4c^2e} + \frac{b(cd+e)^2\log(1-cx)}{4c^2e} + \frac{bex}{2c}$$

[Out] 1/2*b*e*x/c+1/2*(e*x+d)^2*(a+b*arctanh(c*x))/e+1/4*b*(c*d+e)^2*ln(-c*x+1)/c^2/e-1/4*b*(c*d-e)^2*ln(c*x+1)/c^2/e

Rubi [A]   time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5926, 702, 633, 31}

$$\frac{(d+ex)^2\left(a+b\tanh^{-1}(cx)\right)}{2e} - \frac{b(cd-e)^2\log(cx+1)}{4c^2e} + \frac{b(cd+e)^2\log(1-cx)}{4c^2e} + \frac{bex}{2c}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)*(a + b*ArcTanh[c*x]),x]

[Out] (b*e*x)/(2*c) + ((d + e*x)^2*(a + b*ArcTanh[c*x]))/(2*e) + (b*(c*d + e)^2*Log[1 - c*x])/(4*c^2*e) - (b*(c*d - e)^2*Log[1 + c*x])/(4*c^2*e)

Rule 31

Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 633

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]

Rule 702

Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

Rule 5926

Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(

```
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$
\int (d + ex)\left(a + b\tanh^{-1}(cx)\right)\,dx = \frac{(d+ex)^2\left(a + b\tanh^{-1}(cx)\right)}{2e} - \frac{(bc)\int \frac{(d+ex)^2}{1-c^2x^2}\,dx}{2e}
$$

$$
= \frac{(d+ex)^2\left(a + b\tanh^{-1}(cx)\right)}{2e} - \frac{(bc)\int\left(-\frac{e^2}{c^2} + \frac{c^2d^2+e^2+2c^2dex}{c^2(1-c^2x^2)}\right)dx}{2e}
$$

$$
= \frac{bex}{2c} + \frac{(d+ex)^2\left(a + b\tanh^{-1}(cx)\right)}{2e} - \frac{b\int \frac{c^2d^2+e^2+2c^2dex}{1-c^2x^2}\,dx}{2ce}
$$

$$
= \frac{bex}{2c} + \frac{(d+ex)^2\left(a + b\tanh^{-1}(cx)\right)}{2e} + \frac{\left(b(cd-e)^2\right)\int\frac{1}{-c-c^2x}\,dx}{4e} - \frac{\left(b(cd+e)^2\right)}{4e}
$$

$$
= \frac{bex}{2c} + \frac{(d+ex)^2\left(a + b\tanh^{-1}(cx)\right)}{2e} + \frac{b(cd+e)^2\log(1-cx)}{4c^2e} - \frac{b(cd-e)^2\log(}{4c^2e}
$$

Mathematica [A]   time = 0.01, size = 96, normalized size = 1.14

$$
adx + \frac{1}{2}aex^2 + \frac{bd\log\left(1-c^2x^2\right)}{2c} + \frac{be\log(1-cx)}{4c^2} - \frac{be\log(cx+1)}{4c^2} + bdx\tanh^{-1}(cx) + \frac{1}{2}bex^2\tanh^{-1}(cx) + \frac{bex}{2c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] a*d*x + (b*e*x)/(2*c) + (a*e*x^2)/2 + b*d*x*ArcTanh[c*x] + (b*e*x^2*ArcTanh
[c*x])/2 + (b*e*Log[1 - c*x])/(4*c^2) - (b*e*Log[1 + c*x])/(4*c^2) + (b*d*L
og[1 - c^2*x^2])/(2*c)
```

fricas [A]   time = 0.60, size = 98, normalized size = 1.17

$$
\frac{2\,ac^2ex^2 + 2\left(2\,ac^2d + bce\right)x + (2\,bcd - be)\log(cx+1) + (2\,bcd + be)\log(cx-1) + \left(bc^2ex^2 + 2\,bc^2dx\right)\log\left(-\frac{cx+1}{cx-1}\right)}{4\,c^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*c^2*e*x^2 + 2*(2*a*c^2*d + b*c*e)*x + (2*b*c*d - b*e)*log(c*x + 1)
 + (2*b*c*d + b*e)*log(c*x - 1) + (b*c^2*e*x^2 + 2*b*c^2*d*x)*log(-(c*x + 1
)/(c*x - 1)))/c^2
```

giac [B]   time = 0.21, size = 292, normalized size = 3.48

$$-\frac{\left(\dfrac{(cx+1)^2bcd\log\left(-\frac{cx+1}{cx-1}+1\right)}{(cx-1)^2} - \dfrac{2\,(cx+1)bcd\log\left(-\frac{cx+1}{cx-1}+1\right)}{cx-1} + bcd\log\left(-\frac{cx+1}{cx-1}+1\right) - \dfrac{(cx+1)^2bcd\log\left(-\frac{cx+1}{cx-1}\right)}{(cx-1)^2} + \dfrac{(cx+1)bcd\log\left(-\frac{cx+1}{cx-1}\right)}{cx-1} -\right.}{\dfrac{(cx+1)^2c^3}{(cx-1)^2} - \dfrac{2\,(cx+1)c^3}{cx-1} + c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] -((c*x + 1)^2*b*c*d*log(-(c*x + 1)/(c*x - 1) + 1)/(c*x - 1)^2 - 2*(c*x + 1)
*b*c*d*log(-(c*x + 1)/(c*x - 1) + 1)/(c*x - 1) + b*c*d*log(-(c*x + 1)/(c*x
 - 1) + 1) - (c*x + 1)^2*b*c*d*log(-(c*x + 1)/(c*x - 1))/(c*x - 1)^2 + (c*x
 + 1)*b*c*d*log(-(c*x + 1)/(c*x - 1))/(c*x - 1) - 2*(c*x + 1)*a*c*d/(c*x - 1
) + 2*a*c*d - (c*x + 1)*b*e*log(-(c*x + 1)/(c*x - 1))/(c*x - 1) - 2*(c*x +
1)*a*e/(c*x - 1) - (c*x + 1)*b*e/(c*x - 1) + b*e)*c/((c*x + 1)^2*c^3/(c*x -
 1)^2 - 2*(c*x + 1)*c^3/(c*x - 1) + c^3)
```

maple [A]   time = 0.03, size = 92, normalized size = 1.10

$$\frac{a\,x^2e}{2}+adx+\frac{b\,\operatorname{arctanh}(cx)\,x^2e}{2}+b\,\operatorname{arctanh}(cx)\,xd+\frac{bex}{2c}+\frac{b\ln(cx-1)\,d}{2c}+\frac{b\ln(cx-1)\,e}{4c^2}+\frac{b\ln(cx+1)\,d}{2c}-\frac{b\ln(cx}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/2*a*x^2*e+a*d*x+1/2*b*arctanh(c*x)*x^2*e+b*arctanh(c*x)*x*d+1/2*b*e*x/c+1
/2/c*b*ln(c*x-1)*d+1/4/c^2*b*ln(c*x-1)*e+1/2/c*b*ln(c*x+1)*d-1/4/c^2*b*ln(c
*x+1)*e
```

maxima [A]   time = 0.32, size = 83, normalized size = 0.99

$$\frac{1}{2}\,aex^2+\frac{1}{4}\left(2\,x^2\operatorname{artanh}(cx)+c\left(\frac{2\,x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)\right)be+adx+\frac{\left(2\,cx\operatorname{artanh}(cx)+\log\left(-c^2x^2+1\right)\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log
(c*x - 1)/c^3))*b*e + a*d*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*
b*d/c
```

mupad [B]   time = 0.83, size = 67, normalized size = 0.80

$$a\,d\,x + \frac{a\,e\,x^2}{2} + b\,d\,x\,\text{atanh}\,(c\,x) + \frac{b\,e\,x}{2\,c} - \frac{b\,e\,\text{atanh}\,(c\,x)}{2\,c^2} + \frac{b\,e\,x^2\,\text{atanh}\,(c\,x)}{2} + \frac{b\,d\,\ln\left(c^2\,x^2 - 1\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))*(d + e*x),x)
```

```
[Out] a*d*x + (a*e*x^2)/2 + b*d*x*atanh(c*x) + (b*e*x)/(2*c) - (b*e*atanh(c*x))/(
2*c^2) + (b*e*x^2*atanh(c*x))/2 + (b*d*log(c^2*x^2 - 1))/(2*c)
```

sympy [A]   time = 0.67, size = 92, normalized size = 1.10

$$\begin{cases} adx + \frac{aex^2}{2} + bdx\,\text{atanh}\,(cx) + \frac{bex^2\,\text{atanh}\,(cx)}{2} + \frac{bd\log\left(x - \frac{1}{c}\right)}{c} + \frac{bd\,\text{atanh}\,(cx)}{c} + \frac{bex}{2c} - \frac{be\,\text{atanh}\,(cx)}{2c^2} & \text{for } c \neq 0 \\ a\left(dx + \frac{ex^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x**2/2 + b*d*x*atanh(c*x) + b*e*x**2*atanh(c*x)/2 +
b*d*log(x - 1/c)/c + b*d*atanh(c*x)/c + b*e*x/(2*c) - b*e*atanh(c*x)/(2*c**
2), Ne(c, 0)), (a*(d*x + e*x**2/2), True))
```

3.5 $\quad \displaystyle\int \frac{a+b\tanh^{-1}(cx)}{d+ex}\,dx$

Optimal. Leaf size=114

$$\frac{\left(a + b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e} - \frac{\log\left(\frac{2}{cx+1}\right)\left(a + b\tanh^{-1}(cx)\right)}{e} - \frac{b\mathrm{Li}_2\left(1 - \frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2e} + \frac{b\mathrm{Li}_2\left(1 - \frac{2}{cx+1}\right)}{2e}$$

```
[Out] -(a+b*arctanh(c*x))*ln(2/(c*x+1))/e+(a+b*arctanh(c*x))*ln(2*c*(e*x+d)/(c*d+
e)/(c*x+1))/e+1/2*b*polylog(2,1-2/(c*x+1))/e-1/2*b*polylog(2,1-2*c*(e*x+d)/
(c*d+e)/(c*x+1))/e
```

Rubi [A]   time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5920, 2402, 2315, 2447}

$$-\frac{b\mathrm{PolyLog}\left(2,1 - \frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2e} + \frac{b\mathrm{PolyLog}\left(2,1 - \frac{2}{cx+1}\right)}{2e} + \frac{\left(a + b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e} - \frac{\log\left(\frac{2}{cx+1}\right)\left(a + \right.}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(d + e*x),x]
```

```
[Out] -(((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e) + ((a + b*ArcTanh[c*x])*Log[(2
*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e + (b*PolyLog[2, 1 - 2/(1 + c*x)])/(
2*e) - (b*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e)
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```


Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```


Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\int \frac{a + b\tanh^{-1}(cx)}{d + ex}\,dx = -\frac{\left(a + b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{e} + \frac{\left(a + b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e} + \frac{(bc)\int \frac{\log\left(\frac{2}{1+cx}\right)}{1-c^2x^2}}{e}$$

$$= -\frac{\left(a + b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{e} + \frac{\left(a + b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e} - \frac{b\,\mathrm{Li}_2\left(1 - \frac{2c(d+ex)}{(cd+e)}\right)}{2e}$$

$$= -\frac{\left(a + b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{e} + \frac{\left(a + b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e} + \frac{b\,\mathrm{Li}_2\left(1 - \frac{2}{1+cx}\right)}{2e}$$

Mathematica [C]   time = 0.26, size = 257, normalized size = 2.25

$$a\log(d + ex) - \frac{1}{2}ib\left(-\log\left(\frac{2}{\sqrt{1-c^2x^2}}\right)\left(\pi - 2i\tanh^{-1}(cx)\right) - i\mathrm{Li}_2\left(e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right) + i\left(\tanh^{-1}\left(\frac{cd}{e}\right) + \tanh\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d + e*x),x]

[Out] (a*Log[d + e*x] + b*ArcTanh[c*x]*(Log[1 - c^2*x^2]/2 + Log[I*Sinh[ArcTanh[(
c*d)/e] + ArcTanh[c*x]]]) - (I/2)*b*((-1/4*I)*(Pi - (2*I)*ArcTanh[c*x])^2 +
 I*(ArcTanh[(c*d)/e] + ArcTanh[c*x])^2 + (Pi - (2*I)*ArcTanh[c*x])*Log[1 +
E^(2*ArcTanh[c*x])] + (2*I)*(ArcTanh[(c*d)/e] + ArcTanh[c*x])*Log[1 - E^(-2
*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - (Pi - (2*I)*ArcTanh[c*x])*Log[2/Sqrt
[1 - c^2*x^2]] - (2*I)*(ArcTanh[(c*d)/e] + ArcTanh[c*x])*Log[(2*I)*Sinh[Arc
Tanh[(c*d)/e] + ArcTanh[c*x]]] - I*PolyLog[2, -E^(2*ArcTanh[c*x])] - I*Poly
Log[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]))/e
```

fricas [F]   time = 0.77, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b\,\mathrm{artanh}(cx) + a}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(e*x+d),x, algorithm="fricas")
[Out] integral((b*arctanh(c*x) + a)/(e*x + d), x)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{artanh}(cx) + a}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(e*x+d),x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)/(e*x + d), x)
```

maple [A]   time = 0.09, size = 148, normalized size = 1.30

$$\frac{a\ln(cxe + cd)}{e} + \frac{b\ln(cxe + cd)\operatorname{arctanh}(cx)}{e} + \frac{b\ln(cxe + cd)\ln\left(\frac{cxe-e}{-cd-e}\right)}{2e} + \frac{b\operatorname{dilog}\left(\frac{cxe-e}{-cd-e}\right)}{2e} - \frac{b\ln(cxe + cd)\ln\left(\frac{cxe+e}{-cd+e}\right)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(e*x+d),x)
[Out] a*ln(c*e*x+c*d)/e+b*ln(c*e*x+c*d)/e*arctanh(c*x)+1/2*b/e*ln(c*e*x+c*d)*ln((
c*e*x-e)/(-c*d-e))+1/2*b/e*dilog((c*e*x-e)/(-c*d-e))-1/2*b/e*ln(c*e*x+c*d)*
ln((c*e*x+e)/(-c*d+e))-1/2*b/e*dilog((c*e*x+e)/(-c*d+e))
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}b\int \frac{\log(cx + 1) - \log(-cx + 1)}{ex + d}\,dx + \frac{a\log(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(e*x+d),x, algorithm="maxima")
[Out] 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/(e*x + d), x) + a*log(e*x +
d)/e
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\operatorname{atanh}(cx)}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(d + e*x),x)
```

```
[Out] int((a + b*atanh(c*x))/(d + e*x), x)
```

sympy [F]   time $= 0.00$, size $= 0$, normalized size $= 0.00$

$$\int \frac{a + b \operatorname{atanh}(cx)}{d + ex} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(e*x+d),x)
```

```
[Out] Integral((a + b*atanh(c*x))/(d + e*x), x)
```

3.6 $\displaystyle\int \frac{a+b\tanh^{-1}(cx)}{(d+ex)^2}\,dx$

Optimal. Leaf size=93

$$-\frac{a + b\tanh^{-1}(cx)}{e(d + ex)} - \frac{bc\log(d + ex)}{c^2 d^2 - e^2} - \frac{bc\log(1 - cx)}{2e(cd + e)} + \frac{bc\log(cx + 1)}{2e(cd - e)}$$

```
[Out] (-a-b*arctanh(c*x))/e/(e*x+d)-1/2*b*c*ln(-c*x+1)/e/(c*d+e)+1/2*b*c*ln(c*x+1
)/(c*d-e)/e-b*c*ln(e*x+d)/(c^2*d^2-e^2)
```

Rubi [A]   time $= 0.07$, antiderivative size $= 93$, normalized size of antiderivative $= 1.00$, number of steps used $= 6$, number of rules used $= 4$, integrand size $= 16$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used $= \{5926, 706, 31, 633\}$

$$-\frac{a + b\tanh^{-1}(cx)}{e(d + ex)} - \frac{bc\log(d + ex)}{c^2 d^2 - e^2} - \frac{bc\log(1 - cx)}{2e(cd + e)} + \frac{bc\log(cx + 1)}{2e(cd - e)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(d + e*x)^2,x]
```

```
[Out] -((a + b*ArcTanh[c*x])/(e*(d + e*x))) - (b*c*Log[1 - c*x])/(2*e*(c*d + e))
+ (b*c*Log[1 + c*x])/(2*(c*d - e)*e) - (b*c*Log[d + e*x])/(c^2*d^2 - e^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 706

```
Int[1/((((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{a + b \tanh^{-1}(cx)}{(d + ex)^2} \, dx = -\frac{a + b \tanh^{-1}(cx)}{e(d + ex)} + \frac{(bc) \int \frac{1}{(d+ex)(1-c^2x^2)} \, dx}{e}$$

$$= -\frac{a + b \tanh^{-1}(cx)}{e(d + ex)} - \frac{(bc) \int \frac{-c^2d+c^2ex}{1-c^2x^2} \, dx}{e\left(c^2d^2 - e^2\right)} - \frac{(bce) \int \frac{1}{d+ex} \, dx}{c^2d^2 - e^2}$$

$$= -\frac{a + b \tanh^{-1}(cx)}{e(d + ex)} - \frac{bc \log(d + ex)}{c^2d^2 - e^2} - \frac{\left(bc^3\right) \int \frac{1}{-c-c^2x} \, dx}{2(cd - e)e} + \frac{\left(bc^3\right) \int \frac{1}{c-c^2x} \, dx}{2e(cd + e)}$$

$$= -\frac{a + b \tanh^{-1}(cx)}{e(d + ex)} - \frac{bc \log(1 - cx)}{2e(cd + e)} + \frac{bc \log(1 + cx)}{2(cd - e)e} - \frac{bc \log(d + ex)}{c^2d^2 - e^2}$$

Mathematica [A]   time $= 0.12$, size $= 102$, normalized size $= 1.10$

$$-\frac{a}{e(d + ex)} - \frac{bc \log(d + ex)}{c^2d^2 - e^2} - \frac{bc \log(1 - cx)}{2e(cd + e)} - \frac{bc \log(cx + 1)}{2e(e - cd)} - \frac{b \tanh^{-1}(cx)}{e(d + ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d + e*x)^2,x]
```

```
[Out] -(a/(e*(d + e*x))) - (b*ArcTanh[c*x])/(e*(d + e*x)) - (b*c*Log[1 - c*x])/(2
*e*(c*d + e)) - (b*c*Log[1 + c*x])/(2*e*(-(c*d) + e)) - (b*c*Log[d + e*x])/
(c^2*d^2 - e^2)
```

fricas [B]   time $= 0.94$, size $= 182$, normalized size $= 1.96$

$$-\frac{2\,ac^2d^2 - 2\,ae^2 - \left(bc^2d^2 + bcde + \left(bc^2de + bce^2\right)x\right)\log\left(cx + 1\right) + \left(bc^2d^2 - bcde + \left(bc^2de - bce^2\right)x\right)\log\left(cx - 1\right) +}{2\left(c^2d^3e - de^3 + \left(c^2d^2e^2 - e^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*c^2*d^2 - 2*a*e^2 - (b*c^2*d^2 + b*c*d*e + (b*c^2*d*e + b*c*e^2)*
x)*log(c*x + 1) + (b*c^2*d^2 - b*c*d*e + (b*c^2*d*e - b*c*e^2)*x)*log(c*x -
 1) + 2*(b*c*e^2*x + b*c*d*e)*log(e*x + d) + (b*c^2*d^2 - b*e^2)*log(-(c*x
+ 1)/(c*x - 1)))/(c^2*d^3*e - d*e^3 + (c^2*d^2*e^2 - e^4)*x)
```

giac $[\mathbf{B}]$ time $= 0.18$, size $= 374$, normalized size $= 4.02$

$$-\frac{\left(\frac{(cx+1)bcd\log\left(\frac{(cx+1)cd}{cx-1}-cd+\frac{(cx+1)e}{cx-1}+e\right)}{cx-1} - bcd\log\left(\frac{(cx+1)cd}{cx-1} - cd + \frac{(cx+1)e}{cx-1} + e\right) - \frac{(cx+1)bcd\log\left(-\frac{cx+1}{cx-1}\right)}{cx-1} - 2\,acd + \frac{(cx+1)be\log\left(\frac{(c}{}\right.}{}}{\frac{(cx+1)c^3d^3}{cx-1} - c^3d^3 + \frac{(cx+1)c^2d^2e}{cx-1} + c^2d^2e - \frac{(cx+1)cde^2}{cx-1} +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -((c*x + 1)*b*c*d*log((c*x + 1)*c*d/(c*x - 1) - c*d + (c*x + 1)*e/(c*x - 1)
 + e)/(c*x - 1) - b*c*d*log((c*x + 1)*c*d/(c*x - 1) - c*d + (c*x + 1)*e/(c*
x - 1) + e) - (c*x + 1)*b*c*d*log(-(c*x + 1)/(c*x - 1))/(c*x - 1) - 2*a*c*d
 + (c*x + 1)*b*e*log((c*x + 1)*c*d/(c*x - 1) - c*d + (c*x + 1)*e/(c*x - 1)
+ e)/(c*x - 1) + b*e*log((c*x + 1)*c*d/(c*x - 1) - c*d + (c*x + 1)*e/(c*x -
 1) + e) - (c*x + 1)*b*e*log(-(c*x + 1)/(c*x - 1))/(c*x - 1) + 2*a*e)*c/(((c
*x + 1)*c^3*d^3/(c*x - 1) - c^3*d^3 + (c*x + 1)*c^2*d^2*e/(c*x - 1) + c^2*d
^2*e - (c*x + 1)*c*d*e^2/(c*x - 1) + c*d*e^2 - (c*x + 1)*e^3/(c*x - 1) - e^
3)
```

maple $[\mathbf{A}]$ time $= 0.04$, size $= 114$, normalized size $= 1.23$

$$-\frac{ca}{(cxe+cd)\,e} - \frac{cb\arctanh(cx)}{(cxe+cd)\,e} - \frac{cb\ln(cxe+cd)}{(cd+e)(cd-e)} - \frac{cb\ln(cx-1)}{e(2cd+2e)} + \frac{cb\ln(cx+1)}{e(2cd-2e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(e*x+d)^2,x)
```

```
[Out] -c*a/(c*e*x+c*d)/e-c*b/(c*e*x+c*d)/e*arctanh(c*x)-c*b/(c*d+e)/(c*d-e)*ln(c*
e*x+c*d)-c*b/e/(2*c*d+2*e)*ln(c*x-1)+c*b/e/(2*c*d-2*e)*ln(c*x+1)
```

maxima $[\mathbf{A}]$ time $= 0.32$, size $= 99$, normalized size $= 1.06$

$$\frac{1}{2}\left(c\left(\frac{\log(cx+1)}{cde-e^2} - \frac{\log(cx-1)}{cde+e^2} - \frac{2\log(ex+d)}{c^2d^2-e^2}\right) - \frac{2\,\mathrm{artanh}(cx)}{e^2x+de}\right)b - \frac{a}{e^2x+de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c*(log(c*x + 1)/(c*d*e - e^2) - log(c*x - 1)/(c*d*e + e^2) - 2*log(e*x
 + d)/(c^2*d^2 - e^2)) - 2*arctanh(c*x)/(e^2*x + d*e))*b - a/(e^2*x + d*e)
```

mupad $[\mathbf{B}]$ time = 3.60, size = 115, normalized size = 1.24

$$-\frac{d^2\left(\frac{bc\ln\left(c^2x^2-1\right)}{2}-bc\ln\left(d+ex\right)+ac^2x+bc^2x\operatorname{atanh}\left(cx\right)\right)+de\left(b\operatorname{atanh}\left(cx\right)-bcx\ln\left(d+ex\right)+\frac{bcx\ln\left(c^2\,\right.}{2}\right.}{d\left(e^2-c^2d^2\right)\left(d+ex\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(d + e*x)^2,x)
```

```
[Out] -(d^2*((b*c*log(c^2*x^2 - 1))/2 - b*c*log(d + e*x) + a*c^2*x + b*c^2*x*atan
h(c*x)) + d*e*(b*atanh(c*x) - b*c*x*log(d + e*x) + (b*c*x*log(c^2*x^2 - 1))
/2) - a*e^2*x)/(d*(e^2 - c^2*d^2)*(d + e*x))
```

sympy $[\mathbf{A}]$ time = 3.62, size = 690, normalized size = 7.42

$$\begin{cases}\dfrac{ax+bx\operatorname{atanh}\left(cx\right)+\dfrac{b\log\left(x-\frac{1}{c}\right)}{c}+\dfrac{b\operatorname{atanh}\left(cx\right)}{c}}{d^2} \\[4ex] -\dfrac{a}{de+e^2x} \\[3ex] -\dfrac{2ad}{2d^2e+2de^2x}+\dfrac{bd\operatorname{atanh}\left(\frac{ex}{d}\right)}{2d^2e+2de^2x}+\dfrac{bd}{2d^2e+2de^2x}-\dfrac{bex\operatorname{atanh}\left(\frac{ex}{d}\right)}{2d^2e+2de^2x} \\[3ex] -\dfrac{2ad}{2d^2e+2de^2x}-\dfrac{bd\operatorname{atanh}\left(\frac{ex}{d}\right)}{2d^2e+2de^2x}-\dfrac{bd}{2d^2e+2de^2x}+\dfrac{bex\operatorname{atanh}\left(\frac{ex}{d}\right)}{2d^2e+2de^2x} \\[3ex] \tilde{\infty}\left(ax+bx\operatorname{atanh}\left(cx\right)+\dfrac{b\log\left(x-\frac{1}{c}\right)}{c}+\dfrac{b\operatorname{atanh}\left(cx\right)}{c}\right) \\[4ex] -\dfrac{ac^2d^2}{c^2d^3e+c^2d^2e^2x-de^3-e^4x}+\dfrac{ae^2}{c^2d^3e+c^2d^2e^2x-de^3-e^4x}+\dfrac{bc^2dex\operatorname{atanh}\left(cx\right)}{c^2d^3e+c^2d^2e^2x-de^3-e^4x}+\dfrac{bcde\log\left(x-\frac{1}{c}\right)}{c^2d^3e+c^2d^2e^2x-de^3-e^4x}-\dfrac{bcde\log\left(\frac{d}{e}+x\right)}{c^2d^3e+c^2d^2e^2x-de^3-e^4x}+\dfrac{}{c^2d^3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(e*x+d)**2,x)
```

```
[Out] Piecewise(((a*x + b*x*atanh(c*x) + b*log(x - 1/c)/c + b*atanh(c*x)/c)/d**2,
 Eq(e, 0)), (-a/(d*e + e**2*x), Eq(c, 0)), (-2*a*d/(2*d**2*e + 2*d*e**2*x)
+ b*d*atanh(e*x/d)/(2*d**2*e + 2*d*e**2*x) + b*d/(2*d**2*e + 2*d*e**2*x) -
b*e*x*atanh(e*x/d)/(2*d**2*e + 2*d*e**2*x), Eq(c, -e/d)), (-2*a*d/(2*d**2*e
 + 2*d*e**2*x) - b*d*atanh(e*x/d)/(2*d**2*e + 2*d*e**2*x) - b*d/(2*d**2*e +
 2*d*e**2*x) + b*e*x*atanh(e*x/d)/(2*d**2*e + 2*d*e**2*x), Eq(c, e/d)), (zo
```

```
o*(a*x + b*x*atanh(c*x) + b*log(x - 1/c)/c + b*atanh(c*x)/c), Eq(d, -e*x)),
 (-a*c**2*d**2/(c**2*d**3*e + c**2*d**2*e**2*x - d*e**3 - e**4*x) + a*e**2/
(c**2*d**3*e + c**2*d**2*e**2*x - d*e**3 - e**4*x) + b*c**2*d*e*x*atanh(c*x
)/(c**2*d**3*e + c**2*d**2*e**2*x - d*e**3 - e**4*x) + b*c*d*e*log(x - 1/c)
/(c**2*d**3*e + c**2*d**2*e**2*x - d*e**3 - e**4*x) - b*c*d*e*log(d/e + x)/
(c**2*d**3*e + c**2*d**2*e**2*x - d*e**3 - e**4*x) + b*c*d*e*atanh(c*x)/(c*
*2*d**3*e + c**2*d**2*e**2*x - d*e**3 - e**4*x) + b*c*e**2*x*log(x - 1/c)/(
c**2*d**3*e + c**2*d**2*e**2*x - d*e**3 - e**4*x) - b*c*e**2*x*log(d/e + x)
/(c**2*d**3*e + c**2*d**2*e**2*x - d*e**3 - e**4*x) + b*c*e**2*x*atanh(c*x)
/(c**2*d**3*e + c**2*d**2*e**2*x - d*e**3 - e**4*x) + b*e**2*atanh(c*x)/(c*
*2*d**3*e + c**2*d**2*e**2*x - d*e**3 - e**4*x), True))
```

3.7 $\int \dfrac{a+b\tanh^{-1}(cx)}{(d+ex)^3}\,dx$

Optimal. Leaf size=130

$$-\frac{a+b\tanh^{-1}(cx)}{2e(d+ex)^2}+\frac{bc}{2\left(c^2d^2-e^2\right)(d+ex)}-\frac{bc^2\log(1-cx)}{4e(cd+e)^2}+\frac{bc^2\log(cx+1)}{4e(cd-e)^2}-\frac{bc^3d\log(d+ex)}{\left(c^2d^2-e^2\right)^2}$$

[Out] 1/2*b*c/(c^2*d^2-e^2)/(e*x+d)+1/2*(-a-b*arctanh(c*x))/e/(e*x+d)^2-1/4*b*c^2
*ln(-c*x+1)/e/(c*d+e)^2+1/4*b*c^2*ln(c*x+1)/(c*d-e)^2/e-b*c^3*d*ln(e*x+d)/(
c^2*d^2-e^2)^2

Rubi [A]   time = 0.13, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5926, 710, 801}

$$-\frac{a+b\tanh^{-1}(cx)}{2e(d+ex)^2}+\frac{bc}{2\left(c^2d^2-e^2\right)(d+ex)}-\frac{bc^3d\log(d+ex)}{\left(c^2d^2-e^2\right)^2}-\frac{bc^2\log(1-cx)}{4e(cd+e)^2}+\frac{bc^2\log(cx+1)}{4e(cd-e)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*x])/(d + e*x)^3,x]

[Out] (b*c)/(2*(c^2*d^2 - e^2)*(d + e*x)) - (a + b*ArcTanh[c*x])/(2*e*(d + e*x)^2
) - (b*c^2*Log[1 - c*x])/(4*e*(c*d + e)^2) + (b*c^2*Log[1 + c*x])/(4*(c*d -
 e)^2*e) - (b*c^3*d*Log[d + e*x])/(c^2*d^2 - e^2)^2

Rule 710

Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]

Rule 801

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rule 5926

Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(

```
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{a + b\tanh^{-1}(cx)}{(d + ex)^3}\, dx = -\frac{a + b\tanh^{-1}(cx)}{2e(d + ex)^2} + \frac{(bc)\int \frac{1}{(d+ex)^2(1-c^2x^2)}\, dx}{2e}$$

$$= \frac{bc}{2\left(c^2d^2 - e^2\right)(d + ex)} - \frac{a + b\tanh^{-1}(cx)}{2e(d + ex)^2} + \frac{\left(bc^3\right)\int \frac{d-ex}{(d+ex)(1-c^2x^2)}\, dx}{2e\left(c^2d^2 - e^2\right)}$$

$$= \frac{bc}{2\left(c^2d^2 - e^2\right)(d + ex)} - \frac{a + b\tanh^{-1}(cx)}{2e(d + ex)^2} + \frac{\left(bc^3\right)\int \left(\frac{-cd+e}{2(cd+e)(-1+cx)} + \frac{cd+e}{2(cd-e)(1+cx)} + \frac{}{(-cd}\right)}{2e\left(c^2d^2 - e^2\right)}$$

$$= \frac{bc}{2\left(c^2d^2 - e^2\right)(d + ex)} - \frac{a + b\tanh^{-1}(cx)}{2e(d + ex)^2} - \frac{bc^2\log(1 - cx)}{4e(cd + e)^2} + \frac{bc^2\log(1 + cx)}{4(cd - e)^2e} - \frac{bc^3d1}{\left(c^2\right.}$$

Mathematica [A]   time = 0.16, size = 133, normalized size = 1.02

$$\frac{1}{4}\left(-\frac{2a}{e(d + ex)^2} + \frac{2bc}{\left(c^2d^2 - e^2\right)(d + ex)} - \frac{bc^2\log(1 - cx)}{e(cd + e)^2} + \frac{bc^2\log(cx + 1)}{e(e - cd)^2} - \frac{4bc^3d\log(d + ex)}{\left(e^2 - c^2d^2\right)^2} - \frac{2b\tanh^{-1}(cx)}{e(d + ex)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d + e*x)^3,x]

[Out] ((-2*a)/(e*(d + e*x)^2) + (2*b*c)/((c^2*d^2 - e^2)*(d + e*x)) - (2*b*ArcTan
h[c*x])/(e*(d + e*x)^2) - (b*c^2*Log[1 - c*x])/(e*(c*d + e)^2) + (b*c^2*Log
[1 + c*x])/(e*(-(c*d) + e)^2) - (4*b*c^3*d*Log[d + e*x])/(-(c^2*d^2) + e^2)
^2)/4
```

fricas [B]   time = 0.85, size = 454, normalized size = 3.49

$$-\frac{2\,ac^4d^4 - 2\,bc^3d^3e - 4\,ac^2d^2e^2 + 2\,bcde^3 + 2\,ae^4 - 2\left(bc^3d^2e^2 - bce^4\right)x - \left(bc^4d^4 + 2\,bc^3d^3e + bc^2d^2e^2 + \left(bc^4d^2e^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*c^4*d^4 - 2*b*c^3*d^3*e - 4*a*c^2*d^2*e^2 + 2*b*c*d*e^3 + 2*a*e^4
 - 2*(b*c^3*d^2*e^2 - b*c*e^4)*x - (b*c^4*d^4 + 2*b*c^3*d^3*e + b*c^2*d^2*e
^2 + (b*c^4*d^2*e^2 + 2*b*c^3*d*e^3 + b*c^2*e^4)*x^2 + 2*(b*c^4*d^3*e + 2*b
*c^3*d^2*e^2 + b*c^2*d*e^3)*x)*log(c*x + 1) + (b*c^4*d^4 - 2*b*c^3*d^3*e +
b*c^2*d^2*e^2 + (b*c^4*d^2*e^2 - 2*b*c^3*d*e^3 + b*c^2*e^4)*x^2 + 2*(b*c^4*
d^3*e - 2*b*c^3*d^2*e^2 + b*c^2*d*e^3)*x)*log(c*x - 1) + 4*(b*c^3*d*e^3*x^2
 + 2*b*c^3*d^2*e^2*x + b*c^3*d^3*e)*log(e*x + d) + (b*c^4*d^4 - 2*b*c^2*d^2
*e^2 + b*e^4)*log(-(c*x + 1)/(c*x - 1)))/(c^4*d^6*e - 2*c^2*d^4*e^3 + d^2*e
^5 + (c^4*d^4*e^3 - 2*c^2*d^2*e^5 + e^7)*x^2 + 2*(c^4*d^5*e^2 - 2*c^2*d^3*e
^4 + d*e^6)*x)
```

giac [B]   time = 0.19, size = 1197, normalized size = 9.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -((c*x + 1)^2*b*c^4*d^3*log(-(c*x + 1)*c*d/(c*x - 1) + c*d - (c*x + 1)*e/(c
*x - 1) - e)/(c*x - 1)^2 - 2*(c*x + 1)*b*c^4*d^3*log(-(c*x + 1)*c*d/(c*x -
1) + c*d - (c*x + 1)*e/(c*x - 1) - e)/(c*x - 1) + b*c^4*d^3*log(-(c*x + 1)*
c*d/(c*x - 1) + c*d - (c*x + 1)*e/(c*x - 1) - e) - (c*x + 1)^2*b*c^4*d^3*lo
g(-(c*x + 1)/(c*x - 1))/(c*x - 1)^2 + (c*x + 1)*b*c^4*d^3*log(-(c*x + 1)/(c
*x - 1))/(c*x - 1) - 2*(c*x + 1)*a*c^4*d^3/(c*x - 1) + 2*a*c^4*d^3 + 2*(c*x
 + 1)^2*b*c^3*d^2*e*log(-(c*x + 1)*c*d/(c*x - 1) + c*d - (c*x + 1)*e/(c*x -
 1) - e)/(c*x - 1)^2 - 2*b*c^3*d^2*e*log(-(c*x + 1)*c*d/(c*x - 1) + c*d - (
c*x + 1)*e/(c*x - 1) - e) - 2*(c*x + 1)^2*b*c^3*d^2*e*log(-(c*x + 1)/(c*x -
 1))/(c*x - 1)^2 + (c*x + 1)*b*c^3*d^2*e*log(-(c*x + 1)/(c*x - 1))/(c*x - 1
) + 2*(c*x + 1)*a*c^3*d^2*e/(c*x - 1) - 4*a*c^3*d^2*e + (c*x + 1)*b*c^3*d^2
*e/(c*x - 1) - b*c^3*d^2*e + (c*x + 1)^2*b*c^2*d*e^2*log(-(c*x + 1)*c*d/(c*
x - 1) + c*d - (c*x + 1)*e/(c*x - 1) - e)/(c*x - 1)^2 + 2*(c*x + 1)*b*c^2*d
*e^2*log(-(c*x + 1)*c*d/(c*x - 1) + c*d - (c*x + 1)*e/(c*x - 1) - e)/(c*x -
 1) + b*c^2*d*e^2*log(-(c*x + 1)*c*d/(c*x - 1) + c*d - (c*x + 1)*e/(c*x - 1
) - e) - (c*x + 1)^2*b*c^2*d*e^2*log(-(c*x + 1)/(c*x - 1))/(c*x - 1)^2 - (c
*x + 1)*b*c^2*d*e^2*log(-(c*x + 1)/(c*x - 1))/(c*x - 1) + 2*(c*x + 1)*a*c^2
*d*e^2/(c*x - 1) + 2*a*c^2*d*e^2 + 2*b*c^2*d*e^2 - (c*x + 1)*b*c*e^3*log(-(
c*x + 1)/(c*x - 1))/(c*x - 1) - 2*(c*x + 1)*a*c*e^3/(c*x - 1) - (c*x + 1)*b
*c*e^3/(c*x - 1) - b*c*e^3)*c/((c*x + 1)^2*c^6*d^6/(c*x - 1)^2 - 2*(c*x + 1
)*c^6*d^6/(c*x - 1) + c^6*d^6 + 2*(c*x + 1)^2*c^5*d^5*e/(c*x - 1)^2 - 2*c^5
*d^5*e - (c*x + 1)^2*c^4*d^4*e^2/(c*x - 1)^2 + 6*(c*x + 1)*c^4*d^4*e^2/(c*x
 - 1) - c^4*d^4*e^2 - 4*(c*x + 1)^2*c^3*d^3*e^3/(c*x - 1)^2 + 4*c^3*d^3*e^3
 - (c*x + 1)^2*c^2*d^2*e^4/(c*x - 1)^2 - 6*(c*x + 1)*c^2*d^2*e^4/(c*x - 1)
- c^2*d^2*e^4 + 2*(c*x + 1)^2*c*d*e^5/(c*x - 1)^2 - 2*c*d*e^5 + (c*x + 1)^2
*e^6/(c*x - 1)^2 + 2*(c*x + 1)*e^6/(c*x - 1) + e^6)
```

maple [A]   time = 0.04, size = 154, normalized size = 1.18

$$-\frac{c^2 a}{2\,(cxe+cd)^2\,e} - \frac{c^2 b\,\text{arctanh}\,(cx)}{2\,(cxe+cd)^2\,e} + \frac{c^2 b}{2\,(cd+e)\,(cd-e)\,(cxe+cd)} - \frac{c^3 bd\,\ln\,(cxe+cd)}{(cd+e)^2\,(cd-e)^2} - \frac{c^2 b\,\ln\,(cx-1)}{4e\,(cd+e)^2} + \frac{b\,c^2\,\ln\,(cx}{4\,(cd-e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(e*x+d)^3,x)
```

```
[Out] -1/2*c^2*a/(c*e*x+c*d)^2/e-1/2*c^2*b/(c*e*x+c*d)^2/e*arctanh(c*x)+1/2*c^2*b
/(c*d+e)/(c*d-e)/(c*e*x+c*d)-c^3*b*d/(c*d+e)^2/(c*d-e)^2*ln(c*e*x+c*d)-1/4*
c^2*b/e/(c*d+e)^2*ln(c*x-1)+1/4*b*c^2*ln(c*x+1)/(c*d-e)^2/e
```

maxima [A]   time = 0.33, size = 190, normalized size = 1.46

$$-\frac{1}{4}\left(\left(\frac{4\,c^2 d\,\log\,(ex+d)}{c^4 d^4 - 2\,c^2 d^2 e^2 + e^4} - \frac{c\,\log\,(cx+1)}{c^2 d^2 e - 2\,cde^2 + e^3} + \frac{c\,\log\,(cx-1)}{c^2 d^2 e + 2\,cde^2 + e^3} - \frac{2}{c^2 d^3 - de^2 + \left(c^2 d^2 e - e^3\right)x}\right)c + \frac{2\,\text{ar}}{e^3 x^2 + 2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((4*c^2*d*log(e*x + d)/(c^4*d^4 - 2*c^2*d^2*e^2 + e^4) - c*log(c*x + 1
)/(c^2*d^2*e - 2*c*d*e^2 + e^3) + c*log(c*x - 1)/(c^2*d^2*e + 2*c*d*e^2 + e
^3) - 2/(c^2*d^3 - d*e^2 + (c^2*d^2*e - e^3)*x))*c + 2*arctanh(c*x)/(e^3*x^
2 + 2*d*e^2*x + d^2*e))*b - 1/2*a/(e^3*x^2 + 2*d*e^2*x + d^2*e)
```

mupad [B]   time = 4.00, size = 427, normalized size = 3.28

$$\frac{b c^3 d\,\ln\left(c^2 x^2 - 1\right)}{2\left(c^4 d^4 - 2\,c^2 d^2 e^2 + e^4\right)} - \frac{\text{atan}\left(\frac{c^2 x}{\sqrt{-c^2}}\right)\left(b c^5 d^2 + b c^3 e^2\right)}{2 e^5\sqrt{-c^2} - c^2\left(2\,d^4 e\left(-c^2\right)^{3/2} + 4\,d^2 e^3\sqrt{-c^2}\right)} - \frac{b c^3 d\,\ln\left(d + e x\right)}{c^4 d^4 - 2\,c^2 d^2 e^2 + e^4} - \frac{\frac{b\,\text{atanh}(cx)}{2e}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(d + e*x)^3,x)
```

```
[Out] (b*c^3*d*log(c^2*x^2 - 1))/(2*(e^4 + c^4*d^4 - 2*c^2*d^2*e^2)) - (atan((c^2
*x)/(-c^2)^(1/2))*(b*c^5*d^2 + b*c^3*e^2))/(2*e^5*(-c^2)^(1/2) - c^2*(2*d^4
*e*(-c^2)^(3/2) + 4*d^2*e^3*(-c^2)^(1/2))) - (b*c^3*d*log(d + e*x))/(e^4 +
c^4*d^4 - 2*c^2*d^2*e^2) - ((b*atanh(c*x))/(2*e) - (x*(a*e^2 - a*c^2*d^2 +
(b*c*d*e)/2))/(d*(e^2 - c^2*d^2)) - (x^2*((a*e^3)/2 + (b*c*d*e^2)/2 - (a*c^
2*d^2*e)/2))/(d^2*(e^2 - c^2*d^2)) + (x^4*((a*c^2*e^3)/2 - (a*c^4*d^2*e)/2
+ (b*c^3*d*e^2)/2))/(d^2*(e^2 - c^2*d^2)) + (x^3*(a*c^2*e^2 - a*c^4*d^2 + (
```

b*c^3*d*e)/2))/(d*(e^2 - c^2*d^2)) - (b*c^2*x^2*atanh(c*x))/(2*e))/(d^2 + e
^2*x^2 + 2*d*e*x - c^2*d^2*x^2 - c^2*e^2*x^4 - 2*c^2*d*e*x^3)

sympy [**A**]   time = 7.13, size = 3216, normalized size = 24.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x))/(e*x+d)**3,x)

[Out] Piecewise((a*x/d**3, Eq(c, 0) & Eq(e, 0)), (-4*a*d**2/(8*d**4*e + 16*d**3*e
2*x + 8*d2*e**3*x**2) + 3*b*d**2*atanh(e*x/d)/(8*d**4*e + 16*d**3*e**2*
x + 8*d**2*e**3*x**2) + 2*b*d**2/(8*d**4*e + 16*d**3*e**2*x + 8*d**2*e**3*x
2) - 2*b*d*e*x*atanh(e*x/d)/(8*d4*e + 16*d**3*e**2*x + 8*d**2*e**3*x**2
) + b*d*e*x/(8*d**4*e + 16*d**3*e**2*x + 8*d**2*e**3*x**2) - b*e**2*x**2*at
anh(e*x/d)/(8*d**4*e + 16*d**3*e**2*x + 8*d**2*e**3*x**2), Eq(c, -e/d)), (-
4*a*d**2/(8*d**4*e + 16*d**3*e**2*x + 8*d**2*e**3*x**2) - 3*b*d**2*atanh(e*
x/d)/(8*d**4*e + 16*d**3*e**2*x + 8*d**2*e**3*x**2) - 2*b*d**2/(8*d**4*e +
16*d**3*e**2*x + 8*d**2*e**3*x**2) + 2*b*d*e*x*atanh(e*x/d)/(8*d**4*e + 16*
d**3*e**2*x + 8*d**2*e**3*x**2) - b*d*e*x/(8*d**4*e + 16*d**3*e**2*x + 8*d*
*2*e**3*x**2) + b*e**2*x**2*atanh(e*x/d)/(8*d**4*e + 16*d**3*e**2*x + 8*d**
2*e**3*x**2), Eq(c, e/d)), ((a*x + b*x*atanh(c*x) + b*log(x - 1/c)/c + b*at
anh(c*x)/c)/d**3, Eq(e, 0)), (-a/(2*d**2*e + 4*d*e**2*x + 2*e**3*x**2), Eq(
c, 0)), (-a*c**4*d**4/(2*c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4*d**4*e**
3*x**2 - 4*c**2*d**4*e**3 - 8*c**2*d**3*e**4*x - 4*c**2*d**2*e**5*x**2 + 2*
d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 2*a*c**2*d**2*e**2/(2*c**4*d**6*e +
 4*c**4*d**5*e**2*x + 2*c**4*d**4*e**3*x**2 - 4*c**2*d**4*e**3 - 8*c**2*d**
3*e**4*x - 4*c**2*d**2*e**5*x**2 + 2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2)
- a*e**4/(2*c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4*d**4*e**3*x**2 - 4*c*
*2*d**4*e**3 - 8*c**2*d**3*e**4*x - 4*c**2*d**2*e**5*x**2 + 2*d**2*e**5 + 4
*d*e**6*x + 2*e**7*x**2) + 2*b*c**4*d**3*e*x*atanh(c*x)/(2*c**4*d**6*e + 4*
c**4*d**5*e**2*x + 2*c**4*d**4*e**3*x**2 - 4*c**2*d**4*e**3 - 8*c**2*d**3*e
4*x - 4*c2*d**2*e**5*x**2 + 2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + b
*c**4*d**2*e**2*x**2*atanh(c*x)/(2*c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**
4*d**4*e**3*x**2 - 4*c**2*d**4*e**3 - 8*c**2*d**3*e**4*x - 4*c**2*d**2*e**5
*x**2 + 2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 2*b*c**3*d**3*e*log(x - 1
/c)/(2*c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4*d**4*e**3*x**2 - 4*c**2*d*
*4*e**3 - 8*c**2*d**3*e**4*x - 4*c**2*d**2*e**5*x**2 + 2*d**2*e**5 + 4*d*e*
*6*x + 2*e**7*x**2) - 2*b*c**3*d**3*e*log(d/e + x)/(2*c**4*d**6*e + 4*c**4*
d**5*e**2*x + 2*c**4*d**4*e**3*x**2 - 4*c**2*d**4*e**3 - 8*c**2*d**3*e**4*x
 - 4*c**2*d**2*e**5*x**2 + 2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 2*b*c*
*3*d**3*e*atanh(c*x)/(2*c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4*d**4*e**3
*x**2 - 4*c**2*d**4*e**3 - 8*c**2*d**3*e**4*x - 4*c**2*d**2*e**5*x**2 + 2*d
2*e5 + 4*d*e**6*x + 2*e**7*x**2) + b*c**3*d**3*e/(2*c**4*d**6*e + 4*c**
4*d**5*e**2*x + 2*c**4*d**4*e**3*x**2 - 4*c**2*d**4*e**3 - 8*c**2*d**3*e**4

```
*x - 4*c**2*d**2*e**5*x**2 + 2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 4*b*
c**3*d**2*e**2*x*log(x - 1/c)/(2*c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4*
d**4*e**3*x**2 - 4*c**2*d**4*e**3 - 8*c**2*d**3*e**4*x - 4*c**2*d**2*e**5*x
**2 + 2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 4*b*c**3*d**2*e**2*x*log(d/
e + x)/(2*c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4*d**4*e**3*x**2 - 4*c**2
*d**4*e**3 - 8*c**2*d**3*e**4*x - 4*c**2*d**2*e**5*x**2 + 2*d**2*e**5 + 4*d
*e**6*x + 2*e**7*x**2) + 4*b*c**3*d**2*e**2*x*atanh(c*x)/(2*c**4*d**6*e + 4
*c**4*d**5*e**2*x + 2*c**4*d**4*e**3*x**2 - 4*c**2*d**4*e**3 - 8*c**2*d**3*
e**4*x - 4*c**2*d**2*e**5*x**2 + 2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) +
b*c**3*d**2*e**2*x/(2*c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4*d**4*e**3*x
**2 - 4*c**2*d**4*e**3 - 8*c**2*d**3*e**4*x - 4*c**2*d**2*e**5*x**2 + 2*d**
2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 2*b*c**3*d*e**3*x**2*log(x - 1/c)/(2*c
**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4*d**4*e**3*x**2 - 4*c**2*d**4*e**3
- 8*c**2*d**3*e**4*x - 4*c**2*d**2*e**5*x**2 + 2*d**2*e**5 + 4*d*e**6*x + 2
*e**7*x**2) - 2*b*c**3*d*e**3*x**2*log(d/e + x)/(2*c**4*d**6*e + 4*c**4*d**
5*e**2*x + 2*c**4*d**4*e**3*x**2 - 4*c**2*d**4*e**3 - 8*c**2*d**3*e**4*x -
4*c**2*d**2*e**5*x**2 + 2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 2*b*c**3*
d*e**3*x**2*atanh(c*x)/(2*c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4*d**4*e*
*3*x**2 - 4*c**2*d**4*e**3 - 8*c**2*d**3*e**4*x - 4*c**2*d**2*e**5*x**2 + 2
*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 3*b*c**2*d**2*e**2*atanh(c*x)/(2*c
**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4*d**4*e**3*x**2 - 4*c**2*d**4*e**3
- 8*c**2*d**3*e**4*x - 4*c**2*d**2*e**5*x**2 + 2*d**2*e**5 + 4*d*e**6*x + 2
*e**7*x**2) + 2*b*c**2*d*e**3*x*atanh(c*x)/(2*c**4*d**6*e + 4*c**4*d**5*e**
2*x + 2*c**4*d**4*e**3*x**2 - 4*c**2*d**4*e**3 - 8*c**2*d**3*e**4*x - 4*c**
2*d**2*e**5*x**2 + 2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + b*c**2*e**4*x*
*2*atanh(c*x)/(2*c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4*d**4*e**3*x**2 -
 4*c**2*d**4*e**3 - 8*c**2*d**3*e**4*x - 4*c**2*d**2*e**5*x**2 + 2*d**2*e**
5 + 4*d*e**6*x + 2*e**7*x**2) - b*c*d*e**3/(2*c**4*d**6*e + 4*c**4*d**5*e**
2*x + 2*c**4*d**4*e**3*x**2 - 4*c**2*d**4*e**3 - 8*c**2*d**3*e**4*x - 4*c**
2*d**2*e**5*x**2 + 2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - b*c*e**4*x/(2*
c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c**4*d**4*e**3*x**2 - 4*c**2*d**4*e**3
 - 8*c**2*d**3*e**4*x - 4*c**2*d**2*e**5*x**2 + 2*d**2*e**5 + 4*d*e**6*x +
2*e**7*x**2) - b*e**4*atanh(c*x)/(2*c**4*d**6*e + 4*c**4*d**5*e**2*x + 2*c*
*4*d**4*e**3*x**2 - 4*c**2*d**4*e**3 - 8*c**2*d**3*e**4*x - 4*c**2*d**2*e**
5*x**2 + 2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2), True))
```

3.8 $\int \dfrac{a+b\tanh^{-1}(cx)}{(d+ex)^4}\,dx$

Optimal. Leaf size=175

$$-\frac{a+b\tanh^{-1}(cx)}{3e(d+ex)^3}-\frac{bc^3\log(1-cx)}{6e(cd+e)^3}+\frac{bc^3\log(cx+1)}{6e(cd-e)^3}+\frac{bc}{6\left(c^2d^2-e^2\right)(d+ex)^2}+\frac{2bc^3d}{3\left(c^2d^2-e^2\right)^2(d+ex)}-\frac{bc^3\left(3c^2d^2\right.}{3(cd-}$$

[Out] 1/6*b*c/(c^2*d^2-e^2)/(e*x+d)^2+2/3*b*c^3*d/(c^2*d^2-e^2)^2/(e*x+d)+1/3*(-a
-b*arctanh(c*x))/e/(e*x+d)^3-1/6*b*c^3*ln(-c*x+1)/e/(c*d+e)^3+1/6*b*c^3*ln(
c*x+1)/(c*d-e)^3/e-1/3*b*c^3*(3*c^2*d^2+e^2)*ln(e*x+d)/(c^2*d^2-e^2)^3

Rubi [A]   time = 0.19, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5926, 710, 801}

$$-\frac{a+b\tanh^{-1}(cx)}{3e(d+ex)^3}+\frac{2bc^3d}{3\left(c^2d^2-e^2\right)^2(d+ex)}+\frac{bc}{6\left(c^2d^2-e^2\right)(d+ex)^2}-\frac{bc^3\left(3c^2d^2+e^2\right)\log(d+ex)}{3(cd-e)^3(cd+e)^3}-\frac{bc^3\log(1-cx)}{6e(cd+e)^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*x])/(d + e*x)^4,x]

[Out] (b*c)/(6*(c^2*d^2 - e^2)*(d + e*x)^2) + (2*b*c^3*d)/(3*(c^2*d^2 - e^2)^2*(d
 + e*x)) - (a + b*ArcTanh[c*x])/(3*e*(d + e*x)^3) - (b*c^3*Log[1 - c*x])/(6
e(c*d + e)^3) + (b*c^3*Log[1 + c*x])/(6*(c*d - e)^3*e) - (b*c^3*(3*c^2*d^
2 + e^2)*Log[d + e*x])/(3*(c*d - e)^3*(c*d + e)^3)

Rule 710

Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]

Rule 801

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rule 5926

Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(

```
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{a + b\tanh^{-1}(cx)}{(d + ex)^4}\, dx = -\frac{a + b\tanh^{-1}(cx)}{3e(d + ex)^3} + \frac{(bc)\int \frac{1}{(d+ex)^3\left(1-c^2x^2\right)}\, dx}{3e}$$

$$= \frac{bc}{6\left(c^2d^2 - e^2\right)(d + ex)^2} - \frac{a + b\tanh^{-1}(cx)}{3e(d + ex)^3} + \frac{\left(bc^3\right)\int \frac{d-ex}{(d+ex)^2\left(1-c^2x^2\right)}\, dx}{3e\left(c^2d^2 - e^2\right)}$$

$$= \frac{bc}{6\left(c^2d^2 - e^2\right)(d + ex)^2} - \frac{a + b\tanh^{-1}(cx)}{3e(d + ex)^3} + \frac{\left(bc^3\right)\int \left(-\frac{c(cd-e)}{2(cd+e)^2(-1+cx)} + \frac{c(cd+e)}{2(cd-e)^2(1+cx)} + \right.}{3e\left(c^2\right.}$$

$$= \frac{bc}{6\left(c^2d^2 - e^2\right)(d + ex)^2} + \frac{2bc^3d}{3\left(c^2d^2 - e^2\right)^2(d + ex)} - \frac{a + b\tanh^{-1}(cx)}{3e(d + ex)^3} - \frac{bc^3\log(1 - cx)}{6e(cd + e)^3}$$

Mathematica [A]   time = 0.27, size = 173, normalized size = 0.99

$$\frac{1}{6}\left(-\frac{2a}{e(d + ex)^3} - \frac{bc^3\log(1 - cx)}{e(cd + e)^3} + \frac{bc^3\log(cx + 1)}{e(cd - e)^3} + \frac{bc}{\left(c^2d^2 - e^2\right)(d + ex)^2} + \frac{4bc^3d}{\left(e^2 - c^2d^2\right)^2(d + ex)} - \frac{2bc^3\left(3c^2d\right.}{\left(c^2\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d + e*x)^4,x]
```

```
[Out] ((-2*a)/(e*(d + e*x)^3) + (b*c)/((c^2*d^2 - e^2)*(d + e*x)^2) + (4*b*c^3*d)
/((-(c^2*d^2) + e^2)^2*(d + e*x)) - (2*b*ArcTanh[c*x])/(e*(d + e*x)^3) - (b
*c^3*Log[1 - c*x])/(e*(c*d + e)^3) + (b*c^3*Log[1 + c*x])/((c*d - e)^3*e) -
 (2*b*c^3*(3*c^2*d^2 + e^2)*Log[d + e*x])/(c^2*d^2 - e^2)^3)/6
```

fricas [B]   time = 1.70, size = 859, normalized size = 4.91

$$-\frac{2\, ac^6d^6 - 5\, bc^5d^5e - 6\, ac^4d^4e^2 + 6\, bc^3d^3e^3 + 6\, ac^2d^2e^4 - bcde^5 - 2\, ae^6 - 4\left(bc^5d^3e^3 - bc^3de^5\right)x^2 - \left(9\, bc^5d^4e^2 - 1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*c^6*d^6 - 5*b*c^5*d^5*e - 6*a*c^4*d^4*e^2 + 6*b*c^3*d^3*e^3 + 6*a
*c^2*d^2*e^4 - b*c*d*e^5 - 2*a*e^6 - 4*(b*c^5*d^3*e^3 - b*c^3*d*e^5)*x^2 -
(9*b*c^5*d^4*e^2 - 10*b*c^3*d^2*e^4 + b*c*e^6)*x - (b*c^6*d^6 + 3*b*c^5*d^5
*e + 3*b*c^4*d^4*e^2 + b*c^3*d^3*e^3 + (b*c^6*d^3*e^3 + 3*b*c^5*d^2*e^4 + 3
*b*c^4*d*e^5 + b*c^3*e^6)*x^3 + 3*(b*c^6*d^4*e^2 + 3*b*c^5*d^3*e^3 + 3*b*c^
4*d^2*e^4 + b*c^3*d*e^5)*x^2 + 3*(b*c^6*d^5*e + 3*b*c^5*d^4*e^2 + 3*b*c^4*d
^3*e^3 + b*c^3*d^2*e^4)*x)*log(c*x + 1) + (b*c^6*d^6 - 3*b*c^5*d^5*e + 3*b*
c^4*d^4*e^2 - b*c^3*d^3*e^3 + (b*c^6*d^3*e^3 - 3*b*c^5*d^2*e^4 + 3*b*c^4*d*
e^5 - b*c^3*e^6)*x^3 + 3*(b*c^6*d^4*e^2 - 3*b*c^5*d^3*e^3 + 3*b*c^4*d^2*e^4
 - b*c^3*d*e^5)*x^2 + 3*(b*c^6*d^5*e - 3*b*c^5*d^4*e^2 + 3*b*c^4*d^3*e^3 -
b*c^3*d^2*e^4)*x)*log(c*x - 1) + 2*(3*b*c^5*d^5*e + b*c^3*d^3*e^3 + (3*b*c^
5*d^2*e^4 + b*c^3*e^6)*x^3 + 3*(3*b*c^5*d^3*e^3 + b*c^3*d*e^5)*x^2 + 3*(3*b
*c^5*d^4*e^2 + b*c^3*d^2*e^4)*x)*log(e*x + d) + (b*c^6*d^6 - 3*b*c^4*d^4*e^
2 + 3*b*c^2*d^2*e^4 - b*e^6)*log(-(c*x + 1)/(c*x - 1)))/(c^6*d^9*e - 3*c^4*
d^7*e^3 + 3*c^2*d^5*e^5 - d^3*e^7 + (c^6*d^6*e^4 - 3*c^4*d^4*e^6 + 3*c^2*d^
2*e^8 - e^10)*x^3 + 3*(c^6*d^7*e^3 - 3*c^4*d^5*e^5 + 3*c^2*d^3*e^7 - d*e^9)
*x^2 + 3*(c^6*d^8*e^2 - 3*c^4*d^6*e^4 + 3*c^2*d^4*e^6 - d^2*e^8)*x)
```

giac [B]   time = 0.24, size = 3320, normalized size = 18.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(c*x + 1)^3*b*c^7*d^5*log(-(c*x + 1)*c*d/(c*x - 1) + c*d - (c*x + 1
)*e/(c*x - 1) - e)/(c*x - 1)^3 - 9*(c*x + 1)^2*b*c^7*d^5*log(-(c*x + 1)*c*d
/(c*x - 1) + c*d - (c*x + 1)*e/(c*x - 1) - e)/(c*x - 1)^2 + 9*(c*x + 1)*b*c
^7*d^5*log(-(c*x + 1)*c*d/(c*x - 1) + c*d - (c*x + 1)*e/(c*x - 1) - e)/(c*x
 - 1) - 3*b*c^7*d^5*log(-(c*x + 1)*c*d/(c*x - 1) + c*d - (c*x + 1)*e/(c*x -
 1) - e) - 3*(c*x + 1)^3*b*c^7*d^5*log(-(c*x + 1)/(c*x - 1))/(c*x - 1)^3 +
6*(c*x + 1)^2*b*c^7*d^5*log(-(c*x + 1)/(c*x - 1))/(c*x - 1)^2 - 3*(c*x + 1)
*b*c^7*d^5*log(-(c*x + 1)/(c*x - 1))/(c*x - 1) - 6*(c*x + 1)^2*a*c^7*d^5/(c
*x - 1)^2 + 12*(c*x + 1)*a*c^7*d^5/(c*x - 1) - 6*a*c^7*d^5 + 9*(c*x + 1)^3*
b*c^6*d^4*e*log(-(c*x + 1)*c*d/(c*x - 1) + c*d - (c*x + 1)*e/(c*x - 1) - e)
/(c*x - 1)^3 - 9*(c*x + 1)^2*b*c^6*d^4*e*log(-(c*x + 1)*c*d/(c*x - 1) + c*d
 - (c*x + 1)*e/(c*x - 1) - e)/(c*x - 1)^2 - 9*(c*x + 1)*b*c^6*d^4*e*log(-(c
*x + 1)*c*d/(c*x - 1) + c*d - (c*x + 1)*e/(c*x - 1) - e)/(c*x - 1) + 9*b*c^
6*d^4*e*log(-(c*x + 1)*c*d/(c*x - 1) + c*d - (c*x + 1)*e/(c*x - 1) - e) - 9
*(c*x + 1)^3*b*c^6*d^4*e*log(-(c*x + 1)/(c*x - 1))/(c*x - 1)^3 + 12*(c*x +
1)^2*b*c^6*d^4*e*log(-(c*x + 1)/(c*x - 1))/(c*x - 1)^2 - 3*(c*x + 1)*b*c^6*
d^4*e*log(-(c*x + 1)/(c*x - 1))/(c*x - 1) + 6*(c*x + 1)^2*a*c^6*d^4*e/(c*x
 - 1)^2 - 24*(c*x + 1)*a*c^6*d^4*e/(c*x - 1) + 18*a*c^6*d^4*e + 6*(c*x + 1)^
2*b*c^6*d^4*e/(c*x - 1)^2 - 12*(c*x + 1)*b*c^6*d^4*e/(c*x - 1) + 6*b*c^6*d^
4*e + 10*(c*x + 1)^3*b*c^5*d^3*e^2*log(-(c*x + 1)*c*d/(c*x - 1) + c*d - (c*
```

```
x + 1)*e/(c*x - 1) - e)/(c*x - 1)^3 + 6*(c*x + 1)^2*b*c^5*d^3*e^2*log(-(c*x
 + 1)*c*d/(c*x - 1) + c*d - (c*x + 1)*e/(c*x - 1) - e)/(c*x - 1)^2 - 6*(c*x
 + 1)*b*c^5*d^3*e^2*log(-(c*x + 1)*c*d/(c*x - 1) + c*d - (c*x + 1)*e/(c*x -
 1) - e)/(c*x - 1) - 10*b*c^5*d^3*e^2*log(-(c*x + 1)*c*d/(c*x - 1) + c*d -
(c*x + 1)*e/(c*x - 1) - e) - 10*(c*x + 1)^3*b*c^5*d^3*e^2*log(-(c*x + 1)/(c
*x - 1))/(c*x - 1)^3 + 6*(c*x + 1)*b*c^5*d^3*e^2*log(-(c*x + 1)/(c*x - 1))/
(c*x - 1) + 12*(c*x + 1)^2*a*c^5*d^3*e^2/(c*x - 1)^2 - 20*a*c^5*d^3*e^2 + 4
*(c*x + 1)^2*b*c^5*d^3*e^2/(c*x - 1)^2 + 14*(c*x + 1)*b*c^5*d^3*e^2/(c*x -
1) - 18*b*c^5*d^3*e^2 + 6*(c*x + 1)^3*b*c^4*d^2*e^3*log(-(c*x + 1)*c*d/(c*x
 - 1) + c*d - (c*x + 1)*e/(c*x - 1) - e)/(c*x - 1)^3 + 6*(c*x + 1)^2*b*c^4*
d^2*e^3*log(-(c*x + 1)*c*d/(c*x - 1) + c*d - (c*x + 1)*e/(c*x - 1) - e)/(c*
x - 1)^2 + 6*(c*x + 1)*b*c^4*d^2*e^3*log(-(c*x + 1)*c*d/(c*x - 1) + c*d - (
c*x + 1)*e/(c*x - 1) - e)/(c*x - 1) + 6*b*c^4*d^2*e^3*log(-(c*x + 1)*c*d/(c
*x - 1) + c*d - (c*x + 1)*e/(c*x - 1) - e) - 6*(c*x + 1)^3*b*c^4*d^2*e^3*lo
g(-(c*x + 1)/(c*x - 1))/(c*x - 1)^3 - 12*(c*x + 1)^2*b*c^4*d^2*e^3*log(-(c*
x + 1)/(c*x - 1))/(c*x - 1)^2 + 6*(c*x + 1)*b*c^4*d^2*e^3*log(-(c*x + 1)/(c
*x - 1))/(c*x - 1) - 12*(c*x + 1)^2*a*c^4*d^2*e^3/(c*x - 1)^2 + 24*(c*x + 1
)*a*c^4*d^2*e^3/(c*x - 1) + 12*a*c^4*d^2*e^3 - 8*(c*x + 1)^2*b*c^4*d^2*e^3/
(c*x - 1)^2 + 10*(c*x + 1)*b*c^4*d^2*e^3/(c*x - 1) + 18*b*c^4*d^2*e^3 + 3*(
c*x + 1)^3*b*c^3*d*e^4*log(-(c*x + 1)*c*d/(c*x - 1) + c*d - (c*x + 1)*e/(c*
x - 1) - e)/(c*x - 1)^3 + 3*(c*x + 1)^2*b*c^3*d*e^4*log(-(c*x + 1)*c*d/(c*x
 - 1) + c*d - (c*x + 1)*e/(c*x - 1) - e)/(c*x - 1)^2 - 3*(c*x + 1)*b*c^3*d*
e^4*log(-(c*x + 1)*c*d/(c*x - 1) + c*d - (c*x + 1)*e/(c*x -
1) - 3*b*c^3*d*e^4*log(-(c*x + 1)*c*d/(c*x - 1) + c*d - (c*x + 1)*e/(c*x -
1) - e) - 3*(c*x + 1)^3*b*c^3*d*e^4*log(-(c*x + 1)/(c*x - 1))/(c*x - 1)^3 -
 6*(c*x + 1)^2*b*c^3*d*e^4*log(-(c*x + 1)/(c*x - 1))/(c*x - 1)^2 - 3*(c*x +
 1)*b*c^3*d*e^4*log(-(c*x + 1)/(c*x - 1))/(c*x - 1) - 6*(c*x + 1)^2*a*c^3*d
*e^4/(c*x - 1)^2 - 12*(c*x + 1)*a*c^3*d*e^4/(c*x - 1) - 6*a*c^3*d*e^4 - 4*(
c*x + 1)^2*b*c^3*d*e^4/(c*x - 1)^2 - 14*(c*x + 1)*b*c^3*d*e^4/(c*x - 1) - 6
*b*c^3*d*e^4 + (c*x + 1)^3*b*c^2*e^5*log(-(c*x + 1)*c*d/(c*x - 1) + c*d - (
c*x + 1)*e/(c*x - 1) - e)/(c*x - 1)^3 + 3*(c*x + 1)^2*b*c^2*e^5*log(-(c*x +
 1)*c*d/(c*x - 1) + c*d - (c*x + 1)*e/(c*x - 1) - e)/(c*x - 1)^2 + 3*(c*x +
 1)*b*c^2*e^5*log(-(c*x + 1)*c*d/(c*x - 1) + c*d - (c*x + 1)*e/(c*x - 1) -
e)/(c*x - 1) + b*c^2*e^5*log(-(c*x + 1)*c*d/(c*x - 1) + c*d - (c*x + 1)*e/(
c*x - 1) - e) - (c*x + 1)^3*b*c^2*e^5*log(-(c*x + 1)/(c*x - 1))/(c*x - 1)^3
 - 3*(c*x + 1)*b*c^2*e^5*log(-(c*x + 1)/(c*x - 1))/(c*x - 1) + 6*(c*x + 1)^
2*a*c^2*e^5/(c*x - 1)^2 + 2*a*c^2*e^5 + 2*(c*x + 1)^2*b*c^2*e^5/(c*x - 1)^2
 + 2*(c*x + 1)*b*c^2*e^5/(c*x - 1))*c/((c*x + 1)^3*c^9*d^9/(c*x - 1)^3 - 3*
(c*x + 1)^2*c^9*d^9/(c*x - 1)^2 + 3*(c*x + 1)*c^9*d^9/(c*x - 1) - c^9*d^9 +
 3*(c*x + 1)^3*c^8*d^8*e/(c*x - 1)^3 - 3*(c*x + 1)^2*c^8*d^8*e/(c*x - 1)^2
- 3*(c*x + 1)*c^8*d^8*e/(c*x - 1) + 3*c^8*d^8*e + 12*(c*x + 1)^2*c^7*d^7*e^
2/(c*x - 1)^2 - 12*(c*x + 1)*c^7*d^7*e^2/(c*x - 1) - 8*(c*x + 1)^3*c^6*d^6*
e^3/(c*x - 1)^3 + 12*(c*x + 1)^2*c^6*d^6*e^3/(c*x - 1)^2 + 12*(c*x + 1)*c^6
*d^6*e^3/(c*x - 1) - 8*c^6*d^6*e^3 - 6*(c*x + 1)^3*c^5*d^5*e^4/(c*x - 1)^3
- 18*(c*x + 1)^2*c^5*d^5*e^4/(c*x - 1)^2 + 18*(c*x + 1)*c^5*d^5*e^4/(c*x -
```

```
1) + 6*c^5*d^5*e^4 + 6*(c*x + 1)^3*c^4*d^4*e^5/(c*x - 1)^3 - 18*(c*x + 1)^2
*c^4*d^4*e^5/(c*x - 1)^2 - 18*(c*x + 1)*c^4*d^4*e^5/(c*x - 1) + 6*c^4*d^4*e
^5 + 8*(c*x + 1)^3*c^3*d^3*e^6/(c*x - 1)^3 + 12*(c*x + 1)^2*c^3*d^3*e^6/(c*
x - 1)^2 - 12*(c*x + 1)*c^3*d^3*e^6/(c*x - 1) - 8*c^3*d^3*e^6 + 12*(c*x + 1
)^2*c^2*d^2*e^7/(c*x - 1)^2 + 12*(c*x + 1)*c^2*d^2*e^7/(c*x - 1) - 3*(c*x +
 1)^3*c*d*e^8/(c*x - 1)^3 - 3*(c*x + 1)^2*c*d*e^8/(c*x - 1)^2 + 3*(c*x + 1)
*c*d*e^8/(c*x - 1) + 3*c*d*e^8 - (c*x + 1)^3*e^9/(c*x - 1)^3 - 3*(c*x + 1)^
2*e^9/(c*x - 1)^2 - 3*(c*x + 1)*e^9/(c*x - 1) - e^9)
```

maple $[\mathbf{A}]$ time $= 0.04$, size $= 223$, normalized size $= 1.27$

$$-\frac{c^3 a}{3\,(cxe+cd)^3\,e}-\frac{c^3 b\arctanh{(cx)}}{3\,(cxe+cd)^3\,e}+\frac{c^3 b}{6\,(cd+e)\,(cd-e)\,(cxe+cd)^2}-\frac{c^5 b\ln{(cxe+cd)}\,d^2}{(cd+e)^3\,(cd-e)^3}-\frac{c^3 b\,e^2\ln{(cxe+cd)}}{3\,(cd+e)^3\,(cd-e)^3}+\frac{}{3\,(c\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(e*x+d)^4,x)
```

```
[Out] -1/3*c^3*a/(c*e*x+c*d)^3/e-1/3*c^3*b/(c*e*x+c*d)^3/e*arctanh(c*x)+1/6*c^3*b
/(c*d+e)/(c*d-e)/(c*e*x+c*d)^2-c^5*b/(c*d+e)^3/(c*d-e)^3*ln(c*e*x+c*d)*d^2-
1/3*c^3*b*e^2/(c*d+e)^3/(c*d-e)^3*ln(c*e*x+c*d)+2/3*c^4*b*d/(c*d+e)^2/(c*d-
e)^2/(c*e*x+c*d)-1/6*c^3*b/e/(c*d+e)^3*ln(c*x-1)+1/6*b*c^3*ln(c*x+1)/(c*d-e
)^3/e
```

maxima $[\mathbf{B}]$ time $= 0.34$, size $= 339$, normalized size $= 1.94$

$$\frac{1}{6}\left(\left(\frac{c^2\log{(cx+1)}}{c^3d^3e-3\,c^2d^2e^2+3\,cde^3-e^4}-\frac{c^2\log{(cx-1)}}{c^3d^3e+3\,c^2d^2e^2+3\,cde^3+e^4}-\frac{2\left(3\,c^4d^2+c^2e^2\right)\log{(ex+d)}}{c^6d^6-3\,c^4d^4e^2+3\,c^2d^2e^4-e^6}+\frac{}{c^4d^6-2\,c^2\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 1/6*((c^2*log(c*x + 1)/(c^3*d^3*e - 3*c^2*d^2*e^2 + 3*c*d*e^3 - e^4) - c^2*
log(c*x - 1)/(c^3*d^3*e + 3*c^2*d^2*e^2 + 3*c*d*e^3 + e^4) - 2*(3*c^4*d^2 +
 c^2*e^2)*log(e*x + d)/(c^6*d^6 - 3*c^4*d^4*e^2 + 3*c^2*d^2*e^4 - e^6) + (4
*c^2*d*e*x + 5*c^2*d^2 - e^2)/(c^4*d^6 - 2*c^2*d^4*e^2 + d^2*e^4 + (c^4*d^4
*e^2 - 2*c^2*d^2*e^4 + e^6)*x^2 + 2*(c^4*d^5*e - 2*c^2*d^3*e^3 + d*e^5)*x))
*c - 2*arctanh(c*x)/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e))*b - 1/3*
a/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e)
```

mupad $[\mathbf{B}]$ time $= 2.29$, size $= 418$, normalized size $= 2.39$

$$\ln{(d+ex)}\left(\frac{b\,c^3}{6\,e\,(e+c\,d)^3}+\frac{b\,c^3}{6\,e\,(e-c\,d)^3}\right)-\frac{\frac{2\,a\,c^4\,d^4-5\,b\,c^3\,d^3\,e-4\,a\,c^2\,d^2\,e^2+b\,c\,d\,e^3+2\,a\,e^4}{2\left(c^4\,d^4-2\,c^2\,d^2\,e^2+e^4\right)}+\frac{x\left(b\,c\,e^4-9\,b\,c^3\,d^2\,e^2\right)}{2\left(c^4\,d^4-2\,c^2\,d^2\,e^2+e^4\right)}-\frac{2\,b\,c^3\,d\,e^3\,x^2}{c^4\,d^4-2\,c^2\,d^2\,e^2\cdots}}{3\,d^3\,e+9\,d^2\,e^2\,x+9\,d\,e^3\,x^2+3\,e^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x))/(d + e*x)^4,x)

[Out] log(d + e*x)*((b*c^3)/(6*e*(e + c*d)^3) + (b*c^3)/(6*e*(e - c*d)^3)) - ((2*
a*e^4 + 2*a*c^4*d^4 - 4*a*c^2*d^2*e^2 + b*c*d*e^3 - 5*b*c^3*d^3*e)/(2*(e^4
+ c^4*d^4 - 2*c^2*d^2*e^2)) + (x*(b*c*e^4 - 9*b*c^3*d^2*e^2))/(2*(e^4 + c^4
*d^4 - 2*c^2*d^2*e^2)) - (2*b*c^3*d*e^3*x^2)/(e^4 + c^4*d^4 - 2*c^2*d^2*e^2
))/(3*d^3*e + 3*e^4*x^3 + 9*d^2*e^2*x + 9*d*e^3*x^2) - (b*c^3*log(c*x - 1))
/(6*e^4 + 6*c^3*d^3*e + 18*c^2*d^2*e^2 + 18*c*d*e^3) - (b*c^3*log(c*x + 1))
/(6*e^4 - 6*c^3*d^3*e + 18*c^2*d^2*e^2 - 18*c*d*e^3) - (b*log(c*x + 1))/(6*
e*(d^3 + e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x)) + (b*log(1 - c*x))/(3*e*(2*d^3
 + 2*e^3*x^3 + 6*d*e^2*x^2 + 6*d^2*e*x))

sympy [A]   time = 12.91, size = 10946, normalized size = 62.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x))/(e*x+d)**4,x)

[Out] Piecewise((a*x/d**4, Eq(c, 0) & Eq(e, 0)), (-24*a*d**3/(72*d**6*e + 216*d**
5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) + 21*b*d**3*atanh(e*x/d)
/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) + 1
0*b*d**3/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x
3) - 9*b*d2*e*x*atanh(e*x/d)/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e*
*3*x**2 + 72*d**3*e**4*x**3) + 9*b*d**2*e*x/(72*d**6*e + 216*d**5*e**2*x +
216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) - 9*b*d*e**2*x**2*atanh(e*x/d)/(72*
d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) + 3*b*d*
e**2*x**2/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*
x**3) - 3*b*e**3*x**3*atanh(e*x/d)/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*
e**3*x**2 + 72*d**3*e**4*x**3), Eq(c, -e/d)), (-24*a*d**3/(72*d**6*e + 216*
d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) - 21*b*d**3*atanh(e*x
/d)/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3)
- 10*b*d**3/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**
4*x**3) + 9*b*d**2*e*x*atanh(e*x/d)/(72*d**6*e + 216*d**5*e**2*x + 216*d**4
*e**3*x**2 + 72*d**3*e**4*x**3) - 9*b*d**2*e*x/(72*d**6*e + 216*d**5*e**2*x
 + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) + 9*b*d*e**2*x**2*atanh(e*x/d)/(
72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e**4*x**3) - 3*b
*d*e**2*x**2/(72*d**6*e + 216*d**5*e**2*x + 216*d**4*e**3*x**2 + 72*d**3*e*
*4*x**3) + 3*b*e**3*x**3*atanh(e*x/d)/(72*d**6*e + 216*d**5*e**2*x + 216*d*
*4*e**3*x**2 + 72*d**3*e**4*x**3), Eq(c, e/d)), ((a*x + b*x*atanh(c*x) + b*
log(x - 1/c)/c + b*atanh(c*x)/c)/d**4, Eq(e, 0)), (-a/(3*d**3*e + 9*d**2*e*
*2*x + 9*d*e**3*x**2 + 3*e**4*x**3), Eq(c, 0)), (-2*a*c**6*d**6/(6*c**6*d**
9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3

$$- 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d**5*e**5*x**2 - 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 - 18*d**2*e**8*x - 18*d*e**9*x**2 - 6*e**10*x**3) + 6*a*c**4*d**4*e**2/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d**5*e**5*x**2 - 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 - 18*d**2*e**8*x - 18*d*e**9*x**2 - 6*e**10*x**3) - 6*a*c**2*d**2*e**4/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d**5*e**5*x**2 - 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 - 18*d**2*e**8*x - 18*d*e**9*x**2 - 6*e**10*x**3) + 2*a*e**6/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d**5*e**5*x**2 - 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 - 18*d**2*e**8*x - 18*d*e**9*x**2 - 6*e**10*x**3) + 6*b*c**6*d**5*e*x*atanh(c*x)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d**5*e**5*x**2 - 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 - 18*d**2*e**8*x - 18*d*e**9*x**2 - 6*e**10*x**3) + 6*b*c**6*d**4*e**2*x**2*atanh(c*x)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d**5*e**5*x**2 - 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 - 18*d**2*e**8*x - 18*d*e**9*x**2 - 6*e**10*x**3) + 2*b*c**6*d**3*e**3*x**3*atanh(c*x)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d**5*e**5*x**2 - 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 - 18*d**2*e**8*x - 18*d*e**9*x**2 - 6*e**10*x**3) + 6*b*c**5*d**5*e*log(x - 1/c)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d**5*e**5*x**2 - 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 - 18*d**2*e**8*x - 18*d*e**9*x**2 - 6*e**10*x**3) - 6*b*c**5*d**5*e*log(d/e + x)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d**5*e**5*x**2 - 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 - 18*d**2*e**8*x - 18*d*e**9*x**2 - 6*e**10*x**3) + 6*b*c**5*d**5*e*atanh(c*x)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d**5*e**5*x**2 - 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**$$

2*d**4*e**6*x + 54*c**2*d**3e**7*x**2 + 18*c**2*d**2e**8*x**3 − 6*d**3*e*
*7 − 18*d**2*e**8*x − 18*d*e**9*x**2 − 6*e**10*x**3) + 5*b*c**5*d**5*e/(6*c
6*d9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**
4*x**3 − 18*c**4*d**7*e**3 − 54*c**4*d**6*e**4*x − 54*c**4*d**5*e**5*x**2 −
 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2
*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 − 6*d**3*e**7 − 18*d**2*e**8*x − 1
8*d*e**9*x**2 − 6*e**10*x**3) + 18*b*c**5*d**4*e**2*x*log(x − 1/c)/(6*c**6*
d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x*
*3 − 18*c**4*d**7*e**3 − 54*c**4*d**6*e**4*x − 54*c**4*d**5*e**5*x**2 − 18*
c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**
3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 − 6*d**3*e**7 − 18*d**2*e**8*x − 18*d*
e**9*x**2 − 6*e**10*x**3) − 18*b*c**5*d**4*e**2*x*log(d/e + x)/(6*c**6*d**9
*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 −
 18*c**4*d**7*e**3 − 54*c**4*d**6*e**4*x − 54*c**4*d**5*e**5*x**2 − 18*c**4
*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3e*
*7*x**2 + 18*c**2*d**2*e**8*x**3 − 6*d**3*e**7 − 18*d**2*e**8*x − 18*d*e**9
*x**2 − 6*e**10*x**3) + 18*b*c**5*d**4*e**2*x*atanh(c*x)/(6*c**6*d**9*e + 1
8*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 − 18*c*
*4*d**7*e**3 − 54*c**4*d**6*e**4*x − 54*c**4*d**5*e**5*x**2 − 18*c**4*d**4*
e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**
2 + 18*c**2*d**2*e**8*x**3 − 6*d**3*e**7 − 18*d**2*e**8*x − 18*d*e**9*x**2
− 6*e**10*x**3) + 9*b*c**5*d**4*e**2*x/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x
 + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 − 18*c**4*d**7*e**3 − 54*
c**4*d**6*e**4*x − 54*c**4*d**5*e**5*x**2 − 18*c**4*d**4*e**6*x**3 + 18*c**
2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e
8*x3 − 6*d**3*e**7 − 18*d**2*e**8*x − 18*d*e**9*x**2 − 6*e**10*x**3) +
18*b*c**5*d**3*e**3*x**2*log(x − 1/c)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x
+ 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 − 18*c**4*d**7*e**3 − 54*c
4*d6*e**4*x − 54*c**4*d**5*e**5*x**2 − 18*c**4*d**4*e**6*x**3 + 18*c**2
*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e*
*8*x**3 − 6*d**3*e**7 − 18*d**2*e**8*x − 18*d*e**9*x**2 − 6*e**10*x**3) − 1
8*b*c**5*d**3*e**3*x**2*log(d/e + x)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x +
 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 − 18*c**4*d**7*e**3 − 54*c*
*4*d**6*e**4*x − 54*c**4*d**5*e**5*x**2 − 18*c**4*d**4*e**6*x**3 + 18*c**2*
d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**
8*x**3 − 6*d**3*e**7 − 18*d**2*e**8*x − 18*d*e**9*x**2 − 6*e**10*x**3) + 18
*b*c**5*d**3*e**3*x**2*atanh(c*x)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18
*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 − 18*c**4*d**7*e**3 − 54*c**4*
d**6*e**4*x − 54*c**4*d**5*e**5*x**2 − 18*c**4*d**4*e**6*x**3 + 18*c**2*d**
5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x
3 − 6*d3*e**7 − 18*d**2*e**8*x − 18*d*e**9*x**2 − 6*e**10*x**3) + 4*b*c
5*d3*e**3*x**2/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3
*x**2 + 6*c**6*d**6*e**4*x**3 − 18*c**4*d**7*e**3 − 54*c**4*d**6*e**4*x − 5
4*c**4*d**5*e**5*x**2 − 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**
2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 − 6*d**3*e*

```
*7 - 18*d**2*e**8*x - 18*d*e**9*x**2 - 6*e**10*x**3) + 6*b*c**5*d**2*e**4*x
**3*log(x - 1/c)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x
**2 + 6*c**6*d**6*e**4*x**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*
c**4*d**5*e**5*x**2 - 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*
d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7
 - 18*d**2*e**8*x - 18*d*e**9*x**2 - 6*e**10*x**3) - 6*b*c**5*d**2*e**4*x**
3*log(d/e + x)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**
2 + 6*c**6*d**6*e**4*x**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c*
*4*d**5*e**5*x**2 - 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d*
*4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 -
 18*d**2*e**8*x - 18*d*e**9*x**2 - 6*e**10*x**3) + 6*b*c**5*d**2*e**4*x**3*
atanh(c*x)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 +
6*c**6*d**6*e**4*x**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d
**5*e**5*x**2 - 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e
**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 - 18*
d**2*e**8*x - 18*d*e**9*x**2 - 6*e**10*x**3) + 12*b*c**4*d**4*e**2*atanh(c*
x)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d
**6*e**4*x**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d**5*e**5
*x**2 - 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x +
54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 - 18*d**2*e**
8*x - 18*d*e**9*x**2 - 6*e**10*x**3) + 18*b*c**4*d**3*e**3*x*atanh(c*x)/(6*
c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e*
*4*x**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d**5*e**5*x**2
- 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**
2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 - 18*d**2*e**8*x -
18*d*e**9*x**2 - 6*e**10*x**3) + 18*b*c**4*d**2*e**4*x**2*atanh(c*x)/(6*c**
6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*
x**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d**5*e**5*x**2 - 1
8*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d
**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 - 18*d**2*e**8*x - 18*
d*e**9*x**2 - 6*e**10*x**3) + 6*b*c**4*d*e**5*x**3*atanh(c*x)/(6*c**6*d**9*
e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 -
18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d**5*e**5*x**2 - 18*c**4*
d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**
7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 - 18*d**2*e**8*x - 18*d*e**9*
x**2 - 6*e**10*x**3) + 2*b*c**3*d**3*e**3*log(x - 1/c)/(6*c**6*d**9*e + 18*
c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 - 18*c**4
*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d**5*e**5*x**2 - 18*c**4*d**4*e*
*6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2
+ 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 - 18*d**2*e**8*x - 18*d*e**9*x**2 -
6*e**10*x**3) - 2*b*c**3*d**3*e**3*log(d/e + x)/(6*c**6*d**9*e + 18*c**6*d*
*8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 - 18*c**4*d**7*e
**3 - 54*c**4*d**6*e**4*x - 54*c**4*d**5*e**5*x**2 - 18*c**4*d**4*e**6*x**3
 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c*
*2*d**2*e**8*x**3 - 6*d**3*e**7 - 18*d**2*e**8*x - 18*d*e**9*x**2 - 6*e**10
```

*x**3) + 2*b*c**3*d**3*e**3*atanh(c*x)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x
 + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 - 18*c**4*d**7*e**3 - 54*
c**4*d**6*e**4*x - 54*c**4*d**5*e**5*x**2 - 18*c**4*d**4*e**6*x**3 + 18*c**
2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e
8*x3 - 6*d**3*e**7 - 18*d**2*e**8*x - 18*d*e**9*x**2 - 6*e**10*x**3) -
6*b*c**3*d**3*e**3/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3
*x**2 + 6*c**6*d**6*e**4*x**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 5
4*c**4*d**5*e**5*x**2 - 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**
2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e*
*7 - 18*d**2*e**8*x - 18*d*e**9*x**2 - 6*e**10*x**3) + 6*b*c**3*d**2*e**4*x
*log(x - 1/c)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2
 + 6*c**6*d**6*e**4*x**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**
4*d**5*e**5*x**2 - 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**
4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 -
18*d**2*e**8*x - 18*d*e**9*x**2 - 6*e**10*x**3) - 6*b*c**3*d**2*e**4*x*log(
d/e + x)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*
c**6*d**6*e**4*x**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d**
5*e**5*x**2 - 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**
6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 - 18*d*
*2*e**8*x - 18*d*e**9*x**2 - 6*e**10*x**3) + 6*b*c**3*d**2*e**4*x*atanh(c*x
)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d*
*6*e**4*x**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d**5*e**5*
x**2 - 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 5
4*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 - 18*d**2*e**8
*x - 18*d*e**9*x**2 - 6*e**10*x**3) - 10*b*c**3*d**2*e**4*x/(6*c**6*d**9*e
+ 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 - 18
*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d**5*e**5*x**2 - 18*c**4*d*
*4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*
x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 - 18*d**2*e**8*x - 18*d*e**9*x*
*2 - 6*e**10*x**3) + 6*b*c**3*d*e**5*x**2*log(x - 1/c)/(6*c**6*d**9*e + 18*
c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 - 18*c**4
*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d**5*e**5*x**2 - 18*c**4*d**4*e*
*6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2
+ 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 - 18*d**2*e**8*x - 18*d*e**9*x**2 -
6*e**10*x**3) - 6*b*c**3*d*e**5*x**2*log(d/e + x)/(6*c**6*d**9*e + 18*c**6*
d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 - 18*c**4*d**7
*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d**5*e**5*x**2 - 18*c**4*d**4*e**6*x*
*3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*
c**2*d**2*e**8*x**3 - 6*d**3*e**7 - 18*d**2*e**8*x - 18*d*e**9*x**2 - 6*e**
10*x**3) + 6*b*c**3*d*e**5*x**2*atanh(c*x)/(6*c**6*d**9*e + 18*c**6*d**8*e*
*2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 - 18*c**4*d**7*e**3 -
 54*c**4*d**6*e**4*x - 54*c**4*d**5*e**5*x**2 - 18*c**4*d**4*e**6*x**3 + 18
*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d*
*2*e**8*x**3 - 6*d**3*e**7 - 18*d**2*e**8*x - 18*d*e**9*x**2 - 6*e**10*x**3
) - 4*b*c**3*d*e**5*x**2/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**

```
7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4
*x - 54*c**4*d**5*e**5*x**2 - 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 +
54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d
**3*e**7 - 18*d**2*e**8*x - 18*d*e**9*x**2 - 6*e**10*x**3) + 2*b*c**3*e**6*
x**3*log(x - 1/c)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*
x**2 + 6*c**6*d**6*e**4*x**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54
*c**4*d**5*e**5*x**2 - 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2
*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**
7 - 18*d**2*e**8*x - 18*d*e**9*x**2 - 6*e**10*x**3) - 2*b*c**3*e**6*x**3*lo
g(d/e + x)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 +
6*c**6*d**6*e**4*x**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d
**5*e**5*x**2 - 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e
**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 - 18*
d**2*e**8*x - 18*d*e**9*x**2 - 6*e**10*x**3) + 2*b*c**3*e**6*x**3*atanh(c*x
)/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d*
*6*e**4*x**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d**5*e**5*
x**2 - 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 5
4*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 - 18*d**2*e**8
*x - 18*d*e**9*x**2 - 6*e**10*x**3) - 6*b*c**2*d**2*e**4*atanh(c*x)/(6*c**6
*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x
**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d**5*e**5*x**2 - 18
*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d*
*3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 - 18*d**2*e**8*x - 18*d
*e**9*x**2 - 6*e**10*x**3) + b*c*d*e**5/(6*c**6*d**9*e + 18*c**6*d**8*e**2*
x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x**3 - 18*c**4*d**7*e**3 - 54
*c**4*d**6*e**4*x - 54*c**4*d**5*e**5*x**2 - 18*c**4*d**4*e**6*x**3 + 18*c*
*2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*
e**8*x**3 - 6*d**3*e**7 - 18*d**2*e**8*x - 18*d*e**9*x**2 - 6*e**10*x**3) +
 b*c*e**6*x/(6*c**6*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 +
 6*c**6*d**6*e**4*x**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*
d**5*e**5*x**2 - 18*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*
e**6*x + 54*c**2*d**3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 - 18
*d**2*e**8*x - 18*d*e**9*x**2 - 6*e**10*x**3) + 2*b*e**6*atanh(c*x)/(6*c**6
*d**9*e + 18*c**6*d**8*e**2*x + 18*c**6*d**7*e**3*x**2 + 6*c**6*d**6*e**4*x
**3 - 18*c**4*d**7*e**3 - 54*c**4*d**6*e**4*x - 54*c**4*d**5*e**5*x**2 - 18
*c**4*d**4*e**6*x**3 + 18*c**2*d**5*e**5 + 54*c**2*d**4*e**6*x + 54*c**2*d*
*3*e**7*x**2 + 18*c**2*d**2*e**8*x**3 - 6*d**3*e**7 - 18*d**2*e**8*x - 18*d
*e**9*x**2 - 6*e**10*x**3), True))
```

3.9 $\int (d + ex)^3 \left(a + b \tanh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=359

$$-\frac{\left(c^4d^4 + 6c^2d^2e^2 + e^4\right)\left(a + b\tanh^{-1}(cx)\right)^2}{4c^4e} + \frac{abex\left(6c^2d^2 + e^2\right)}{2c^3} + \frac{d\left(c^2d^2 + e^2\right)\left(a + b\tanh^{-1}(cx)\right)^2}{c^3} - \frac{2bd\left(c^2d^2 + \right.}{}$$

```
[Out] b^2*d*e^2*x/c^2+1/2*a*b*e*(6*c^2*d^2+e^2)*x/c^3+1/12*b^2*e^3*x^2/c^2-b^2*d*
e^2*arctanh(c*x)/c^3+1/2*b^2*e*(6*c^2*d^2+e^2)*x*arctanh(c*x)/c^3+b*d*e^2*x
^2*(a+b*arctanh(c*x))/c+1/6*b*e^3*x^3*(a+b*arctanh(c*x))/c+d*(c^2*d^2+e^2)*
(a+b*arctanh(c*x))^2/c^3-1/4*(c^4*d^4+6*c^2*d^2*e^2+e^4)*(a+b*arctanh(c*x))
^2/c^4/e+1/4*(e*x+d)^4*(a+b*arctanh(c*x))^2/e-2*b*d*(c^2*d^2+e^2)*(a+b*arct
anh(c*x))*ln(2/(-c*x+1))/c^3+1/12*b^2*e^3*ln(-c^2*x^2+1)/c^4+1/4*b^2*e*(6*c
^2*d^2+e^2)*ln(-c^2*x^2+1)/c^4-b^2*d*(c^2*d^2+e^2)*polylog(2,1-2/(-c*x+1))/
c^3
```

Rubi [A]   time = 0.53, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {5928, 5910, 260, 5916, 321, 206, 266, 43, 6048, 5948, 5984, 5918, 2402, 2315}

$$-\frac{b^2d\left(c^2d^2 + e^2\right)\text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)}{c^3} + \frac{abex\left(6c^2d^2 + e^2\right)}{2c^3} + \frac{d\left(c^2d^2 + e^2\right)\left(a + b\tanh^{-1}(cx)\right)^2}{c^3} - \frac{\left(6c^2d^2e^2 + c^4d^4 + \right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (b^2*d*e^2*x)/c^2 + (a*b*e*(6*c^2*d^2 + e^2)*x)/(2*c^3) + (b^2*e^3*x^2)/(12
*c^2) - (b^2*d*e^2*ArcTanh[c*x])/c^3 + (b^2*e*(6*c^2*d^2 + e^2)*x*ArcTanh[c
*x])/(2*c^3) + (b*d*e^2*x^2*(a + b*ArcTanh[c*x]))/c + (b*e^3*x^3*(a + b*Arc
Tanh[c*x]))/(6*c) + (d*(c^2*d^2 + e^2)*(a + b*ArcTanh[c*x])^2)/c^3 - ((c^4*
d^4 + 6*c^2*d^2*e^2 + e^4)*(a + b*ArcTanh[c*x])^2)/(4*c^4*e) + ((d + e*x)^4
*(a + b*ArcTanh[c*x])^2)/(4*e) - (2*b*d*(c^2*d^2 + e^2)*(a + b*ArcTanh[c*x]
)*Log[2/(1 - c*x)])/c^3 + (b^2*e^3*Log[1 - c^2*x^2])/(12*c^4) + (b^2*e*(6*c
^2*d^2 + e^2)*Log[1 - c^2*x^2])/(4*c^4) - (b^2*d*(c^2*d^2 + e^2)*PolyLog[2,
 1 - 2/(1 - c*x)])/c^3
```

<u>Rule 43</u>

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
p)/(d(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]

Rule 5918

Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]

Rule 5928

Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]

Rule 5948

Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]

Rule 5984

Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]

Rule 6048

Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]

Rubi steps

$$\int (d + ex)^3 \left(a + b \tanh^{-1}(cx)\right)^2 \, dx = \frac{(d + ex)^4 \left(a + b \tanh^{-1}(cx)\right)^2}{4e} - \frac{(bc) \int \left(-\frac{e^2\left(6c^2d^2+e^2\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4} - \frac{4de^3x\left(a+b\right.}{}\right)}{}$$

$$= \frac{(d + ex)^4 \left(a + b \tanh^{-1}(cx)\right)^2}{4e} - \frac{b \int \frac{\left(c^4d^4+6c^2d^2e^2+e^4+4c^2de\left(c^2d^2+e^2\right)x\right)\left(a+b\tanh^{-1}(c\right.}{1-c^2x^2}}{2c^3e}$$

$$= \frac{abe\left(6c^2d^2 + e^2\right)x}{2c^3} + \frac{bde^2x^2\left(a + b\tanh^{-1}(cx)\right)}{c} + \frac{be^3x^3\left(a + b\tanh^{-1}(cx)\right)}{6c}$$

$$= \frac{b^2de^2x}{c^2} + \frac{abe\left(6c^2d^2 + e^2\right)x}{2c^3} + \frac{b^2e\left(6c^2d^2 + e^2\right)x\tanh^{-1}(cx)}{2c^3} + \frac{bde^2x^2\left(a + \right.}{}$$

$$= \frac{b^2de^2x}{c^2} + \frac{abe\left(6c^2d^2 + e^2\right)x}{2c^3} - \frac{b^2de^2\tanh^{-1}(cx)}{c^3} + \frac{b^2e\left(6c^2d^2 + e^2\right)x\tanh^{-}}{2c^3}$$

$$= \frac{b^2de^2x}{c^2} + \frac{abe\left(6c^2d^2 + e^2\right)x}{2c^3} + \frac{b^2e^3x^2}{12c^2} - \frac{b^2de^2\tanh^{-1}(cx)}{c^3} + \frac{b^2e\left(6c^2d^2 + e^2\right.}{2}$$

$$= \frac{b^2de^2x}{c^2} + \frac{abe\left(6c^2d^2 + e^2\right)x}{2c^3} + \frac{b^2e^3x^2}{12c^2} - \frac{b^2de^2\tanh^{-1}(cx)}{c^3} + \frac{b^2e\left(6c^2d^2 + e^2\right.}{2}$$

$$= \frac{b^2de^2x}{c^2} + \frac{abe\left(6c^2d^2 + e^2\right)x}{2c^3} + \frac{b^2e^3x^2}{12c^2} - \frac{b^2de^2\tanh^{-1}(cx)}{c^3} + \frac{b^2e\left(6c^2d^2 + e^2\right.}{2}$$

Mathematica [A]   time = 0.93, size = 506, normalized size = 1.41

$$12a^2c^4d^3x + 18a^2c^4d^2ex^2 + 12a^2c^4de^2x^3 + 3a^2c^4e^3x^4 + 36abc^3d^2ex + 12abc^3de^2x^2 + 2abc^3e^3x^3 + 18abc^2d^2e\log(1$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^3*(a + b*ArcTanh[c*x])^2,x]

[Out] (-(b^2*e^3) + 12*a^2*c^4*d^3*x + 36*a*b*c^3*d^2*e*x + 12*b^2*c^2*d*e^2*x +
6*a*b*c*e^3*x + 18*a^2*c^4*d^2*e*x^2 + 12*a*b*c^3*d*e^2*x^2 + b^2*c^2*e^3*x
^2 + 12*a^2*c^4*d*e^2*x^3 + 2*a*b*c^3*e^3*x^3 + 3*a^2*c^4*e^3*x^4 + 3*b^2*(
-4*c^3*d^3 - 6*c^2*d^2*e - 4*c*d*e^2 - e^3 + c^4*x*(4*d^3 + 6*d^2*e*x + 4*d
*e^2*x^2 + e^3*x^3))*ArcTanh[c*x]^2 + 2*b*c*ArcTanh[c*x]*(3*a*c^3*x*(4*d^3
+ 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + b*e*(18*c^2*d^2*x + 6*d*e*(-1 + c^2*
```

```
x^2) + e^2*x*(3 + c^2*x^2)) - 12*b*d*(c^2*d^2 + e^2)*Log[1 + E^(-2*ArcTanh[
c*x])]) + 18*a*b*c^2*d^2*e*Log[1 - c*x] + 3*a*b*e^3*Log[1 - c*x] - 18*a*b*c
^2*d^2*e*Log[1 + c*x] - 3*a*b*e^3*Log[1 + c*x] + 12*a*b*c^3*d^3*Log[1 - c^2
*x^2] + 18*b^2*c^2*d^2*e*Log[1 - c^2*x^2] + 4*b^2*e^3*Log[1 - c^2*x^2] + 12
*a*b*c*d*e^2*Log[-1 + c^2*x^2] + 12*b^2*c*d*(c^2*d^2 + e^2)*PolyLog[2, -E^(
-2*ArcTanh[c*x])])/(12*c^4)
```

__fricas__ **[F]**   time $= 1.11$, size $= 0$, normalized size $= 0.00$

$$\text{integral} \left(a^2 e^3 x^3 + 3\,a^2 d e^2 x^2 + 3\,a^2 d^2 e x + a^2 d^3 + \left(b^2 e^3 x^3 + 3\,b^2 d e^2 x^2 + 3\,b^2 d^2 e x + b^2 d^3 \right) \text{artanh}\,(cx)^2 + 2\,\left(a b e^3 \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="fricas")

[Out] integral(a^2*e^3*x^3 + 3*a^2*d*e^2*x^2 + 3*a^2*d^2*e*x + a^2*d^3 + (b^2*e^3
*x^3 + 3*b^2*d*e^2*x^2 + 3*b^2*d^2*e*x + b^2*d^3)*arctanh(c*x)^2 + 2*(a*b*e
^3*x^3 + 3*a*b*d*e^2*x^2 + 3*a*b*d^2*e*x + a*b*d^3)*arctanh(c*x), x)

__giac__ **[F(-1)]**   time $= 0.00$, size $= 0$, normalized size $= 0.00$

<div align="center">Timed out</div>

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="giac")

[Out] Timed out

__maple__ **[B]**   time $= 0.07$, size $= 1430$, normalized size $= 3.98$

<div align="center">result too large to display</div>

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^3*(a+b*arctanh(c*x))^2,x)

[Out] 1/4*a^2/e*d^4+a^2*e^2*x^3*d+3/2*a^2*e*x^2*d^2+b^2*arctanh(c*x)^2*x*d^3+1/4*
b^2/e*arctanh(c*x)^2*d^4+1/16*b^2/e*ln(c*x-1)^2*d^4+1/16*b^2/e*ln(c*x+1)^2*
d^4+1/4*b^2*e^3*arctanh(c*x)^2*x^4+1/16/c^4*b^2*e^3*ln(c*x+1)^2+1/3/c^4*b^2
*e^3*ln(c*x-1)+1/3/c^4*b^2*e^3*ln(c*x+1)+1/16/c^4*b^2*e^3*ln(c*x-1)^2+1/4/c
*b^2*ln(c*x-1)^2*d^3-1/c*b^2*dilog(1/2+1/2*c*x)*d^3-1/4/c*b^2*ln(c*x+1)^2*d
^3-1/2/c*b^2*ln(1/2+1/2*c*x)*ln(-1/2*c*x+1/2)*d^3-1/2/c*b^2*ln(c*x-1)*ln(1/
2+1/2*c*x)*d^3+1/2*a*b/e*arctanh(c*x)*d^4+1/4*a*b/e*ln(c*x-1)*d^4-1/4*a*b/e
*ln(c*x+1)*d^4-1/8*b^2/e*ln(c*x+1)*ln(-1/2*c*x+1/2)*d^4+1/8*b^2/e*ln(1/2+1/
2*c*x)*ln(-1/2*c*x+1/2)*d^4+1/c*b^2*arctanh(c*x)*ln(c*x-1)*d^3+1/2*a*b*e^3*
arctanh(c*x)*x^4+1/2/c*b^2*ln(c*x+1)*ln(-1/2*c*x+1/2)*d^3+1/6/c*b^2*e^3*arc
```

```
tanh(c*x)*x^3+1/c*b^2*arctanh(c*x)*ln(c*x+1)*d^3+1/2*a*b/c^3*x*e^3+1/c*a*b*
ln(c*x+1)*d^3+1/c*a*b*ln(c*x-1)*d^3+1/4/c^3*b^2*e^2*ln(c*x-1)^2*d-1/c^3*b^2
*e^2*dilog(1/2+1/2*c*x)*d+1/2/c^3*b^2*e^3*arctanh(c*x)*x-1/4/c^3*b^2*e^2*ln
(c*x+1)^2*d+3/8/c^2*b^2*e*ln(c*x+1)^2*d^2-1/4/c^4*a*b*e^3*ln(c*x+1)+1/4/c^4
*a*b*e^3*ln(c*x-1)+3/2/c^2*b^2*e*ln(c*x-1)*d^2+3/2/c^2*b^2*e*ln(c*x+1)*d^2+
1/2/c^3*b^2*e^2*ln(c*x-1)*d-1/2/c^3*b^2*e^2*ln(c*x+1)*d+b^2*e^2*arctanh(c*x
)^2*x^3*d+3/2*b^2*e*arctanh(c*x)^2*x^2*d^2+1/12*b^2*e^3*x^2/c^2-1/8/c^4*b^2
*e^3*ln(c*x+1)*ln(-1/2*c*x+1/2)+1/8/c^4*b^2*e^3*ln(1/2+1/2*c*x)*ln(-1/2*c*x
+1/2)+1/4/c^4*b^2*e^3*arctanh(c*x)*ln(c*x-1)-1/4/c^4*b^2*e^3*arctanh(c*x)*l
n(c*x+1)-1/8/c^4*b^2*e^3*ln(c*x-1)*ln(1/2+1/2*c*x)+1/6/c*a*b*x^3*e^3+3/8/c^
2*b^2*e*ln(c*x-1)^2*d^2+a^2*x*d^3+1/4*a^2*e^3*x^4+1/4*b^2/e*arctanh(c*x)*ln
(c*x-1)*d^4-1/8*b^2/e*ln(c*x-1)*ln(1/2+1/2*c*x)*d^4-1/4*b^2/e*arctanh(c*x)*
ln(c*x+1)*d^4+2*a*b*arctanh(c*x)*x*d^3+b^2*d*e^2*x/c^2+1/c^3*b^2*e^2*arctan
h(c*x)*ln(c*x+1)*d+3/2/c^2*b^2*e*arctanh(c*x)*ln(c*x-1)*d^2+3/4/c^2*b^2*e*l
n(1/2+1/2*c*x)*ln(-1/2*c*x+1/2)*d^2-1/2/c^3*b^2*e^2*ln(1/2+1/2*c*x)*ln(-1/2
*c*x+1/2)*d-3/4/c^2*b^2*e*ln(c*x-1)*ln(1/2+1/2*c*x)*d^2-1/2/c^3*b^2*e^2*ln(
c*x-1)*ln(1/2+1/2*c*x)*d+3/c*b^2*e*arctanh(c*x)*x*d^2+1/c*b^2*e^2*arctanh(c
*x)*x^2*d+3*a*b/c*e*x*d^2+1/c*a*b*e^2*x^2*d+2*a*b*e^2*arctanh(c*x)*x^3*d+3*
a*b*e*arctanh(c*x)*x^2*d^2+3/2/c^2*a*b*e*ln(c*x-1)*d^2+1/c^3*a*b*e^2*ln(c*x
-1)*d-3/2/c^2*a*b*e*ln(c*x+1)*d^2+1/c^3*a*b*e^2*ln(c*x+1)*d-3/4/c^2*b^2*e*l
n(c*x+1)*ln(-1/2*c*x+1/2)*d^2+1/2/c^3*b^2*e^2*ln(c*x+1)*ln(-1/2*c*x+1/2)*d+
1/c^3*b^2*e^2*arctanh(c*x)*ln(c*x-1)*d-3/2/c^2*b^2*e*arctanh(c*x)*ln(c*x+1)
*d^2
```

maxima **[B]**   time = 0.56, size = 782, normalized size = 2.18

$$\frac{1}{4}\,a^2e^3x^4+a^2de^2x^3+\frac{3}{2}\,a^2d^2ex^2+\frac{3}{2}\left(2\,x^2\operatorname{artanh}(cx)+c\left(\frac{2\,x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)\right)abd^2e+\left(2\,x^3\operatorname{artanh}(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="maxima")

[Out] 1/4*a^2*e^3*x^4 + a^2*d*e^2*x^3 + 3/2*a^2*d^2*e*x^2 + 3/2*(2*x^2*arctanh(c*
x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*a*b*d^2*e + (2*x^3*
arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*a*b*d*e^2 + 1/12*(6*x^4*
arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x -
1)/c^5))*a*b*e^3 + a^2*d^3*x + (2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*a*b
*d^3/c + (c^2*d^3 + d*e^2)*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*
x + 1/2))*b^2/c^3 + 1/6*(9*c^2*d^2*e - 3*c*d*e^2 + 2*e^3)*b^2*log(c*x + 1)/
c^4 + 1/6*(9*c^2*d^2*e + 3*c*d*e^2 + 2*e^3)*b^2*log(c*x - 1)/c^4 + 1/48*(4*
b^2*c^2*e^3*x^2 + 48*b^2*c^2*d*e^2*x + 3*(b^2*c^4*e^3*x^4 + 4*b^2*c^4*d*e^2
*x^3 + 6*b^2*c^4*d^2*e*x^2 + 4*b^2*c^4*d^3*x + (4*c^3*d^3 - 6*c^2*d^2*e + 4
*c*d*e^2 - e^3)*b^2)*log(c*x + 1)^2 + 3*(b^2*c^4*e^3*x^4 + 4*b^2*c^4*d*e^2*
x^3 + 6*b^2*c^4*d^2*e*x^2 + 4*b^2*c^4*d^3*x - (4*c^3*d^3 + 6*c^2*d^2*e + 4*
```

```
c*d*e^2 + e^3)*b^2)*log(-c*x + 1)^2 + 4*(b^2*c^3*e^3*x^3 + 6*b^2*c^3*d*e^2*
x^2 + 3*(6*c^3*d^2*e + c*e^3)*b^2*x)*log(c*x + 1) - 2*(2*b^2*c^3*e^3*x^3 +
12*b^2*c^3*d*e^2*x^2 + 6*(6*c^3*d^2*e + c*e^3)*b^2*x + 3*(b^2*c^4*e^3*x^4 +
 4*b^2*c^4*d*e^2*x^3 + 6*b^2*c^4*d^2*e*x^2 + 4*b^2*c^4*d^3*x + (4*c^3*d^3 -
 6*c^2*d^2*e + 4*c*d*e^2 - e^3)*b^2)*log(c*x + 1))*log(-c*x + 1))/c^4
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \operatorname{atanh}(c\,x))^2 (d + e\,x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x))^2*(d + e*x)^3,x)

[Out] int((a + b*atanh(c*x))^2*(d + e*x)^3, x)

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}(cx))^2 (d + ex)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**3*(a+b*atanh(c*x))**2,x)

[Out] Integral((a + b*atanh(c*x))**2*(d + e*x)**3, x)

3.10 $\int (d + ex)^2 \left(a + b \tanh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=257

$$-\frac{d\left(\frac{3e^2}{c^2} + d^2\right)\left(a + b\tanh^{-1}(cx)\right)^2}{3e} + \frac{\left(3c^2d^2 + e^2\right)\left(a + b\tanh^{-1}(cx)\right)^2}{3c^3} - \frac{2b\left(3c^2d^2 + e^2\right)\log\left(\frac{2}{1-cx}\right)\left(a + b\tanh^{-1}(cx)\right)}{3c^3}$$

[Out] 2*a*b*d*e*x/c+1/3*b^2*e^2*x/c^2-1/3*b^2*e^2*arctanh(c*x)/c^3+2*b^2*d*e*x*ar
ctanh(c*x)/c+1/3*b*e^2*x^2*(a+b*arctanh(c*x))/c+1/3*(3*c^2*d^2+e^2)*(a+b*ar
ctanh(c*x))^2/c^3-1/3*d*(d^2+3*e^2/c^2)*(a+b*arctanh(c*x))^2/e+1/3*(e*x+d)^
3*(a+b*arctanh(c*x))^2/e-2/3*b*(3*c^2*d^2+e^2)*(a+b*arctanh(c*x))*ln(2/(-c*
x+1))/c^3+b^2*d*e*ln(-c^2*x^2+1)/c^2-1/3*b^2*(3*c^2*d^2+e^2)*polylog(2,1-2/
(-c*x+1))/c^3

Rubi [A]   time = 0.41, antiderivative size = 257, normalized size of antiderivative
= 1.00, number of steps used = 15, number of rules used = 12, integrand size = 18,
$\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5928, 5910, 260, 5916, 321, 206, 6048, 5948, 5984, 5918,
2402, 2315}

$$-\frac{b^2\left(3c^2d^2 + e^2\right)\text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)}{3c^3} + \frac{\left(3c^2d^2 + e^2\right)\left(a + b\tanh^{-1}(cx)\right)^2}{3c^3} - \frac{d\left(\frac{3e^2}{c^2} + d^2\right)\left(a + b\tanh^{-1}(cx)\right)^2}{3e} - \frac{2b\Big($$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^2*(a + b*ArcTanh[c*x])^2,x]

[Out] (2*a*b*d*e*x)/c + (b^2*e^2*x)/(3*c^2) - (b^2*e^2*ArcTanh[c*x])/(3*c^3) + (2
*b^2*d*e*x*ArcTanh[c*x])/c + (b*e^2*x^2*(a + b*ArcTanh[c*x]))/(3*c) + ((3*c
^2*d^2 + e^2)*(a + b*ArcTanh[c*x])^2)/(3*c^3) - (d*(d^2 + (3*e^2)/c^2)*(a +
 b*ArcTanh[c*x])^2)/(3*e) + ((d + e*x)^3*(a + b*ArcTanh[c*x])^2)/(3*e) - (2
b(3*c^2*d^2 + e^2)*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^3) + (b^2*
d*e*Log[1 - c^2*x^2])/c^2 - (b^2*(3*c^2*d^2 + e^2)*PolyLog[2, 1 - 2/(1 - c*
x)])/(3*c^3)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
```

```
  Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6048

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int (d+ex)^2 \left(a + b\tanh^{-1}(cx)\right)^2 dx = \frac{(d+ex)^3 \left(a + b\tanh^{-1}(cx)\right)^2}{3e} - \frac{(2bc)\int \left(-\frac{3de^2\left(a+b\tanh^{-1}(cx)\right)}{c^2} - \frac{e^3 x\left(a+b\tanh^{-1}\right)}{c^2}\right)}{}$$

$$= \frac{(d+ex)^3 \left(a + b\tanh^{-1}(cx)\right)^2}{3e} - \frac{(2b)\int \frac{\left(c^2d^3+3de^2+e\left(3c^2d^2+e^2\right)x\right)\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}}{3ce}$$

$$= \frac{2abdex}{c} + \frac{be^2x^2 \left(a + b\tanh^{-1}(cx)\right)}{3c} + \frac{(d+ex)^3 \left(a + b\tanh^{-1}(cx)\right)^2}{3e} - \frac{(2b)}{}$$

$$= \frac{2abdex}{c} + \frac{b^2e^2x}{3c^2} + \frac{2b^2dex\tanh^{-1}(cx)}{c} + \frac{be^2x^2 \left(a + b\tanh^{-1}(cx)\right)}{3c} + \frac{(d+}{}$$

$$= \frac{2abdex}{c} + \frac{b^2e^2x}{3c^2} - \frac{b^2e^2\tanh^{-1}(cx)}{3c^3} + \frac{2b^2dex\tanh^{-1}(cx)}{c} + \frac{be^2x^2 \left(a + b\,\text{ta}}{3c}$$

$$= \frac{2abdex}{c} + \frac{b^2e^2x}{3c^2} - \frac{b^2e^2\tanh^{-1}(cx)}{3c^3} + \frac{2b^2dex\tanh^{-1}(cx)}{c} + \frac{be^2x^2 \left(a + b\,\text{ta}}{3c}$$

$$= \frac{2abdex}{c} + \frac{b^2e^2x}{3c^2} - \frac{b^2e^2\tanh^{-1}(cx)}{3c^3} + \frac{2b^2dex\tanh^{-1}(cx)}{c} + \frac{be^2x^2 \left(a + b\,\text{ta}}{3c}$$

$$= \frac{2abdex}{c} + \frac{b^2e^2x}{3c^2} - \frac{b^2e^2\tanh^{-1}(cx)}{3c^3} + \frac{2b^2dex\tanh^{-1}(cx)}{c} + \frac{be^2x^2 \left(a + b\,\text{ta}}{3c}$$

Mathematica [A]   time = 0.65, size = 319, normalized size = 1.24

$$3a^2c^3d^2x + 3a^2c^3dex^2 + a^2c^3e^2x^3 + 3abc^2d^2 \log\left(1 - c^2x^2\right) + 6abc^2dex + abc^2e^2x^2 + abe^2 \log\left(c^2x^2 - 1\right) + b\tanh$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcTanh[c*x])^2,x]

[Out] (3*a^2*c^3*d^2*x + 6*a*b*c^2*d*e*x + b^2*c*e^2*x + 3*a^2*c^3*d*e*x^2 + a*b*
c^2*e^2*x^2 + a^2*c^3*e^2*x^3 + b^2*(-1 + c*x)*(e^2 + c*e*(3*d + e*x) + c^2
*(3*d^2 + 3*d*e*x + e^2*x^2))*ArcTanh[c*x]^2 + b*ArcTanh[c*x]*(b*e*(-e + 6*
c^2*d*x + c^2*e*x^2) + 2*a*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2) - 2*b*(3*c^2*d
^2 + e^2)*Log[1 + E^(-2*ArcTanh[c*x])]) + 3*a*b*c*d*e*Log[1 - c*x] - 3*a*b*
c*d*e*Log[1 + c*x] + 3*a*b*c^2*d^2*Log[1 - c^2*x^2] + 3*b^2*c*d*e*Log[1 - c
```

```
^2*x^2] + a*b*e^2*Log[-1 + c^2*x^2] + b^2*(3*c^2*d^2 + e^2)*PolyLog[2, -E^(
-2*ArcTanh[c*x])])])/(3*c^3)
```

fricas [F]   time = 1.74, size = 0, normalized size = 0.00

integral $\left(a^2e^2x^2 + 2\,a^2dex + a^2d^2 + \left(b^2e^2x^2 + 2\,b^2dex + b^2d^2\right) \operatorname{artanh} (cx)^2 + 2\left(abe^2x^2 + 2\,abdex + abd^2\right) \operatorname{artanh}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="fricas")

[Out] integral(a^2*e^2*x^2 + 2*a^2*d*e*x + a^2*d^2 + (b^2*e^2*x^2 + 2*b^2*d*e*x +
 b^2*d^2)*arctanh(c*x)^2 + 2*(a*b*e^2*x^2 + 2*a*b*d*e*x + a*b*d^2)*arctanh(
c*x), x)

giac [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="giac")

[Out] Timed out

maple [B]   time = 0.07, size = 1050, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^2*(a+b*arctanh(c*x))^2,x)

[Out] 1/3*b^2*e^2*arctanh(c*x)^2*x^3-1/12/c^3*b^2*e^2*ln(c*x+1)^2+a^2*e*x^2*d+1/1
2/c^3*b^2*e^2*ln(c*x-1)^2-1/3/c^3*b^2*e^2*dilog(1/2+1/2*c*x)-1/c*b^2*dilog(
1/2+1/2*c*x)*d^2-1/4/c*b^2*ln(c*x+1)^2*d^2+1/4/c*b^2*ln(c*x-1)^2*d^2+1/12*b
^2/e*ln(c*x-1)^2*d^3+1/3*b^2/e*arctanh(c*x)^2*d^3+1/12*b^2/e*ln(c*x+1)^2*d^
3+b^2*arctanh(c*x)^2*x*d^2+1/6/c^3*b^2*e^2*ln(c*x-1)-1/6/c^3*b^2*e^2*ln(c*x
+1)+1/3*a^2/e*d^3+1/3/c^3*a*b*e^2*ln(c*x+1)+1/3*b^2*e^2*x/c^2+2*a*b*e*arcta
nh(c*x)*x^2*d+1/c^2*a*b*e*ln(c*x-1)*d-1/c^2*a*b*e*ln(c*x+1)*d+1/c^2*b^2*e*a
rctanh(c*x)*ln(c*x-1)*d-1/2/c^2*b^2*e*ln(c*x-1)*ln(1/2+1/2*c*x)*d+1/2/c^2*b
^2*e*ln(1/2+1/2*c*x)*ln(-1/2*c*x+1/2)*d-1/2/c^2*b^2*e*ln(c*x+1)*ln(-1/2*c*x
+1/2)*d-1/c^2*b^2*e*arctanh(c*x)*ln(c*x+1)*d+1/4/c^2*b^2*e*ln(c*x-1)^2*d+1/
c^2*b^2*e*ln(c*x-1)*d+1/c^2*b^2*e*ln(c*x+1)*d+1/4/c^2*b^2*e*ln(c*x+1)^2*d-1
/6/c^3*b^2*e^2*ln(c*x-1)*ln(1/2+1/2*c*x)+1/3/c^3*b^2*e^2*arctanh(c*x)*ln(c*
x-1)+1/3/c^3*b^2*e^2*arctanh(c*x)*ln(c*x+1)-1/6/c^3*b^2*e^2*ln(1/2+1/2*c*x)
*ln(-1/2*c*x+1/2)+1/6/c^3*b^2*e^2*ln(c*x+1)*ln(-1/2*c*x+1/2)+1/3*a*b/e*ln(c
```

```
*x-1)*d^3+2/3*a*b/e*arctanh(c*x)*d^3-1/3*a*b/e*ln(c*x+1)*d^3+1/3/c^3*a*b*e^
2*ln(c*x-1)+1/3*a^2*e^2*x^3+a^2*x*d^2+b^2*e*arctanh(c*x)^2*x^2*d+2/3*a*b*e^
2*arctanh(c*x)*x^3-1/3*b^2/e*arctanh(c*x)*ln(c*x+1)*d^3+2*a*b*arctanh(c*x)*
x*d^2-1/6*b^2/e*ln(c*x+1)*ln(-1/2*c*x+1/2)*d^3+1/6*b^2/e*ln(1/2+1/2*c*x)*ln
(-1/2*c*x+1/2)*d^3-1/6*b^2/e*ln(c*x-1)*ln(1/2+1/2*c*x)*d^3+1/3*b^2/e*arctan
h(c*x)*ln(c*x-1)*d^3-1/2/c*b^2*ln(1/2+1/2*c*x)*ln(-1/2*c*x+1/2)*d^2+1/3/c*b
^2*e^2*arctanh(c*x)*x^2+1/2/c*b^2*ln(c*x+1)*ln(-1/2*c*x+1/2)*d^2-1/2/c*b^2*
ln(c*x-1)*ln(1/2+1/2*c*x)*d^2+1/c*b^2*arctanh(c*x)*ln(c*x+1)*d^2+1/3/c*a*b*
x^2*e^2+1/c*b^2*arctanh(c*x)*ln(c*x-1)*d^2+1/c*a*b*ln(c*x-1)*d^2+1/c*a*b*ln
(c*x+1)*d^2+2*a*b*d*e*x/c+2*b^2*d*e*x*arctanh(c*x)/c
```

maxima [B]   time = 0.57, size = 524, normalized size = 2.04

$$\frac{1}{3}\, a^2 e^2 x^3 + a^2 dex^2 + \left(2\,x^2 \operatorname{artanh}(cx) + c\left(\frac{2\,x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3}\right)\right)abde + \frac{1}{3}\left(2\,x^3 \operatorname{artanh}(cx) + c\left(\frac{x^2}{c^2} + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="maxima")

[Out] 1/3*a^2*e^2*x^3 + a^2*d*e*x^2 + (2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x
+ 1)/c^3 + log(c*x - 1)/c^3))*a*b*d*e + 1/3*(2*x^3*arctanh(c*x) + c*(x^2/c^
2 + log(c^2*x^2 - 1)/c^4))*a*b*e^2 + a^2*d^2*x + (2*c*x*arctanh(c*x) + log(
-c^2*x^2 + 1))*a*b*d^2/c + 1/3*(3*c^2*d^2 + e^2)*(log(c*x + 1)*log(-1/2*c*x
+ 1/2) + dilog(1/2*c*x + 1/2))*b^2/c^3 + 1/6*(6*c*d*e - e^2)*b^2*log(c*x +
1)/c^3 + 1/6*(6*c*d*e + e^2)*b^2*log(c*x - 1)/c^3 + 1/12*(4*b^2*c*e^2*x +
(b^2*c^3*e^2*x^3 + 3*b^2*c^3*d*e*x^2 + 3*b^2*c^3*d^2*x + (3*c^2*d^2 - 3*c*d
*e + e^2)*b^2)*log(c*x + 1)^2 + (b^2*c^3*e^2*x^3 + 3*b^2*c^3*d*e*x^2 + 3*b^
2*c^3*d^2*x - (3*c^2*d^2 + 3*c*d*e + e^2)*b^2)*log(-c*x + 1)^2 + 2*(b^2*c^2
*e^2*x^2 + 6*b^2*c^2*d*e*x)*log(c*x + 1) - 2*(b^2*c^2*e^2*x^2 + 6*b^2*c^2*d
*e*x + (b^2*c^3*e^2*x^3 + 3*b^2*c^3*d*e*x^2 + 3*b^2*c^3*d^2*x + (3*c^2*d^2
- 3*c*d*e + e^2)*b^2)*log(c*x + 1))*log(-c*x + 1))/c^3
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\operatorname{atanh}(c\,x))^2\,(d + e\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2*(d + e*x)^2,x)

[Out] int((a + b*atanh(c*x))^2*(d + e*x)^2, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\operatorname{atanh}(cx))^2\,(d + ex)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*atanh(c*x))**2,x)
[Out] Integral((a + b*atanh(c*x))**2*(d + e*x)**2, x)
```

3.11 $\int (d + ex)\left(a + b\tanh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=160

$$-\frac{\left(\frac{e^2}{c^2} + d^2\right)\left(a + b\tanh^{-1}(cx)\right)^2}{2e} + \frac{(d + ex)^2 \left(a + b\tanh^{-1}(cx)\right)^2}{2e} + \frac{d\left(a + b\tanh^{-1}(cx)\right)^2}{c} - \frac{2bd\log\left(\frac{2}{1-cx}\right)\left(a + b\tan\right)}{c}$$

```
[Out] a*b*e*x/c+b^2*e*x*arctanh(c*x)/c+d*(a+b*arctanh(c*x))^2/c-1/2*(d^2+e^2/c^2)
*(a+b*arctanh(c*x))^2/e+1/2*(e*x+d)^2*(a+b*arctanh(c*x))^2/e-2*b*d*(a+b*arc
tanh(c*x))*ln(2/(-c*x+1))/c+1/2*b^2*e*ln(-c^2*x^2+1)/c^2-b^2*d*polylog(2,1-
2/(-c*x+1))/c
```

Rubi [A]   time = 0.33, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5928, 5910, 260, 6048, 5948, 5984, 5918, 2402, 2315}

$$-\frac{b^2 d\,\text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)}{c} - \frac{\left(\frac{e^2}{c^2} + d^2\right)\left(a + b\tanh^{-1}(cx)\right)^2}{2e} + \frac{(d + ex)^2 \left(a + b\tanh^{-1}(cx)\right)^2}{2e} + \frac{d\left(a + b\tanh^{-1}(cx)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (a*b*e*x)/c + (b^2*e*x*ArcTanh[c*x])/c + (d*(a + b*ArcTanh[c*x])^2)/c - ((d
^2 + e^2/c^2)*(a + b*ArcTanh[c*x])^2)/(2*e) + ((d + e*x)^2*(a + b*ArcTanh[c
*x])^2)/(2*e) - (2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c + (b^2*e*Lo
g[1 - c^2*x^2])/(2*c^2) - (b^2*d*PolyLog[2, 1 - 2/(1 - c*x)])/c
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6048

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.)))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int (d + ex)\left(a + b\tanh^{-1}(cx)\right)^2\,dx = \frac{(d+ex)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2e} - \frac{(bc)\int\left(-\frac{e^2\left(a+b\tanh^{-1}(cx)\right)}{c^2} + \frac{\left(c^2d^2+e^2+2c^2dex\right)\left(a\dots}{c^2\left(1-c^2\dots\right.}\right)}{e}$$

$$= \frac{(d+ex)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2e} - \frac{b\int\frac{\left(c^2d^2+e^2+2c^2dex\right)\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}\,dx}{ce} + \frac{(be)\int\dots}{\dots}$$

$$= \frac{abex}{c} + \frac{(d+ex)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2e} - \frac{b\int\left(\frac{c^2d^2\left(1+\frac{e^2}{c^2d^2}\right)\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2} + \frac{2c^2\dots}{\dots}\right)}{ce}$$

$$= \frac{abex}{c} + \frac{b^2ex\tanh^{-1}(cx)}{c} + \frac{(d+ex)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2e} - (2bcd)\int\frac{x\left(a+\dots\right.}{\dots}$$

$$= \frac{abex}{c} + \frac{b^2ex\tanh^{-1}(cx)}{c} + \frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{c} - \frac{\left(d^2+\frac{e^2}{c^2}\right)\left(a+b\tanh^{-1}\dots\right.}{2e}$$

$$= \frac{abex}{c} + \frac{b^2ex\tanh^{-1}(cx)}{c} + \frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{c} - \frac{\left(d^2+\frac{e^2}{c^2}\right)\left(a+b\tanh^{-1}\dots\right.}{2e}$$

$$= \frac{abex}{c} + \frac{b^2ex\tanh^{-1}(cx)}{c} + \frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{c} - \frac{\left(d^2+\frac{e^2}{c^2}\right)\left(a+b\tanh^{-1}\dots\right.}{2e}$$

$$= \frac{abex}{c} + \frac{b^2ex\tanh^{-1}(cx)}{c} + \frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{c} - \frac{\left(d^2+\frac{e^2}{c^2}\right)\left(a+b\tanh^{-1}\dots\right.}{2e}$$

Mathematica [A]   time = 0.44, size = 174, normalized size = 1.09

$$\frac{2a^2c^2dx + a^2c^2ex^2 + 2abcd\log\left(1-c^2x^2\right) + 2bc\tanh^{-1}(cx)\left(acx(2d+ex) - 2bd\log\left(e^{-2\tanh^{-1}(cx)}+1\right) + bex\right) + \dots}{\dots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)*(a + b*ArcTanh[c*x])^2,x]

[Out] (2*a^2*c^2*d*x + 2*a*b*c*e*x + a^2*c^2*e*x^2 + b^2*(-1 + c*x)*(2*c*d + e +
c*e*x)*ArcTanh[c*x]^2 + 2*b*c*ArcTanh[c*x]*(b*e*x + a*c*x*(2*d + e*x) - 2*b
*d*Log[1 + E^(-2*ArcTanh[c*x])]]) + a*b*e*Log[1 - c*x] - a*b*e*Log[1 + c*x]
+ 2*a*b*c*d*Log[1 - c^2*x^2] + b^2*e*Log[1 - c^2*x^2] + 2*b^2*c*d*PolyLog[2
, -E^(-2*ArcTanh[c*x])]])/(2*c^2)
```

fricas [F]   time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(a^2ex + a^2d + \left(b^2ex + b^2d\right)\operatorname{artanh}(cx)^2 + 2\left(abex + abd\right)\operatorname{artanh}(cx),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*e*x + a^2*d + (b^2*e*x + b^2*d)*arctanh(c*x)^2 + 2*(a*b*e*x +
a*b*d)*arctanh(c*x), x)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (ex + d)(b\,\text{artanh}\,(cx) + a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*arctanh(c*x) + a)^2, x)
```

maple [B]   time = 0.06, size = 462, normalized size = 2.89

$$-\frac{b^2\ln{(cx+1)}^2 d}{4c} + \frac{b^2\ln{(cx+1)}e}{2c^2} - \frac{b^2\,\text{dilog}\left(\frac{1}{2}+\frac{cx}{2}\right)d}{c} + b^2\arctanh{(cx)}^2\,xd + \frac{b^2\arctanh{(cx)}^2\,x^2e}{2} + \frac{b^2\ln{(cx+1)}}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arctanh(c*x))^2,x)
```

```
[Out] -1/4/c*b^2*ln(c*x+1)^2*d+1/2/c^2*b^2*ln(c*x+1)*e-1/c*b^2*dilog(1/2+1/2*c*x)
*d+b^2*arctanh(c*x)^2*x*d+1/2*b^2*arctanh(c*x)^2*x^2*e+1/8/c^2*b^2*ln(c*x+1
)^2*e+1/8/c^2*b^2*ln(c*x-1)^2*e+1/2/c^2*b^2*ln(c*x-1)*e+1/4/c*b^2*ln(c*x-1)
^2*d+a^2*d*x+a*b*e*x/c+b^2*e*x*arctanh(c*x)/c-1/2/c^2*a*b*ln(c*x+1)*e-1/2/c
^2*b^2*arctanh(c*x)*ln(c*x+1)*e-1/4/c^2*b^2*ln(c*x+1)*ln(-1/2*c*x+1/2)*e+1/
4/c^2*b^2*ln(1/2+1/2*c*x)*ln(-1/2*c*x+1/2)*e+1/2/c^2*b^2*arctanh(c*x)*ln(c*
x-1)*e+1/2/c^2*a*b*ln(c*x-1)*e+1/2/c*b^2*ln(c*x+1)*ln(-1/2*c*x+1/2)*d-1/2/c
*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)*d+1/c*b^2*arctanh(c*x)*ln(c*x-1)*d+1/c*b^2*a
rctanh(c*x)*ln(c*x+1)*d-1/2/c*b^2*ln(1/2+1/2*c*x)*ln(-1/2*c*x+1/2)*d+1/c*a*
b*ln(c*x+1)*d+1/c*a*b*ln(c*x-1)*d-1/4/c^2*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)*e+1
/2*a^2*x^2*e+a*b*arctanh(c*x)*x^2*e+2*a*b*arctanh(c*x)*x*d
```

maxima [B]   time = 0.56, size = 313, normalized size = 1.96

$$\frac{1}{2}a^2ex^2 + \frac{1}{2}\left(2x^2\,\text{artanh}\,(cx) + c\left(\frac{2x}{c^2} - \frac{\log{(cx+1)}}{c^3} + \frac{\log{(cx-1)}}{c^3}\right)\right)abe + a^2dx + \frac{\left(2cx\,\text{artanh}\,(cx) + \log\left(-c^2x^2 + \right.\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*e*x^2 + 1/2*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + l
og(c*x - 1)/c^3))*a*b*e + a^2*d*x + (2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1)
)*a*b*d/c + (log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*d
/c + 1/2*b^2*e*log(c*x + 1)/c^2 + 1/2*b^2*e*log(c*x - 1)/c^2 + 1/8*(4*b^2*c
*e*x*log(c*x + 1) + (b^2*c^2*e*x^2 + 2*b^2*c^2*d*x + (2*c*d - e)*b^2)*log(c
*x + 1)^2 + (b^2*c^2*e*x^2 + 2*b^2*c^2*d*x - (2*c*d + e)*b^2)*log(-c*x + 1)
^2 - 2*(2*b^2*c*e*x + (b^2*c^2*e*x^2 + 2*b^2*c^2*d*x + (2*c*d - e)*b^2)*log
(c*x + 1))*log(-c*x + 1))/c^2
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{atanh}(c\,x))^2 \, (d + e\,x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2*(d + e*x),x)
```

```
[Out] int((a + b*atanh(c*x))^2*(d + e*x), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}(cx))^2 \, (d + ex) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*atanh(c*x))**2,x)
```

```
[Out] Integral((a + b*atanh(c*x))**2*(d + e*x), x)
```

3.12 $\displaystyle\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+ex}\,dx$

Optimal. Leaf size=188

$$-\frac{b\left(a+b\tanh^{-1}(cx)\right)\mathrm{Li}_2\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{e}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e}+\frac{b\mathrm{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(}{e}$$

[Out] -(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/e+(a+b*arctanh(c*x))^2*ln(2*c*(e*x+d)/(
c*d+e)/(c*x+1))/e+b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/e-b*(a+b*arct
anh(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e+1/2*b^2*polylog(3,1-2/
(c*x+1))/e-1/2*b^2*polylog(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e

Rubi [A]   time = 0.05, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {5922}

$$-\frac{b\left(a+b\tanh^{-1}(cx)\right)\mathrm{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e}+\frac{b\,\mathrm{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e}-\frac{b^2\mathrm{PolyLog}\left(3,1\right.}{2e}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*x])^2/(d + e*x),x]

[Out] -(((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e) + ((a + b*ArcTanh[c*x])^2*Lo
g[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e + (b*(a + b*ArcTanh[c*x])*PolyL
og[2, 1 - 2/(1 + c*x)])/e - (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - (2*c*(d
+ e*x))/((c*d + e)*(1 + c*x))])/e + (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e)
 - (b^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e)

Rule 5922

Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 -Simp[((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e, x] + (Simp[((a + b*ArcT
anh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Simp[(b*(a
+ b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e, x] - Simp[(b*(a + b*ArcTa
nh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Sim
p[(b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e), x] - Simp[(b^2*PolyLog[3, 1 - (2
c(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e), x]) /; FreeQ[{a, b, c, d, e},
x] && NeQ[c^2*d^2 - e^2, 0]

Rubi steps

$$\int \frac{\left(a + b\tanh^{-1}(cx)\right)^2}{d + ex} \, dx = -\frac{\left(a + b\tanh^{-1}(cx)\right)^2 \log\left(\frac{2}{1+cx}\right)}{e} + \frac{\left(a + b\tanh^{-1}(cx)\right)^2 \log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e} + \frac{b\left(a + b\right.}{}$$

Mathematica [C]   time = 10.61, size = 759, normalized size = 4.04

$$6a^2 \log(d + ex) - 6iab\left(-\log\left(\frac{2}{\sqrt{1-c^2x^2}}\right)\left(\pi - 2i\tanh^{-1}(cx)\right) - i\mathrm{Li}_2\left(e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right) + i\left(\tanh^{-1}\left(\frac{cd}{e}\right) + \right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(d + e*x),x]
```

```
[Out] (6*a^2*Log[d + e*x] + 6*a*b*ArcTanh[c*x]*(Log[1 - c^2*x^2] + 2*Log[I*Sinh[A
rcTanh[(c*d)/e] + ArcTanh[c*x]]]) - (6*I)*a*b*((-1/4*I)*(Pi - (2*I)*ArcTanh
[c*x])^2 + I*(ArcTanh[(c*d)/e] + ArcTanh[c*x])^2 + (Pi - (2*I)*ArcTanh[c*x]
)*Log[1 + E^(2*ArcTanh[c*x])] + (2*I)*(ArcTanh[(c*d)/e] + ArcTanh[c*x])*Log
[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - (Pi - (2*I)*ArcTanh[c*x])*
Log[2/Sqrt[1 - c^2*x^2]] - (2*I)*(ArcTanh[(c*d)/e] + ArcTanh[c*x])*Log[(2*I
)*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - I*PolyLog[2, -E^(2*ArcTanh[c*x])
] - I*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]) + (b^2*(-8*c*d*
ArcTanh[c*x]^3 + 4*e*ArcTanh[c*x]^3 - (4*Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[
c*x]^3)/E^ArcTanh[(c*d)/e] - 6*c*d*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x
])] - (6*I)*c*d*Pi*ArcTanh[c*x]*Log[(E^(-ArcTanh[c*x]) + E^ArcTanh[c*x])/2]
 + 6*c*d*ArcTanh[c*x]^2*Log[1 - E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 6*c*
d*ArcTanh[c*x]^2*Log[1 + E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 12*c*d*ArcT
anh[(c*d)/e]*ArcTanh[c*x]*Log[(I/2)*E^(-ArcTanh[(c*d)/e] - ArcTanh[c*x])*(-
1 + E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x])))] + 6*c*d*ArcTanh[c*x]^2*Log[(e
*(-1 + E^(2*ArcTanh[c*x])) + c*d*(1 + E^(2*ArcTanh[c*x])))/(2*E^ArcTanh[c*x
])] - 6*c*d*ArcTanh[c*x]^2*Log[(c*(d + e*x))/Sqrt[1 - c^2*x^2]] - (3*I)*c*d
*Pi*ArcTanh[c*x]*Log[1 - c^2*x^2] - 12*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x]*Lo
g[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] + 6*c*d*ArcTanh[c*x]*PolyLog[2,
-E^(-2*ArcTanh[c*x])] + 12*c*d*ArcTanh[c*x]*PolyLog[2, -E^(ArcTanh[(c*d)/e]
 + ArcTanh[c*x])] + 12*c*d*ArcTanh[c*x]*PolyLog[2, E^(ArcTanh[(c*d)/e] + Ar
cTanh[c*x])] + 3*c*d*PolyLog[3, -E^(-2*ArcTanh[c*x])] - 12*c*d*PolyLog[3, -
E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] - 12*c*d*PolyLog[3, E^(ArcTanh[(c*d)/e
] + ArcTanh[c*x])]))/(c*d))/(6*e)
```

fricas [F]   time = 0.55, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^2 \operatorname{artanh}(cx)^2 + 2\,ab \operatorname{artanh}(cx) + a^2}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(e*x + d), x)
```

giac [**F**]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{artanh}\,(cx) + a)^2}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/(e*x + d), x)
```

maple [**C**]   time = 0.74, size = 1170, normalized size = 6.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/(e*x+d),x)
```

```
[Out] a^2*ln(c*e*x+c*d)/e+b^2*ln(c*e*x+c*d)/e*arctanh(c*x)^2-b^2/e*arctanh(c*x)^2
*ln(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1)))-1/2*I*b^2/
e*arctanh(c*x)^2*Pi*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(
-c^2*x^2+1)))/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)
-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1))))-1/2*I*b^2/e*arctanh(c*x)^2*Pi*csgn(I
/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c
*x+1)^2/(-c^2*x^2+1)))/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+1/2*I*b^2/e*arctanh(c*
x)^2*Pi*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1))
)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3+1/2*I*b^2/e*arctanh(c*x)^2*Pi*csgn(I/(1+(c*
x+1)^2/(-c^2*x^2+1)))*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2
/(-c^2*x^2+1)))/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)
-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1))))-b^2/e*arctanh(c*x)*polylog(2,-(c*x+1
)^2/(-c^2*x^2+1))+1/2*b^2/e*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+b^2/(c*d+e)*
arctanh(c*x)^2*ln(1-(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+b^2/(c*d+e)*ar
ctanh(c*x)*polylog(2,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-1/2*b^2/(c*d+
e)*polylog(3,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+c*b^2/e*d/(c*d+e)*arc
tanh(c*x)^2*ln(1-(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+c*b^2/e*d/(c*d+e)
*arctanh(c*x)*polylog(2,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-1/2*c*b^2/
e*d/(c*d+e)*polylog(3,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+2*a*b*ln(c*e
*x+c*d)/e*arctanh(c*x)+a*b/e*ln(c*e*x+c*d)*ln((c*e*x-e)/(-c*d-e))+a*b/e*dil
og((c*e*x-e)/(-c*d-e))-a*b/e*ln(c*e*x+c*d)*ln((c*e*x+e)/(-c*d+e))-a*b/e*dil
og((c*e*x+e)/(-c*d+e))
```

maxima **[F]**   time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 \log (ex + d)}{e} + \int \frac{b^2 \big(\log (cx + 1) - \log (-cx + 1)\big)^2}{4 (ex + d)} + \frac{ab\big(\log (cx + 1) - \log (-cx + 1)\big)}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="maxima")

[Out] a^2*log(e*x + d)/e + integrate(1/4*b^2*(log(c*x + 1) - log(-c*x + 1))^2/(e*x + d) + a*b*(log(c*x + 1) - log(-c*x + 1))/(e*x + d), x)

mupad **[F]**   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atanh}\,(c\,x))^2}{d + e\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x))^2/(d + e*x),x)

[Out] int((a + b*atanh(c*x))^2/(d + e*x), x)

sympy **[F]**   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atanh}\,(cx))^2}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x))**2/(e*x+d),x)

[Out] Integral((a + b*atanh(c*x))**2/(d + e*x), x)

3.13 $\int \dfrac{\left(a+b\tanh^{-1}(cx)\right)^2}{(d+ex)^2}\,dx$

Optimal. Leaf size=321

$$\frac{2bc\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2-e^2}-\frac{2bc\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{c^2d^2-e^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{e(d+ex)}+\frac{bc\log\left(\frac{2}{1-cx}\right)(\,}{e(c\,}$$

[Out] -(a+b*arctanh(c*x))^2/e/(e*x+d)+b*c*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/e/(c*
d+e)-b*c*(a+b*arctanh(c*x))*ln(2/(c*x+1))/(c*d-e)/e+2*b*c*(a+b*arctanh(c*x)
)*ln(2/(c*x+1))/(c^2*d^2-e^2)-2*b*c*(a+b*arctanh(c*x))*ln(2*c*(e*x+d)/(c*d+
e)/(c*x+1))/(c^2*d^2-e^2)+1/2*b^2*c*polylog(2,1-2/(-c*x+1))/e/(c*d+e)+1/2*b
^2*c*polylog(2,1-2/(c*x+1))/(c*d-e)/e-b^2*c*polylog(2,1-2/(c*x+1))/(c^2*d^2
-e^2)+b^2*c*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/(c^2*d^2-e^2)

Rubi [A] time = 0.31, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 18, $\dfrac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5928, 5918, 2402, 2315, 5920, 2447}

$$-\frac{b^2c\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{c^2d^2-e^2}+\frac{b^2c\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{c^2d^2-e^2}+\frac{b^2c\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2e(cd+e)}+\frac{b^2c\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2e(cd-e)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*x])^2/(d + e*x)^2,x]

[Out] -((a + b*ArcTanh[c*x])^2/(e*(d + e*x))) + (b*c*(a + b*ArcTanh[c*x])*Log[2/(
1 - c*x)])/(e*(c*d + e)) - (b*c*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/((c*
d - e)*e) + (2*b*c*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c^2*d^2 - e^2) -
 (2*b*c*(a + b*ArcTanh[c*x])*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(c
^2*d^2 - e^2) + (b^2*c*PolyLog[2, 1 - 2/(1 - c*x)])/(2*e*(c*d + e)) + (b^2*
c*PolyLog[2, 1 - 2/(1 + c*x)])/(2*(c*d - e)*e) - (b^2*c*PolyLog[2, 1 - 2/(1
 + c*x)])/(c^2*d^2 - e^2) + (b^2*c*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e
)*(1 + c*x))])/(c^2*d^2 - e^2)

Rule 2315

Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]

Rule 2402

Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{\left(a + b\tanh^{-1}(cx)\right)^2}{(d + ex)^2}\, dx = -\frac{\left(a + b\tanh^{-1}(cx)\right)^2}{e(d + ex)} + \frac{(2bc)\int\left(-\frac{c\left(a+b\tanh^{-1}(cx)\right)}{2(cd+e)(-1+cx)} + \frac{c\left(a+b\tanh^{-1}(cx)\right)}{2(cd-e)(1+cx)} + \frac{e^2\left(a+b\tanh^{-1}(cx)\right)}{(-cd+e)(cd+e)(d+ex}\right)}{e}$$

$$= -\frac{\left(a + b\tanh^{-1}(cx)\right)^2}{e(d + ex)} + \frac{(bc^2)\int\frac{a+b\tanh^{-1}(cx)}{1+cx}\, dx}{(cd - e)e} - \frac{(bc^2)\int\frac{a+b\tanh^{-1}(cx)}{-1+cx}\, dx}{e(cd + e)} + \frac{(2bce)\int}{(-cd}$$

$$= -\frac{\left(a + b\tanh^{-1}(cx)\right)^2}{e(d + ex)} + \frac{bc\left(a + b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{e(cd + e)} - \frac{bc\left(a + b\tanh^{-1}(cx)\right)\log}{(cd - e)e}$$

$$= -\frac{\left(a + b\tanh^{-1}(cx)\right)^2}{e(d + ex)} + \frac{bc\left(a + b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{e(cd + e)} - \frac{bc\left(a + b\tanh^{-1}(cx)\right)\log}{(cd - e)e}$$

$$= -\frac{\left(a + b\tanh^{-1}(cx)\right)^2}{e(d + ex)} + \frac{bc\left(a + b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{e(cd + e)} - \frac{bc\left(a + b\tanh^{-1}(cx)\right)\log}{(cd - e)e}$$

Mathematica [C]　　time = 4.60, size = 317, normalized size = 0.99

$$-\frac{a^2}{e(d + ex)} + \frac{abc\left(\frac{(e-cd)\log(1-cx)+(cd+e)\log(cx+1)-2e\log(c(d+ex))}{(cd-e)(cd+e)} - \frac{2\tanh^{-1}(cx)}{cd+cex}\right)}{e} + \frac{b^2\left(\frac{cd\left(-i\pi\left(\tanh^{-1}(cx)-\frac{1}{2}\log(1-c^2x^2)\right)+\text{Li}_2\left(e^{-2(\tan}\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(d + e*x)^2,x]

[Out] -(a^2/(e*(d + e*x))) + (a*b*c*((-2*ArcTanh[c*x])/(c*d + c*e*x) + ((-(c*d) +
 e)*Log[1 - c*x] + (c*d + e)*Log[1 + c*x] - 2*e*Log[c*(d + e*x)])/((c*d - e
)*(c*d + e))))/e + (b^2*(-(ArcTanh[c*x]^2/(Sqrt[1 - (c^2*d^2)/e^2]*e*E^ArcT
anh[(c*d)/e])) + (x*ArcTanh[c*x]^2)/(d + e*x) + (c*d*(I*Pi*Log[1 + E^(2*Arc
Tanh[c*x])] - 2*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]
))] - I*Pi*(ArcTanh[c*x] - Log[1 - c^2*x^2]/2) - 2*ArcTanh[(c*d)/e]*(ArcTan
h[c*x] + Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - Log[I*Sinh[Arc
Tanh[(c*d)/e] + ArcTanh[c*x]]]) + PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcT
anh[c*x]))]))/(c^2*d^2 - e^2)))/d
```

fricas [F]　　time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\operatorname{artanh}(cx)^2 + 2\,ab\operatorname{artanh}(cx) + a^2}{e^2x^2 + 2\,dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))^2/(e*x+d)^2,x, algorithm="fricas")

[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(e^2*x^2 + 2*d*e*x
 + d^2), x)

giac $[\mathbf{F}]$ time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{artanh}(cx) + a)^2}{(ex + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))^2/(e*x+d)^2,x, algorithm="giac")

[Out] integrate((b*arctanh(c*x) + a)^2/(e*x + d)^2, x)

maple $[\mathbf{A}]$ time = 0.07, size = 605, normalized size = 1.88

$$-\frac{c\,a^2}{(cxe + cd)\,e} - \frac{c\,b^2\arctanh{(cx)}^2}{(cxe + cd)\,e} - \frac{2c\,b^2\arctanh{(cx)}\ln{(cxe + cd)}}{(cd + e)(cd - e)} - \frac{2c\,b^2\arctanh{(cx)}\ln{(cx - 1)}}{e(2cd + 2e)} + \frac{2c\,b^2\arctanh}{e(2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(c*x))^2/(e*x+d)^2,x)

[Out] -c*a^2/(c*e*x+c*d)/e-c*b^2/(c*e*x+c*d)/e*arctanh(c*x)^2-2*c*b^2*arctanh(c*x
)/(c*d+e)/(c*d-e)*ln(c*e*x+c*d)-2*c*b^2/e*arctanh(c*x)/(2*c*d+2*e)*ln(c*x-1
)+2*c*b^2/e*arctanh(c*x)/(2*c*d-2*e)*ln(c*x+1)-c*b^2/(c*d+e)/(c*d-e)*ln(c*e
*x+c*d)*ln((c*e*x-e)/(-c*d-e))-c*b^2/(c*d+e)/(c*d-e)*dilog((c*e*x-e)/(-c*d-
e))+c*b^2/(c*d+e)/(c*d-e)*ln(c*e*x+c*d)*ln((c*e*x+e)/(-c*d+e))+c*b^2/(c*d+e
)/(c*d-e)*dilog((c*e*x+e)/(-c*d+e))-1/4*c*b^2/e/(c*d-e)*ln(c*x+1)^2+1/2*c*b
^2/e/(c*d-e)*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/2*c*b^2/e/(c*d-e)*ln(-1/2*c*x+1/2
)*ln(1/2+1/2*c*x)-1/2*c*b^2/e/(c*d-e)*dilog(1/2+1/2*c*x)-1/4*c*b^2/e/(c*d+e
)*ln(c*x-1)^2+1/2*c*b^2/e/(c*d+e)*dilog(1/2+1/2*c*x)+1/2*c*b^2/e/(c*d+e)*ln
(c*x-1)*ln(1/2+1/2*c*x)-2*c*a*b/(c*e*x+c*d)/e*arctanh(c*x)-2*c*a*b/(c*d+e)/
(c*d-e)*ln(c*e*x+c*d)-2*c*a*b/(2*c*d+2*e)*ln(c*x-1)+2*c*a*b/e/(2*c*d-2*e)
*ln(c*x+1)

maxima $[\mathbf{F}]$ time = 0.00, size = 0, normalized size = 0.00

$$\left(c\left(\frac{\log{(cx + 1)}}{cde - e^2} - \frac{\log{(cx - 1)}}{cde + e^2} - \frac{2\log{(ex + d)}}{c^2d^2 - e^2}\right) - \frac{2\operatorname{artanh}{(cx)}}{e^2x + de}\right)ab - \frac{1}{4}b^2\left(\frac{\log{(-cx + 1)}^2}{e^2x + de} + \int -\frac{(cex - e)\log{(}}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] (c*(log(c*x + 1)/(c*d*e - e^2) - log(c*x - 1)/(c*d*e + e^2) - 2*log(e*x + d
)/(c^2*d^2 - e^2)) - 2*arctanh(c*x)/(e^2*x + d*e))*a*b - 1/4*b^2*(log(-c*x
+ 1)^2/(e^2*x + d*e) + integrate(-((c*e*x - e)*log(c*x + 1)^2 + 2*(c*e*x +
c*d - (c*e*x - e)*log(c*x + 1))*log(-c*x + 1))/(c*e^3*x^3 - d^2*e + (2*c*d*
e^2 - e^3)*x^2 + (c*d^2*e - 2*d*e^2)*x), x)) - a^2/(e^2*x + d*e)
```

mupad **[F]**   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{atanh}(c\,x))^2}{(d + e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(d + e*x)^2,x)
```

```
[Out] int((a + b*atanh(c*x))^2/(d + e*x)^2, x)
```

sympy **[F]**   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(cx))^2}{(d + ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*atanh(c*x))**2/(d + e*x)**2, x)
```

3.14 $\displaystyle\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(d+ex)^3}\,dx$

Optimal. Leaf size=480

$$\frac{2bc^3d\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{(cd-e)^2(cd+e)^2}-\frac{2bc^3d\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{(cd-e)^2(cd+e)^2}+\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{\left(c^2d^2-e^2\right)(d+ex)}+\frac{bc^2\log}{}$$

```
[Out] b*c*(a+b*arctanh(c*x))/(c^2*d^2-e^2)/(e*x+d)-1/2*(a+b*arctanh(c*x))^2/e/(e*
x+d)^2+1/2*b*c^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/e/(c*d+e)^2+1/2*b^2*c^2*
ln(-c*x+1)/(c*d-e)/(c*d+e)^2-1/2*b*c^2*(a+b*arctanh(c*x))*ln(2/(c*x+1))/(c*
d-e)^2/e+2*b*c^3*d*(a+b*arctanh(c*x))*ln(2/(c*x+1))/(c^2*d^2-e^2)^2-1/2*b^2
*c^2*ln(c*x+1)/(c*d-e)^2/(c*d+e)+b^2*c^2*e*ln(e*x+d)/(c^2*d^2-e^2)^2-2*b*c^
3*d*(a+b*arctanh(c*x))*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/(c^2*d^2-e^2)^2+1/4*
b^2*c^2*polylog(2,1-2/(-c*x+1))/e/(c*d+e)^2+1/4*b^2*c^2*polylog(2,1-2/(c*x+
1))/(c*d-e)^2/e-b^2*c^3*d*polylog(2,1-2/(c*x+1))/(c^2*d^2-e^2)^2+b^2*c^3*d*
polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/(c^2*d^2-e^2)^2
```

Rubi [A]   time = 0.50, antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {5928, 5918, 2402, 2315, 5926, 706, 31, 633, 5920, 2447}

$$-\frac{b^2c^3d\,\mathrm{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{(cd-e)^2(cd+e)^2}+\frac{b^2c^3d\,\mathrm{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{(cd-e)^2(cd+e)^2}+\frac{b^2c^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{4e(cd+e)^2}+\frac{b^2c^2\,\mathrm{PolyLog}\left(2,\right.}{4e(cd-e)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(d + e*x)^3,x]
```

```
[Out] (b*c*(a + b*ArcTanh[c*x]))/((c^2*d^2 - e^2)*(d + e*x)) - (a + b*ArcTanh[c*x
])^2/(2*e*(d + e*x)^2) + (b*c^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(2*e
*(c*d + e)^2) + (b^2*c^2*Log[1 - c*x])/(2*(c*d - e)*(c*d + e)^2) - (b*c^2*(
a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(2*(c*d - e)^2*e) + (2*b*c^3*d*(a + b
*ArcTanh[c*x])*Log[2/(1 + c*x)])/((c*d - e)^2*(c*d + e)^2) - (b^2*c^2*Log[1
 + c*x])/(2*(c*d - e)^2*(c*d + e)) + (b^2*c^2*e*Log[d + e*x])/((c*d - e)^2*
(c*d + e)^2) - (2*b*c^3*d*(a + b*ArcTanh[c*x])*Log[(2*c*(d + e*x))/((c*d +
e)*(1 + c*x))])/((c*d - e)^2*(c*d + e)^2) + (b^2*c^2*PolyLog[2, 1 - 2/(1 -
c*x)])/(4*e*(c*d + e)^2) + (b^2*c^2*PolyLog[2, 1 - 2/(1 + c*x)])/(4*(c*d -
e)^2*e) - (b^2*c^3*d*PolyLog[2, 1 - 2/(1 + c*x)])/((c*d - e)^2*(c*d + e)^2)
 + (b^2*c^3*d*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/((c*d
- e)^2*(c*d + e)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{\left(a + b \tanh^{-1}(cx)\right)^2}{(d + ex)^3}\, dx = -\frac{\left(a + b \tanh^{-1}(cx)\right)^2}{2e(d + ex)^2} + \frac{(bc)\int\left(-\frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{2(cd+e)^2(-1+cx)} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{2(cd-e)^2(1+cx)} + \frac{e^2\left(a+b\tanh^{-1}(cx)\right)}{(-cd+e)(cd+e)(d+ex)}\right)}{e}$$

$$= -\frac{\left(a + b\tanh^{-1}(cx)\right)^2}{2e(d + ex)^2} + \frac{\left(bc^3\right)\int \frac{a+b\tanh^{-1}(cx)}{1+cx}\,dx}{2(cd - e)^2 e} - \frac{\left(bc^3\right)\int \frac{a+b\tanh^{-1}(cx)}{-1+cx}\,dx}{2e(cd + e)^2} - \frac{\left(2bc^3de\right)}{(cd}$$

$$= \frac{bc\left(a + b\tanh^{-1}(cx)\right)}{(cd - e)(cd + e)(d + ex)} - \frac{\left(a + b\tanh^{-1}(cx)\right)^2}{2e(d + ex)^2} + \frac{bc^2\left(a + b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{2e(cd + e)^2} -$$

$$= \frac{bc\left(a + b\tanh^{-1}(cx)\right)}{(cd - e)(cd + e)(d + ex)} - \frac{\left(a + b\tanh^{-1}(cx)\right)^2}{2e(d + ex)^2} + \frac{bc^2\left(a + b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{2e(cd + e)^2} -$$

$$= \frac{bc\left(a + b\tanh^{-1}(cx)\right)}{(cd - e)(cd + e)(d + ex)} - \frac{\left(a + b\tanh^{-1}(cx)\right)^2}{2e(d + ex)^2} + \frac{bc^2\left(a + b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{2e(cd + e)^2} -$$

$$= \frac{bc\left(a + b\tanh^{-1}(cx)\right)}{(cd - e)(cd + e)(d + ex)} - \frac{\left(a + b\tanh^{-1}(cx)\right)^2}{2e(d + ex)^2} + \frac{bc^2\left(a + b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{2e(cd + e)^2} +$$

Mathematica [C]   time = 7.06, size = 470, normalized size = 0.98

$$-\frac{a^2}{2e(d + ex)^2} - \frac{abc^2\left(\frac{\frac{2e(c^2(-d^2)+2c^2d(d+ex)\log(c(d+ex))+e^2)}{c(cd+e)^2(d+ex)}-\log(cx+1)}{(e-cd)^2} + \frac{\log(1-cx)}{(cd+e)^2} + \frac{2\tanh^{-1}(cx)}{(cd+cex)^2}\right)}{2e} + \frac{b^2c^2\left(\frac{2cd\left(-i\pi\left(\tanh^{-1}(cx)-\frac{1}{2}\log\left(1-c\right.\right.\right.}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(d + e*x)^3,x]

[Out] -1/2*a^2/(e*(d + e*x)^2) - (a*b*c^2*((2*ArcTanh[c*x])/(c*d + c*e*x)^2 + Log
[1 - c*x]/(c*d + e)^2 + (-Log[1 + c*x] + (2*e*(-(c^2*d^2) + e^2 + 2*c^2*d*(
d + e*x)*Log[c*(d + e*x)]))/(c*(c*d + e)^2*(d + e*x)))/(-(c*d) + e)^2))/(2*
e) + (b^2*c^2*((-2*ArcTanh[c*x]^2)/(Sqrt[1 - (c^2*d^2)/e^2]*e*E^ArcTanh[(c*
d)/e]) - (e*(-1 + c^2*x^2)*ArcTanh[c*x]^2)/(c^2*(d + e*x)^2) + (2*x*ArcTanh
[c*x]*(-e + c*d*ArcTanh[c*x]))/(c*d*(d + e*x)) + (2*e*(-(e*ArcTanh[c*x]) +
c*d*Log[(c*(d + e*x))/Sqrt[1 - c^2*x^2]]))/(c^3*d^3 - c*d*e^2) + (2*c*d*(I*
Pi*Log[1 + E^(2*ArcTanh[c*x])] - 2*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d
)/e] + ArcTanh[c*x]))] - I*Pi*(ArcTanh[c*x] - Log[1 - c^2*x^2]/2) - 2*ArcTa
nh[(c*d)/e]*(ArcTanh[c*x] + Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x])
```

```
)] - Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]]) + PolyLog[2, E^(-2*(ArcT
anh[(c*d)/e] + ArcTanh[c*x]))]]))/(c^2*d^2 - e^2)))/(2*(c*d - e)*(c*d + e))
```

fricas $[\mathbf{F}]$ time $= 0.61$, size $= 0$, normalized size $= 0.00$

$$\text{integral}\left(\frac{b^2 \operatorname{artanh}(cx)^2 + 2\,ab\operatorname{artanh}(cx) + a^2}{e^3 x^3 + 3\,de^2 x^2 + 3\,d^2 ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))^2/(e*x+d)^3,x, algorithm="fricas")

[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(e^3*x^3 + 3*d*e^2
*x^2 + 3*d^2*e*x + d^3), x)

giac $[\mathbf{F}]$ time $= 0.00$, size $= 0$, normalized size $= 0.00$

$$\int \frac{(b\operatorname{artanh}(cx) + a)^2}{(ex + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))^2/(e*x+d)^3,x, algorithm="giac")

[Out] integrate((b*arctanh(c*x) + a)^2/(e*x + d)^3, x)

maple $[\mathbf{A}]$ time $= 0.08$, size $= 824$, normalized size $= 1.72$

$$\frac{c^2 ab}{(cd + e)(cd - e)(cxe + cd)} + \frac{c^2 ab\ln(cx + 1)}{2e(cd - e)^2} + \frac{c^2 b^2 \ln(cx - 1)\ln\left(\frac{1}{2} + \frac{cx}{2}\right)}{4e(cd + e)^2} + \frac{c^2 b^2 \ln\left(-\frac{cx}{2} + \frac{1}{2}\right)\ln(cx + 1)}{4e(cd - e)^2} - \frac{c^2 b^2 \ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(c*x))^2/(e*x+d)^3,x)

[Out] c^2*a*b/(c*d+e)/(c*d-e)/(c*e*x+c*d)+1/2*c^2*a*b/e/(c*d-e)^2*ln(c*x+1)+c^2*b
^2*arctanh(c*x)/(c*d+e)/(c*d-e)/(c*e*x+c*d)-1/2*c^2*b^2/e*arctanh(c*x)/(c*d
+e)^2*ln(c*x-1)+1/2*c^2*b^2/e*arctanh(c*x)/(c*d-e)^2*ln(c*x+1)+1/4*c^2*b^2/
e/(c*d+e)^2*ln(c*x-1)*ln(1/2+1/2*c*x)+1/4*c^2*b^2/e/(c*d-e)^2*ln(-1/2*c*x+1
/2)*ln(c*x+1)-1/4*c^2*b^2/e/(c*d-e)^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-c^2*
a*b/(c*e*x+c*d)^2/e*arctanh(c*x)-c^3*b^2*d/(c*d+e)^2/(c*d-e)^2*dilog((c*e*x
-e)/(-c*d-e))+c^3*b^2*d/(c*d+e)^2/(c*d-e)^2*dilog((c*e*x+e)/(-c*d+e))-1/2*c
^2*a*b/e/(c*d+e)^2*ln(c*x-1)+c^2*b^2*e/(c*d+e)^2/(c*d-e)^2*ln(c*e*x+c*d)+c^
2*b^2/(c*d+e)/(c*d-e)/(2*c*d+2*e)*ln(c*x-1)-c^2*b^2/(c*d+e)/(c*d-e)/(2*c*d-
2*e)*ln(c*x+1)-1/2*c^2*a^2/(c*e*x+c*d)^2/e-2*c^3*a*b*d/(c*d+e)^2/(c*d-e)^2*
ln(c*e*x+c*d)-2*c^3*b^2*arctanh(c*x)*d/(c*d+e)^2/(c*d-e)^2*ln(c*e*x+c*d)-c^

```
3*b^2*d/(c*d+e)^2/(c*d-e)^2*ln(c*e*x+c*d)*ln((c*e*x-e)/(-c*d-e))+c^3*b^2*d/
(c*d+e)^2/(c*d-e)^2*ln(c*e*x+c*d)*ln((c*e*x+e)/(-c*d+e))+1/4*c^2*b^2/e/(c*d
+e)^2*dilog(1/2+1/2*c*x)-1/8*c^2*b^2/e/(c*d-e)^2*ln(c*x+1)^2-1/4*c^2*b^2/e/
(c*d-e)^2*dilog(1/2+1/2*c*x)-1/2*c^2*b^2/(c*e*x+c*d)^2/e*arctanh(c*x)^2-1/8
*c^2*b^2/e/(c*d+e)^2*ln(c*x-1)^2
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(\left(\frac{4\,c^2 d\log{(ex+d)}}{c^4 d^4 - 2\,c^2 d^2 e^2 + e^4} - \frac{c\log{(cx+1)}}{c^2 d^2 e - 2\,cde^2 + e^3} + \frac{c\log{(cx-1)}}{c^2 d^2 e + 2\,cde^2 + e^3} - \frac{2}{c^2 d^3 - de^2 + \left(c^2 d^2 e - e^3\right)x}\right)c + \frac{2\ \text{arta}}{e^3 x^2 + 2\,}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(e*x+d)^3,x, algorithm="maxima")

[Out] -1/2*((4*c^2*d*log(e*x + d)/(c^4*d^4 - 2*c^2*d^2*e^2 + e^4) - c*log(c*x + 1
)/(c^2*d^2*e - 2*c*d*e^2 + e^3) + c*log(c*x - 1)/(c^2*d^2*e + 2*c*d*e^2 + e
^3) - 2/(c^2*d^3 - d*e^2 + (c^2*d^2*e - e^3)*x))*c + 2*arctanh(c*x)/(e^3*x^
2 + 2*d*e^2*x + d^2*e))*a*b - 1/8*b^2*(log(-c*x + 1)^2/(e^3*x^2 + 2*d*e^2*x
 + d^2*e) + 2*integrate(-((c*e*x - e)*log(c*x + 1)^2 + (c*e*x + c*d - 2*(c*
e*x - e)*log(c*x + 1))*log(-c*x + 1))/(c*e^4*x^4 - d^3*e + (3*c*d*e^3 - e^4
)*x^3 + 3*(c*d^2*e^2 - d*e^3)*x^2 + (c*d^3*e - 3*d^2*e^2)*x), x)) - 1/2*a^2
/(e^3*x^2 + 2*d*e^2*x + d^2*e)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\text{atanh}\,(c\,x))^2}{(d + e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/(d + e*x)^3,x)

[Out] int((a + b*atanh(c*x))^2/(d + e*x)^3, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\text{atanh}\,(cx))^2}{(d + ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/(e*x+d)**3,x)

[Out] Integral((a + b*atanh(c*x))**2/(d + e*x)**3, x)
```

3.15 $\int (d + ex)^3 \left(a + b \tanh^{-1}(cx)\right)^3 dx$

Optimal. Leaf size=614

$$-\frac{b^2 e^3 \log\left(\frac{2}{1-cx}\right)\left(a + b\tanh^{-1}(cx)\right)}{2c^4} + \frac{3ab^2 de^2 x}{c^2} + \frac{b^2 e^3 x^2 \left(a + b\tanh^{-1}(cx)\right)}{4c^2} - \frac{3b^2 e\left(6c^2 d^2 + e^2\right)\log\left(\frac{2}{1-cx}\right)\left(a + b t\right.}{2c^4}$$

```
[Out] 3*a*b^2*d*e^2*x/c^2+1/4*b^3*e^3*x/c^3-1/4*b^3*e^3*arctanh(c*x)/c^4+3*b^3*d*
e^2*x*arctanh(c*x)/c^2+1/4*b^2*e^3*x^2*(a+b*arctanh(c*x))/c^2-3/2*b*d*e^2*(
a+b*arctanh(c*x))^2/c^3+1/4*b*e^3*(a+b*arctanh(c*x))^2/c^4+3/4*b*e*(6*c^2*d
^2+e^2)*(a+b*arctanh(c*x))^2/c^4+3/4*b*e*(6*c^2*d^2+e^2)*x*(a+b*arctanh(c*x
))^2/c^3+3/2*b*d*e^2*x^2*(a+b*arctanh(c*x))^2/c+1/4*b*e^3*x^3*(a+b*arctanh(
c*x))^2/c+d*(c^2*d^2+e^2)*(a+b*arctanh(c*x))^3/c^3-1/4*(c^4*d^4+6*c^2*d^2*e
^2+e^4)*(a+b*arctanh(c*x))^3/c^4/e+1/4*(e*x+d)^4*(a+b*arctanh(c*x))^3/e-1/2
*b^2*e^3*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^4-3/2*b^2*e*(6*c^2*d^2+e^2)*(a
+b*arctanh(c*x))*ln(2/(-c*x+1))/c^4-3*b*d*(c^2*d^2+e^2)*(a+b*arctanh(c*x))^
2*ln(2/(-c*x+1))/c^3+3/2*b^3*d*e^2*ln(-c^2*x^2+1)/c^3-1/4*b^3*e^3*polylog(2
,1-2/(-c*x+1))/c^4-3/4*b^3*e*(6*c^2*d^2+e^2)*polylog(2,1-2/(-c*x+1))/c^4-3*
b^2*d*(c^2*d^2+e^2)*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))/c^3+3/2*b^3*
d*(c^2*d^2+e^2)*polylog(3,1-2/(-c*x+1))/c^3
```

Rubi [A] time = 1.18, antiderivative size = 614, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 15, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5928, 5910, 5984, 5918, 2402, 2315, 5916, 5980, 260, 5948, 321, 206, 6048, 6058, 6610}

$$-\frac{3b^2 d\left(c^2 d^2 + e^2\right)\text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)\left(a + b\tanh^{-1}(cx)\right)}{c^3} - \frac{3b^3 e\left(6c^2 d^2 + e^2\right)\text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)}{4c^4} + \frac{3b^3 d\left(c^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (3*a*b^2*d*e^2*x)/c^2 + (b^3*e^3*x)/(4*c^3) - (b^3*e^3*ArcTanh[c*x])/(4*c^4
) + (3*b^3*d*e^2*x*ArcTanh[c*x])/c^2 + (b^2*e^3*x^2*(a + b*ArcTanh[c*x]))/(
4*c^2) - (3*b*d*e^2*(a + b*ArcTanh[c*x])^2)/(2*c^3) + (b*e^3*(a + b*ArcTanh
[c*x])^2)/(4*c^4) + (3*b*e*(6*c^2*d^2 + e^2)*(a + b*ArcTanh[c*x])^2)/(4*c^4
) + (3*b*e*(6*c^2*d^2 + e^2)*x*(a + b*ArcTanh[c*x])^2)/(4*c^3) + (3*b*d*e^2
*x^2*(a + b*ArcTanh[c*x])^2)/(2*c) + (b*e^3*x^3*(a + b*ArcTanh[c*x])^2)/(4*
c) + (d*(c^2*d^2 + e^2)*(a + b*ArcTanh[c*x])^3)/c^3 - ((c^4*d^4 + 6*c^2*d^2
*e^2 + e^4)*(a + b*ArcTanh[c*x])^3)/(4*c^4*e) + ((d + e*x)^4*(a + b*ArcTanh
[c*x])^3)/(4*e) - (b^2*e^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(2*c^4) -
 (3*b^2*e*(6*c^2*d^2 + e^2)*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(2*c^4)
```

```
- (3*b*d*(c^2*d^2 + e^2)*(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x)])/c^3 + (3*
b^3*d*e^2*Log[1 - c^2*x^2])/(2*c^3) - (b^3*e^3*PolyLog[2, 1 - 2/(1 - c*x)])
/(4*c^4) - (3*b^3*e*(6*c^2*d^2 + e^2)*PolyLog[2, 1 - 2/(1 - c*x)])/(4*c^4)
- (3*b^2*d*(c^2*d^2 + e^2)*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)]
)/c^3 + (3*b^3*d*(c^2*d^2 + e^2)*PolyLog[3, 1 - 2/(1 - c*x)])/(2*c^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
```

```
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6048

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])
```

```
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (d + ex)^3 \left(a + b\tanh^{-1}(cx)\right)^3 dx = \frac{(d+ex)^4 \left(a + b\tanh^{-1}(cx)\right)^3}{4e} - \frac{(3bc) \int \left(-\frac{e^2\left(6c^2d^2+e^2\right)\left(a+b\tanh^{-1}(cx)\right)^2}{c^4} - \frac{4de^3x}{\cdots}\right.}{\cdots}$$

$$= \frac{(d+ex)^4 \left(a + b\tanh^{-1}(cx)\right)^3}{4e} - \frac{(3b) \int \frac{\left(c^4d^4+6c^2d^2e^2+e^4+4c^2de\left(c^2d^2+e^2\right)x\right)\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}}{4c^3e}$$

$$= \frac{3be \left(6c^2d^2 + e^2\right) x \left(a + b\tanh^{-1}(cx)\right)^2}{4c^3} + \frac{3bde^2x^2 \left(a + b\tanh^{-1}(cx)\right)^2}{2c} + \cdots$$

$$= \frac{3be \left(6c^2d^2 + e^2\right) \left(a + b\tanh^{-1}(cx)\right)^2}{4c^4} + \frac{3be \left(6c^2d^2 + e^2\right) x \left(a + b\tanh^{-1}(cx)\right)^2}{4c^3} + \cdots$$

$$= \frac{3ab^2de^2x}{c^2} + \frac{b^2e^3x^2 \left(a + b\tanh^{-1}(cx)\right)}{4c^2} - \frac{3bde^2 \left(a + b\tanh^{-1}(cx)\right)^2}{2c^3} + \frac{be^3}{\cdots}$$

$$= \frac{3ab^2de^2x}{c^2} + \frac{b^3e^3x}{4c^3} + \frac{3b^3de^2x\tanh^{-1}(cx)}{c^2} + \frac{b^2e^3x^2 \left(a + b\tanh^{-1}(cx)\right)}{4c^2} - \cdots$$

$$= \frac{3ab^2de^2x}{c^2} + \frac{b^3e^3x}{4c^3} - \frac{b^3e^3\tanh^{-1}(cx)}{4c^4} + \frac{3b^3de^2x\tanh^{-1}(cx)}{c^2} + \frac{b^2e^3x^2 \left(a + \cdots\right)}{\cdots}$$

$$= \frac{3ab^2de^2x}{c^2} + \frac{b^3e^3x}{4c^3} - \frac{b^3e^3\tanh^{-1}(cx)}{4c^4} + \frac{3b^3de^2x\tanh^{-1}(cx)}{c^2} + \frac{b^2e^3x^2 \left(a + \cdots\right)}{\cdots}$$

Mathematica [A]   time = 2.11, size = 830, normalized size = 1.35

$$\frac{2a^3e^3x^4c^4 + 6a^2bx\left(4d^3 + 6exd^2 + 4e^2x^2d + e^3x^3\right)\tanh^{-1}(cx)c^4 + 2a^2e^2(4acd + be)x^3c^3 + 12a^2de(acd + be)x^2c^3 \cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^3*(a + b*ArcTanh[c*x])^3,x]

[Out] (2*a^2*c*(4*a*c^3*d^3 + 3*b*e*(6*c^2*d^2 + e^2))*x + 12*a^2*c^3*d*e*(a*c*d
+ b*e)*x^2 + 2*a^2*c^3*e^2*(4*a*c*d + b*e)*x^3 + 2*a^3*c^4*e^3*x^4 + 6*a^2*
b*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)*ArcTanh[c*x] + 3*a^2*b*
(4*c^3*d^3 + 6*c^2*d^2*e + 4*c*d*e^2 + e^3)*Log[1 - c*x] + 3*a^2*b*(4*c^3*d
^3 - 6*c^2*d^2*e + 4*c*d*e^2 - e^3)*Log[1 + c*x] + 36*a*b^2*c^2*d^2*e*(2*c*
```

```
x*ArcTanh[c*x] + (-1 + c^2*x^2)*ArcTanh[c*x]^2 + Log[1 - c^2*x^2]) + 2*a*b^
2*e^3*(-1 + c^2*x^2 + 2*c*x*(3 + c^2*x^2)*ArcTanh[c*x] + 3*(-1 + c^4*x^4)*A
rcTanh[c*x]^2 + 4*Log[1 - c^2*x^2]) - 12*b^3*c^2*d^2*e*(ArcTanh[c*x]*((3 -
3*c*x)*ArcTanh[c*x] + (1 - c^2*x^2)*ArcTanh[c*x]^2 + 6*Log[1 + E^(-2*ArcTan
h[c*x])]) - 3*PolyLog[2, -E^(-2*ArcTanh[c*x])]) + 24*a*b^2*c*d*e^2*(c*x + (
-1 + c^3*x^3)*ArcTanh[c*x]^2 + ArcTanh[c*x]*(-1 + c^2*x^2 - 2*Log[1 + E^(-2
*ArcTanh[c*x])]) + PolyLog[2, -E^(-2*ArcTanh[c*x])]) + 24*a*b^2*c^3*d^3*(Ar
cTanh[c*x]*((-1 + c*x)*ArcTanh[c*x] - 2*Log[1 + E^(-2*ArcTanh[c*x])]) + Pol
yLog[2, -E^(-2*ArcTanh[c*x])]) + 2*b^3*e^3*(c*x + (-4 + 3*c*x + c^3*x^3)*Ar
cTanh[c*x]^2 + (-1 + c^4*x^4)*ArcTanh[c*x]^3 + ArcTanh[c*x]*(-1 + c^2*x^2 -
 8*Log[1 + E^(-2*ArcTanh[c*x])]) + 4*PolyLog[2, -E^(-2*ArcTanh[c*x])]) + 8*
b^3*c^3*d^3*(ArcTanh[c*x]^2*((-1 + c*x)*ArcTanh[c*x] - 3*Log[1 + E^(-2*ArcT
anh[c*x])]) + 3*ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + (3*PolyLog[
3, -E^(-2*ArcTanh[c*x])])/2) + 4*b^3*c*d*e^2*(6*c*x*ArcTanh[c*x] - 3*ArcTan
h[c*x]^2 + 3*c^2*x^2*ArcTanh[c*x]^2 - 2*ArcTanh[c*x]^3 + 2*c^3*x^3*ArcTanh[
c*x]^3 - 6*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 3*Log[1 - c^2*x^2]
 + 6*ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 3*PolyLog[3, -E^(-2*Ar
cTanh[c*x])])))/(8*c^4)
```

fricas [F]   time = 1.22, size = 0, normalized size = 0.00

$$\text{integral} \left(a^3 e^3 x^3 + 3\,a^3 d e^2 x^2 + 3\,a^3 d^2 e x + a^3 d^3 + \left(b^3 e^3 x^3 + 3\,b^3 d e^2 x^2 + 3\,b^3 d^2 e x + b^3 d^3 \right) \operatorname{artanh}(cx)^3 + 3\left(ab^2 e^3 x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(a+b*arctanh(c*x))^3,x, algorithm="fricas")

[Out] integral(a^3*e^3*x^3 + 3*a^3*d*e^2*x^2 + 3*a^3*d^2*e*x + a^3*d^3 + (b^3*e^3
*x^3 + 3*b^3*d*e^2*x^2 + 3*b^3*d^2*e*x + b^3*d^3)*arctanh(c*x)^3 + 3*(a*b^2
*e^3*x^3 + 3*a*b^2*d*e^2*x^2 + 3*a*b^2*d^2*e*x + a*b^2*d^3)*arctanh(c*x)^2
+ 3*(a^2*b*e^3*x^3 + 3*a^2*b*d*e^2*x^2 + 3*a^2*b*d^2*e*x + a^2*b*d^3)*arcta
nh(c*x), x)

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (ex + d)^3 (b\operatorname{artanh}(cx) + a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(a+b*arctanh(c*x))^3,x, algorithm="giac")

[Out] integrate((e*x + d)^3*(b*arctanh(c*x) + a)^3, x)

maple [C]   time = 11.29, size = 6104, normalized size = 9.94

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*arctanh(c*x))^3,x)
```

```
[Out] result too large to display
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

<center>result too large to display</center>

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^3*e^3*x^4 + a^3*d*e^2*x^3 + 3/2*a^3*d^2*e*x^2 + 9/4*(2*x^2*arctanh(c*
x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*a^2*b*d^2*e + 3/2*(
2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*a^2*b*d*e^2 + 1/8*
(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log
(c*x - 1)/c^5))*a^2*b*e^3 + a^3*d^3*x + 3/2*(2*c*x*arctanh(c*x) + log(-c^2*
x^2 + 1))*a^2*b*d^3/c - 1/32*((b^3*c^4*e^3*x^4 + 4*b^3*c^4*d*e^2*x^3 + 6*b^
3*c^4*d^2*e*x^2 + 4*b^3*c^4*d^3*x - (4*c^3*d^3 + 6*c^2*d^2*e + 4*c*d*e^2 +
e^3)*b^3)*log(-c*x + 1)^3 - (6*a*b^2*c^4*e^3*x^4 + 2*(12*a*b^2*c^4*d*e^2 +
b^3*c^3*e^3)*x^3 + 12*(3*a*b^2*c^4*d^2*e + b^3*c^3*d*e^2)*x^2 + 6*(4*a*b^2*
c^4*d^3 + (6*c^3*d^2*e + c*e^3)*b^3)*x + 3*(b^3*c^4*e^3*x^4 + 4*b^3*c^4*d*e
^2*x^3 + 6*b^3*c^4*d^2*e*x^2 + 4*b^3*c^4*d^3*x + (4*c^3*d^3 - 6*c^2*d^2*e +
 4*c*d*e^2 - e^3)*b^3)*log(c*x + 1))*log(-c*x + 1)^2)/c^4 - integrate(-1/16
*(2*(b^3*c^4*e^3*x^4 - b^3*c^3*d^3 + (3*c^4*d*e^2 - c^3*e^3)*b^3*x^3 + 3*(c
^4*d^2*e - c^3*d*e^2)*b^3*x^2 + (c^4*d^3 - 3*c^3*d^2*e)*b^3*x)*log(c*x + 1)
^3 + 12*(a*b^2*c^4*e^3*x^4 - a*b^2*c^3*d^3 + (3*c^4*d*e^2 - c^3*e^3)*a*b^2*
x^3 + 3*(c^4*d^2*e - c^3*d*e^2)*a*b^2*x^2 + (c^4*d^3 - 3*c^3*d^2*e)*a*b^2*x
)*log(c*x + 1)^2 - (6*a*b^2*c^4*e^3*x^4 + 2*(12*a*b^2*c^4*d*e^2 + b^3*c^3*e
^3)*x^3 + 12*(3*a*b^2*c^4*d^2*e + b^3*c^3*d*e^2)*x^2 + 6*(b^3*c^4*e^3*x^4 -
 b^3*c^3*d^3 + (3*c^4*d*e^2 - c^3*e^3)*b^3*x^3 + 3*(c^4*d^2*e - c^3*d*e^2)*
b^3*x^2 + (c^4*d^3 - 3*c^3*d^2*e)*b^3*x)*log(c*x + 1)^2 + 6*(4*a*b^2*c^4*d^
3 + (6*c^3*d^2*e + c*e^3)*b^3)*x - 3*(8*a*b^2*c^3*d^3 - (8*a*b^2*c^4*e^3 +
b^3*c^4*e^3)*x^4 - (4*c^3*d^3 - 6*c^2*d^2*e + 4*c*d*e^2 - e^3)*b^3 - 4*(b^3
*c^4*d*e^2 + 2*(3*c^4*d*e^2 - c^3*e^3)*a*b^2)*x^3 - 6*(b^3*c^4*d^2*e + 4*(c
^4*d^2*e - c^3*d*e^2)*a*b^2)*x^2 - 4*(b^3*c^4*d^3 + 2*(c^4*d^3 - 3*c^3*d^2*
e)*a*b^2)*x)*log(c*x + 1))*log(-c*x + 1))/(c^4*x - c^3), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + b \operatorname{atanh}\left(c\,x\right)\right)^3 \left(d + e\,x\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3*(d + e*x)^3,x)
```

```
[Out] int((a + b*atanh(c*x))^3*(d + e*x)^3, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}(cx))^3 (d + ex)^3 \ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3*(d + e*x)**3, x)
```

3.16 $\int (d + ex)^2 \left(a + b \tanh^{-1}(cx)\right)^3 dx$

Optimal. Leaf size=387

$$-\frac{6b^2de\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2}+\frac{ab^2e^2x}{c^2}-\frac{b^2\left(3c^2d^2+e^2\right)\mathrm{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3}-\frac{be^2\left(a+b\tanh^{-1}\right)}{2c^3}$$

```
[Out] a*b^2*e^2*x/c^2+b^3*e^2*x*arctanh(c*x)/c^2+3*b*d*e*(a+b*arctanh(c*x))^2/c^2
-1/2*b*e^2*(a+b*arctanh(c*x))^2/c^3+3*b*d*e*x*(a+b*arctanh(c*x))^2/c+1/2*b*
e^2*x^2*(a+b*arctanh(c*x))^2/c+1/3*(3*c^2*d^2+e^2)*(a+b*arctanh(c*x))^3/c^3
-1/3*d*(d^2+3*e^2/c^2)*(a+b*arctanh(c*x))^3/e+1/3*(e*x+d)^3*(a+b*arctanh(c*
x))^3/e-6*b^2*d*e*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^2-b*(3*c^2*d^2+e^2)*(
a+b*arctanh(c*x))^2*ln(2/(-c*x+1))/c^3+1/2*b^3*e^2*ln(-c^2*x^2+1)/c^3-3*b^3
*d*e*polylog(2,1-2/(-c*x+1))/c^2-b^2*(3*c^2*d^2+e^2)*(a+b*arctanh(c*x))*pol
ylog(2,1-2/(-c*x+1))/c^3+1/2*b^3*(3*c^2*d^2+e^2)*polylog(3,1-2/(-c*x+1))/c^
3
```

Rubi [A]   time = 0.80, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {5928, 5910, 5984, 5918, 2402, 2315, 5916, 5980, 260, 5948, 6048, 6058, 6610}

$$-\frac{b^2\left(3c^2d^2+e^2\right)\mathrm{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3}+\frac{b^3\left(3c^2d^2+e^2\right)\mathrm{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2c^3}-\frac{3b^3de\,\mathrm{PolyL}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (a*b^2*e^2*x)/c^2 + (b^3*e^2*x*ArcTanh[c*x])/c^2 + (3*b*d*e*(a + b*ArcTanh[
c*x])^2)/c^2 - (b*e^2*(a + b*ArcTanh[c*x])^2)/(2*c^3) + (3*b*d*e*x*(a + b*A
rcTanh[c*x])^2)/c + (b*e^2*x^2*(a + b*ArcTanh[c*x])^2)/(2*c) + ((3*c^2*d^2
+ e^2)*(a + b*ArcTanh[c*x])^3)/(3*c^3) - (d*(d^2 + (3*e^2)/c^2)*(a + b*ArcT
anh[c*x])^3)/(3*e) + ((d + e*x)^3*(a + b*ArcTanh[c*x])^3)/(3*e) - (6*b^2*d*
e*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c^2 - (b*(3*c^2*d^2 + e^2)*(a + b*
ArcTanh[c*x])^2*Log[2/(1 - c*x)])/c^3 + (b^3*e^2*Log[1 - c^2*x^2])/(2*c^3)
- (3*b^3*d*e*PolyLog[2, 1 - 2/(1 - c*x)])/c^2 - (b^2*(3*c^2*d^2 + e^2)*(a +
 b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)])/c^3 + (b^3*(3*c^2*d^2 + e^2)*
PolyLog[3, 1 - 2/(1 - c*x)])/(2*c^3)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6048

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (d + ex)^2 \left(a + b \tanh^{-1}(cx)\right)^3 \, dx = \frac{(d + ex)^3 \left(a + b \tanh^{-1}(cx)\right)^3}{3e} - \frac{(bc) \int \left(-\frac{3de^2\left(a+b\tanh^{-1}(cx)\right)^2}{c^2} - \frac{e^3 x\left(a+b\tanh^{-1}(c\ldots\right)}{c^2}\right)}{}$$

$$= \frac{(d + ex)^3 \left(a + b \tanh^{-1}(cx)\right)^3}{3e} - \frac{b \int \frac{\left(c^2 d^3 + 3de^2 + e\left(3c^2 d^2 + e^2\right)x\right)\left(a+b\tanh^{-1}(cx)\right)^2}{1 - c^2 x^2} \, dx}{ce}$$

$$= \frac{3bdex \left(a + b \tanh^{-1}(cx)\right)^2}{c} + \frac{be^2 x^2 \left(a + b \tanh^{-1}(cx)\right)^2}{2c} + \frac{(d + ex)^3 \left(a + b \tanh\right.}{3e}$$

$$= \frac{3bde \left(a + b \tanh^{-1}(cx)\right)^2}{c^2} + \frac{3bdex \left(a + b \tanh^{-1}(cx)\right)^2}{c} + \frac{be^2 x^2 \left(a + b \tanh^{-1}\right.}{2c}$$

$$= \frac{ab^2 e^2 x}{c^2} + \frac{3bde \left(a + b \tanh^{-1}(cx)\right)^2}{c^2} - \frac{be^2 \left(a + b \tanh^{-1}(cx)\right)^2}{2c^3} + \frac{3bdex \left(a + \right.}{}$$

$$= \frac{ab^2 e^2 x}{c^2} + \frac{b^3 e^2 x \tanh^{-1}(cx)}{c^2} + \frac{3bde \left(a + b \tanh^{-1}(cx)\right)^2}{c^2} - \frac{be^2 \left(a + b \tanh^{-1}\right.}{2c^3}$$

$$= \frac{ab^2 e^2 x}{c^2} + \frac{b^3 e^2 x \tanh^{-1}(cx)}{c^2} + \frac{3bde \left(a + b \tanh^{-1}(cx)\right)^2}{c^2} - \frac{be^2 \left(a + b \tanh^{-1}\right.}{2c^3}$$

$$= \frac{ab^2 e^2 x}{c^2} + \frac{b^3 e^2 x \tanh^{-1}(cx)}{c^2} + \frac{3bde \left(a + b \tanh^{-1}(cx)\right)^2}{c^2} - \frac{be^2 \left(a + b \tanh^{-1}\right.}{2c^3}$$

Mathematica [A]   time = 1.31, size = 591, normalized size = 1.53

$$2a^3 c^3 e^2 x^3 + 6a^2 bc^3 x \tanh^{-1}(cx)\left(3d^2 + 3dex + e^2 x^2\right) + 3a^2 b \left(3c^2 d^2 + 3cde + e^2\right)\log(1 - cx) + 3a^2 b \left(3c^2 d^2 - 3cde\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d + e*x)^2*(a + b*ArcTanh[c*x])^3,x]

[Out] (6*a^2*c^2*d*(a*c*d + 3*b*e)*x + 3*a^2*c^2*e*(2*a*c*d + b*e)*x^2 + 2*a^3*c^
3*e^2*x^3 + 6*a^2*b*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2)*ArcTanh[c*x] + 3*a^2*
b*(3*c^2*d^2 + 3*c*d*e + e^2)*Log[1 - c*x] + 3*a^2*b*(3*c^2*d^2 - 3*c*d*e +
 e^2)*Log[1 + c*x] + 18*a*b^2*c*d*e*(2*c*x*ArcTanh[c*x] + (-1 + c^2*x^2)*Ar
cTanh[c*x]^2 + Log[1 - c^2*x^2]) - 6*b^3*c*d*e*(ArcTanh[c*x]*((3 - 3*c*x)*A
```

```
rcTanh[c*x] + (1 - c^2*x^2)*ArcTanh[c*x]^2 + 6*Log[1 + E^(-2*ArcTanh[c*x])]
) - 3*PolyLog[2, -E^(-2*ArcTanh[c*x])]) + 6*a*b^2*e^2*(c*x + (-1 + c^3*x^3)
*ArcTanh[c*x]^2 + ArcTanh[c*x]*(-1 + c^2*x^2 - 2*Log[1 + E^(-2*ArcTanh[c*x]
)]) + PolyLog[2, -E^(-2*ArcTanh[c*x])]) + 18*a*b^2*c^2*d^2*(ArcTanh[c*x]*((
-1 + c*x)*ArcTanh[c*x] - 2*Log[1 + E^(-2*ArcTanh[c*x])]) + PolyLog[2, -E^(-
2*ArcTanh[c*x])]) + 6*b^3*c^2*d^2*(ArcTanh[c*x]^2*((-1 + c*x)*ArcTanh[c*x]
 - 3*Log[1 + E^(-2*ArcTanh[c*x])]) + 3*ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTan
h[c*x])] + (3*PolyLog[3, -E^(-2*ArcTanh[c*x])])/2) + b^3*e^2*(6*c*x*ArcTanh
[c*x] - 3*ArcTanh[c*x]^2 + 3*c^2*x^2*ArcTanh[c*x]^2 - 2*ArcTanh[c*x]^3 + 2*
c^3*x^3*ArcTanh[c*x]^3 - 6*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 3*
Log[1 - c^2*x^2] + 6*ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 3*Poly
Log[3, -E^(-2*ArcTanh[c*x])]]))/(6*c^3)
```

fricas [**F**]   time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(a^3 e^2 x^2 + 2\,a^3 dex + a^3 d^2 + \left(b^3 e^2 x^2 + 2\,b^3 dex + b^3 d^2\right)\operatorname{artanh}\left(cx\right)^3 + 3\left(ab^2 e^2 x^2 + 2\,ab^2 dex + ab^2 d^2\right)\text{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*e^2*x^2 + 2*a^3*d*e*x + a^3*d^2 + (b^3*e^2*x^2 + 2*b^3*d*e*x +
 b^3*d^2)*arctanh(c*x)^3 + 3*(a*b^2*e^2*x^2 + 2*a*b^2*d*e*x + a*b^2*d^2)*ar
ctanh(c*x)^2 + 3*(a^2*b*e^2*x^2 + 2*a^2*b*d*e*x + a^2*b*d^2)*arctanh(c*x),
x)
```

giac [**F**]   time = 0.00, size = 0, normalized size = 0.00

$$\int (ex + d)^2 (b\operatorname{artanh}(cx) + a)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*arctanh(c*x) + a)^3, x)
```

maple [**C**]   time = 6.21, size = 4600, normalized size = 11.89

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a+b*arctanh(c*x))^3,x)
```

```
[Out] d*x^2*a^3*e+1/2/c^3*a^2*b*e^2*ln(c*x-1)+1/2/c^3*a^2*b*e^2*ln(c*x+1)-6/c^2*b
^3*e*d*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-6/c^2*b^3*e*d*dilog(1-I*(c*x+1
```

```
)/(-c^2*x^2+1)^(1/2))-1/c^2*b^3*e*d*arctanh(c*x)^3+1/4*a*b^2/e*ln(c*x+1)^2*
d^3+a*b^2*e^2*x/c^2+b^3*e^2*x*arctanh(c*x)/c^2-3/4*I/c^2*b^3*e*Pi*d*csgn(I/
(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/
(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2+a^2*b*e^2*arctanh(c*
x)*x^3+a*b^2*e^2*arctanh(c*x)^2*x^3+b^3*e*arctanh(c*x)^3*x^2*d+3*a*b^2*arct
anh(c*x)^2*x*d^2+1/2*b^3/e*arctanh(c*x)^2*ln(c*x-1)*d^3-1/2*b^3/e*arctanh(c
*x)^2*ln(c*x+1)*d^3+b^3/e*ln((c*x+1)/(-c^2*x^2+1)^(1/2))*arctanh(c*x)^2*d^3
+3*a^2*b*arctanh(c*x)*x*d^2-1/2*a^2*b/e*ln(c*x+1)*d^3+1/4*a*b^2/e*ln(c*x-1)
^2*d^3+a*b^2/e*arctanh(c*x)^2*d^3+a^2*b/e*arctanh(c*x)*d^3+1/2*a^2*b/e*ln(c
*x-1)*d^3+1/2/c*a^2*b*x^2*e^2+3/c^2*b^3*e*arctanh(c*x)^2*d+1/2/c^3*a*b^2*e^
2*ln(c*x-1)-1/2/c^3*a*b^2*e^2*ln(c*x+1)+3/2/c*b^3*arctanh(c*x)^2*ln(c*x-1)*
d^2-3/c*b^3*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))*arctanh(c*x)*d^2-3/c*b^3*ln(
(c*x+1)/(-c^2*x^2+1)^(1/2))*arctanh(c*x)^2*d^2+3/2/c*b^3*arctanh(c*x)^2*ln(
c*x+1)*d^2-3/4/c*a*b^2*ln(c*x+1)^2*d^2+3/4/c*a*b^2*ln(c*x-1)^2*d^2-3/c*a*b^
2*dilog(1/2+1/2*c*x)*d^2-3/c*b^3*ln(2)*d^2*arctanh(c*x)^2+1/2/c*b^3*e^2*arc
tanh(c*x)^2*x^2+3/2/c*a^2*b*ln(c*x-1)*d^2+3/2/c*a^2*b*ln(c*x+1)*d^2-1/c^3*b
^3*e^2*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))*arctanh(c*x)-1/c^3*b^3*e^2*ln((c*
x+1)/(-c^2*x^2+1)^(1/2))*arctanh(c*x)^2+1/2/c^3*b^3*e^2*arctanh(c*x)^2*ln(c
*x-1)+1/2/c^3*b^3*e^2*arctanh(c*x)^2*ln(c*x+1)+1/4/c^3*a*b^2*e^2*ln(c*x-1)^
2-1/c^3*a*b^2*e^2*dilog(1/2+1/2*c*x)-1/4/c^3*a*b^2*e^2*ln(c*x+1)^2-1/c^3*b^
3*e^2*ln(2)*arctanh(c*x)^2+3/4*I/c^2*b^3*e*Pi*d*csgn(I*(c*x+1)^2/(c^2*x^2-1
)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2-3/2*I/c*b^3*Pi*d^2*csgn(I*(c
*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*arctanh(c*x)^2-3/
4*I/c*b^3*Pi*d^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x
^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+3/4*I/c*b^3*Pi*d^2*csgn(
I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^
2+1)))^2*arctanh(c*x)^2-3/4*I/c*b^3*Pi*d^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2
))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2+1/4*I/c^3*b^3*e^2*Pi*csgn
(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x
^2+1)))^2*arctanh(c*x)^2+1/4*I*b^3/e*Pi*d^3*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1
)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)
^2+1/4*I*b^3/e*Pi*d^3*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2
/(c^2*x^2-1))*arctanh(c*x)^2+1/2*I*b^3/e*Pi*d^3*csgn(I*(c*x+1)/(-c^2*x^2+1)
^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*arctanh(c*x)^2-1/4*I*b^3/e*Pi*d^3*c
sgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^
2*x^2+1)))^2*arctanh(c*x)^2-1/4*I/c^3*b^3*e^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*
x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh
(c*x)^2-1/2*I/c^3*b^3*e^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x
+1)^2/(c^2*x^2-1))^2*arctanh(c*x)^2-1/4*I/c^3*b^3*e^2*Pi*csgn(I*(c*x+1)/(-c
^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2+3/4*I/c^2*b
^3*e*Pi*d*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2+3/2*I/c^2*b^3*e*Pi
*d*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-3/2*I/c^2*b^3*e*Pi*d
*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2+6/c*a*b^2*e*arctanh(c*
x)*x*d-3/2/c^2*a*b^2*e*ln(c*x+1)*ln(-1/2*c*x+1/2)*d+3/2/c^2*a*b^2*e*ln(-1/2
*c*x+1/2)*ln(1/2+1/2*c*x)*d-3/2/c^2*a*b^2*e*ln(c*x-1)*ln(1/2+1/2*c*x)*d+3/c
```

```
^2*a*b^2*e*arctanh(c*x)*ln(c*x-1)*d-3/c^2*a*b^2*e*arctanh(c*x)*ln(c*x+1)*d-
1/2*I/c^3*b^3*e^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2-1/
4*I/c^3*b^3*e^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))
^3*arctanh(c*x)^2+1/2*I/c^3*b^3*e^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2
*arctanh(c*x)^2-1/4*I/c^3*b^3*e^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctan
h(c*x)^2-3/2*I/c^2*b^3*e*Pi*d*arctanh(c*x)^2-3/4*I/c*b^3*Pi*d^2*csgn(I*(c*x
+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2-3/4*I/c*b^3*Pi*d^2*csgn(I*(c*x+1)^2/(c^
2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2+3/2*I/c*b^3*Pi*d^2*cs
gn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-3/2*I/c*b^3*Pi*d^2*csgn(I
/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2-1/2*I*b^3/e*Pi*d^3*csgn(I/(1+
(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2+1/2*I*b^3/e*Pi*d^3*csgn(I/(1+(c*x
+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+1/4*I*b^3/e*Pi*d^3*csgn(I*(c*x+1)^2/(
c^2*x^2-1))^3*arctanh(c*x)^2+1/4*I*b^3/e*Pi*d^3*csgn(I*(c*x+1)^2/(c^2*x^2-1
)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2+a^3*x*d^2+1/3*a^3*e^2*x^3-1/
2/c^3*a*b^2*e^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-1/2/c^3*a*b^2*e^2*ln(c*x-1
)*ln(1/2+1/2*c*x)+1/c^3*a*b^2*e^2*arctanh(c*x)*ln(c*x-1)+1/c^3*a*b^2*e^2*ar
ctanh(c*x)*ln(c*x+1)+1/2/c^3*a*b^2*e^2*ln(c*x+1)*ln(-1/2*c*x+1/2)-6/c^2*b^3
*e*d*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-3/2/c^2*b^3*e*arctanh(
c*x)^2*ln(c*x+1)*d+3/2/c^2*b^3*e*arctanh(c*x)^2*ln(c*x-1)*d-6/c^2*b^3*e*d*a
rctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+3/c^2*b^3*e*ln((c*x+1)/(-c^2
*x^2+1)^(1/2))*arctanh(c*x)^2*d+3/2/c^2*a^2*b*e*ln(c*x-1)*d-3/2/c^2*a^2*b*e
*ln(c*x+1)*d+3/c^2*a*b^2*e*ln(c*x+1)*d+3/c^2*a*b^2*e*ln(c*x-1)*d+3/4/c^2*a*
b^2*e*ln(c*x+1)^2*d+3/4/c^2*a*b^2*e*ln(c*x-1)^2*d-1/2*I*b^3/e*Pi*d^3*arctan
h(c*x)^2-3/2*I/c*b^3*Pi*d^2*arctanh(c*x)^2-1/2*I/c^3*b^3*e^2*Pi*arctanh(c*x
)^2+3*a^2*b/c*x*d*e+3*a^2*b*e*arctanh(c*x)*x^2*d+a*b^2/e*arctanh(c*x)*ln(c*
x-1)*d^3-a*b^2/e*arctanh(c*x)*ln(c*x+1)*d^3-1/2*a*b^2/e*ln(c*x+1)*ln(-1/2*c
*x+1/2)*d^3+1/2*a*b^2/e*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)*d^3-1/2*a*b^2/e*ln
(c*x-1)*ln(1/2+1/2*c*x)*d^3+3*a*b^2*e*arctanh(c*x)^2*x^2*d+1/c*a*b^2*e^2*ar
ctanh(c*x)*x^2+3/c*b^3*e*arctanh(c*x)^2*x*d+3/2/c*a*b^2*ln(c*x+1)*ln(-1/2*c
*x+1/2)*d^2-3/2/c*a*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)*d^2-3/2/c*a*b^2*ln
(c*x-1)*ln(1/2+1/2*c*x)*d^2+3/c*a*b^2*arctanh(c*x)*ln(c*x-1)*d^2+3/c*a*b^2*
arctanh(c*x)*ln(c*x+1)*d^2+1/3*b^3*e^2*arctanh(c*x)^3*x^3+b^3*arctanh(c*x)^
3*x*d^2+1/c*b^3*d^2*arctanh(c*x)^3+3/2/c*b^3*polylog(3,-(c*x+1)^2/(-c^2*x^2
+1))*d^2+1/3/c^3*b^3*e^2*arctanh(c*x)^3-1/2/c^3*b^3*e^2*arctanh(c*x)^2+1/2/
c^3*b^3*e^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-1/c^3*b^3*e^2*ln(1+(c*x+1)^2
/(-c^2*x^2+1))+1/c^3*b^3*e^2*arctanh(c*x)+1/3*a^3/e*d^3+3/4*I/c*b^3*Pi*d^2*
csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*
x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2-3/4*I/c^2*b^3
*e*Pi*d*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+
1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+3/4*I/c^2*b^3*e*Pi*d*csgn(I*(c*x+1)/(-
c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2+3/4*I/c^2*
b^3*e*Pi*d*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/
(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+1/4*I/c^3*b^3*e^2*Pi*csgn(I/(1
+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c
^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2+3/2*I/c^2*b^3*e*Pi*d*c
```

```
sgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*arctanh(c
*x)^2-1/4*I*b^3/e*Pi*d^3*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^
2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arc
tanh(c*x)^2
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,a^3e^2x^3 + a^3dex^2 + \frac{3}{2}\left(2\,x^2\,\text{artanh}\,(cx) + c\left(\frac{2\,x}{c^2} - \frac{\log\,(cx+1)}{c^3} + \frac{\log\,(cx-1)}{c^3}\right)\right)a^2bde + \frac{1}{2}\left(2\,x^3\,\text{artanh}\,(cx) + c\left(\frac{x^2}{c^2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(a+b*arctanh(c*x))^3,x, algorithm="maxima")

```
[Out] 1/3*a^3*e^2*x^3 + a^3*d*e*x^2 + 3/2*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(
c*x + 1)/c^3 + log(c*x - 1)/c^3))*a^2*b*d*e + 1/2*(2*x^3*arctanh(c*x) + c*(
x^2/c^2 + log(c^2*x^2 - 1)/c^4))*a^2*b*e^2 + a^3*d^2*x + 3/2*(2*c*x*arctanh
(c*x) + log(-c^2*x^2 + 1))*a^2*b*d^2/c - 1/24*((b^3*c^3*e^2*x^3 + 3*b^3*c^3
*d*e*x^2 + 3*b^3*c^3*d^2*x - (3*c^2*d^2 + 3*c*d*e + e^2)*b^3)*log(-c*x + 1)
^3 - 3*(2*a*b^2*c^3*e^2*x^3 + (6*a*b^2*c^3*d*e + b^3*c^2*e^2)*x^2 + 6*(a*b^
2*c^3*d^2 + b^3*c^2*d*e)*x + (b^3*c^3*e^2*x^3 + 3*b^3*c^3*d*e*x^2 + 3*b^3*c
^3*d^2*x + (3*c^2*d^2 - 3*c*d*e + e^2)*b^3)*log(c*x + 1))*log(-c*x + 1)^2)/
c^3 - integrate(-1/8*((b^3*c^3*e^2*x^3 - b^3*c^2*d^2 + (2*c^3*d*e - c^2*e^2
)*b^3*x^2 + (c^3*d^2 - 2*c^2*d*e)*b^3*x)*log(c*x + 1)^3 + 6*(a*b^2*c^3*e^2*
x^3 - a*b^2*c^2*d^2 + (2*c^3*d*e - c^2*e^2)*a*b^2*x^2 + (c^3*d^2 - 2*c^2*d*
e)*a*b^2*x)*log(c*x + 1)^2 - (4*a*b^2*c^3*e^2*x^3 + 2*(6*a*b^2*c^3*d*e + b^
3*c^2*e^2)*x^2 + 3*(b^3*c^3*e^2*x^3 - b^3*c^2*d^2 + (2*c^3*d*e - c^2*e^2)*b
^3*x^2 + (c^3*d^2 - 2*c^2*d*e)*b^3*x)*log(c*x + 1)^2 + 12*(a*b^2*c^3*d^2 +
b^3*c^2*d*e)*x - 2*(6*a*b^2*c^2*d^2 - (3*c^2*d^2 - 3*c*d*e + e^2)*b^3 - (6*
a*b^2*c^3*e^2 + b^3*c^3*e^2)*x^3 - 3*(b^3*c^3*d*e + 2*(2*c^3*d*e - c^2*e^2)
*a*b^2)*x^2 - 3*(b^3*c^3*d^2 + 2*(c^3*d^2 - 2*c^2*d*e)*a*b^2)*x)*log(c*x +
1))*log(-c*x + 1))/(c^3*x - c^2), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\text{atanh}\,(c\,x))^3\,(d + e\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x))^3*(d + e*x)^2,x)

[Out] int((a + b*atanh(c*x))^3*(d + e*x)^2, x)

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\text{atanh}\,(cx))^3\,(d + ex)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*atanh(c*x))**3,x)
[Out] Integral((a + b*atanh(c*x))**3*(d + e*x)**2, x)
```

3.17 $\int (d + ex)\left(a + b\tanh^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=244

$$-\frac{3b^2e\log\left(\frac{2}{1-cx}\right)\left(a + b\tanh^{-1}(cx)\right)}{c^2} - \frac{3b^2d\mathrm{Li}_2\left(1 - \frac{2}{1-cx}\right)\left(a + b\tanh^{-1}(cx)\right)}{c} - \frac{\left(\frac{e^2}{c^2} + d^2\right)\left(a + b\tanh^{-1}(cx)\right)^3}{2e} + \frac{3be\big(}{}$$

[Out] 3/2*b*e*(a+b*arctanh(c*x))^2/c^2+3/2*b*e*x*(a+b*arctanh(c*x))^2/c+d*(a+b*ar
ctanh(c*x))^3/c-1/2*(d^2+e^2/c^2)*(a+b*arctanh(c*x))^3/e+1/2*(e*x+d)^2*(a+b
*arctanh(c*x))^3/e-3*b^2*e*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^2-3*b*d*(a+b
*arctanh(c*x))^2*ln(2/(-c*x+1))/c-3/2*b^3*e*polylog(2,1-2/(-c*x+1))/c^2-3*b
^2*d*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))/c+3/2*b^3*d*polylog(3,1-2/(
-c*x+1))/c

Rubi [A]   time = 0.60, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5928, 5910, 5984, 5918, 2402, 2315, 6048, 5948, 6058, 6610}

$$-\frac{3b^2d\mathrm{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)\left(a + b\tanh^{-1}(cx)\right)}{c} - \frac{3b^3e\mathrm{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)}{2c^2} + \frac{3b^3d\mathrm{PolyLog}\left(3, 1 - \frac{2}{1-cx}\right)}{2c} - \frac{3b^2e\,\mathrm{lo}}{}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)*(a + b*ArcTanh[c*x])^3,x]

[Out] (3*b*e*(a + b*ArcTanh[c*x])^2)/(2*c^2) + (3*b*e*x*(a + b*ArcTanh[c*x])^2)/(
2*c) + (d*(a + b*ArcTanh[c*x])^3)/c - ((d^2 + e^2/c^2)*(a + b*ArcTanh[c*x])
^3)/(2*e) + ((d + e*x)^2*(a + b*ArcTanh[c*x])^3)/(2*e) - (3*b^2*e*(a + b*Ar
cTanh[c*x])*Log[2/(1 - c*x)])/c^2 - (3*b*d*(a + b*ArcTanh[c*x])^2*Log[2/(1
- c*x)])/c - (3*b^3*e*PolyLog[2, 1 - 2/(1 - c*x)])/(2*c^2) - (3*b^2*d*(a +
b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)])/c + (3*b^3*d*PolyLog[3, 1 - 2/
(1 - c*x)])/(2*c)

Rule 2315

Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]

Rule 2402

Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6048

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
```

```
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (d + ex)\left(a + b\tanh^{-1}(cx)\right)^3 dx = \frac{(d + ex)^2\left(a + b\tanh^{-1}(cx)\right)^3}{2e} - \frac{(3bc)\int\left(-\frac{e^2\left(a+b\tanh^{-1}(cx)\right)^2}{c^2} + \frac{\left(c^2d^2+e^2+2c^2dex\right)\left(\cdots\right)}{c^2\left(1-c\cdots\right)}\right)}{2e}$$

$$= \frac{(d + ex)^2\left(a + b\tanh^{-1}(cx)\right)^3}{2e} - \frac{(3b)\int\frac{\left(c^2d^2+e^2+2c^2dex\right)\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}\,dx}{2ce} + (3b\cdots$$

$$= \frac{3bex\left(a + b\tanh^{-1}(cx)\right)^2}{2c} + \frac{(d + ex)^2\left(a + b\tanh^{-1}(cx)\right)^3}{2e} - \frac{(3b)\int\left(\frac{c^2d^2\left(1+\frac{e\cdots}{c^2\cdots}\right)}{\cdots}\right)}{\cdots}$$

$$= \frac{3be\left(a + b\tanh^{-1}(cx)\right)^2}{2c^2} + \frac{3bex\left(a + b\tanh^{-1}(cx)\right)^2}{2c} + \frac{(d + ex)^2\left(a + b\tanh^{-}\cdots\right)}{2e}$$

$$= \frac{3be\left(a + b\tanh^{-1}(cx)\right)^2}{2c^2} + \frac{3bex\left(a + b\tanh^{-1}(cx)\right)^2}{2c} + \frac{d\left(a + b\tanh^{-1}(cx)\right)^3}{c}$$

$$= \frac{3be\left(a + b\tanh^{-1}(cx)\right)^2}{2c^2} + \frac{3bex\left(a + b\tanh^{-1}(cx)\right)^2}{2c} + \frac{d\left(a + b\tanh^{-1}(cx)\right)^3}{c}$$

$$= \frac{3be\left(a + b\tanh^{-1}(cx)\right)^2}{2c^2} + \frac{3bex\left(a + b\tanh^{-1}(cx)\right)^2}{2c} + \frac{d\left(a + b\tanh^{-1}(cx)\right)^3}{c}$$

$$= \frac{3be\left(a + b\tanh^{-1}(cx)\right)^2}{2c^2} + \frac{3bex\left(a + b\tanh^{-1}(cx)\right)^2}{2c} + \frac{d\left(a + b\tanh^{-1}(cx)\right)^3}{c}$$

Mathematica [A]   time = 0.75, size = 331, normalized size = 1.36

$$2a^3c^2ex^2 + 6a^2bc^2x\tanh^{-1}(cx)(2d + ex) + 2a^2cx(2acd + 3be) + 3a^2b(2cd + e)\log(1 - cx) + 3a^2b(2cd - e)\log(cx)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (2*a^2*c*(2*a*c*d + 3*b*e)*x + 2*a^3*c^2*e*x^2 + 6*a^2*b*c^2*x*(2*d + e*x)*
ArcTanh[c*x] + 3*a^2*b*(2*c*d + e)*Log[1 - c*x] + 3*a^2*b*(2*c*d - e)*Log[1
 + c*x] + 6*a*b^2*e*(2*c*x*ArcTanh[c*x] + (-1 + c^2*x^2)*ArcTanh[c*x]^2 + L
og[1 - c^2*x^2]) - 2*b^3*e*(ArcTanh[c*x]*((3 - 3*c*x)*ArcTanh[c*x] + (1 - c
^2*x^2)*ArcTanh[c*x]^2 + 6*Log[1 + E^(-2*ArcTanh[c*x])]) - 3*PolyLog[2, -E^
(-2*ArcTanh[c*x])]) + 12*a*b^2*c*d*(ArcTanh[c*x]*((-1 + c*x)*ArcTanh[c*x] -
 2*Log[1 + E^(-2*ArcTanh[c*x])]) + PolyLog[2, -E^(-2*ArcTanh[c*x])]) + 4*b^
3*c*d*(ArcTanh[c*x]^2*((-1 + c*x)*ArcTanh[c*x] - 3*Log[1 + E^(-2*ArcTanh[c*
x])]) + 3*ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + (3*PolyLog[3, -E^
(-2*ArcTanh[c*x])])/2))/(4*c^2)
```

fricas [F]   time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(a^3ex + a^3d + \left(b^3ex + b^3d\right)\operatorname{artanh}(cx)^3 + 3\left(ab^2ex + ab^2d\right)\operatorname{artanh}(cx)^2 + 3\left(a^2bex + a^2bd\right)\operatorname{artanh}(cx)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*e*x + a^3*d + (b^3*e*x + b^3*d)*arctanh(c*x)^3 + 3*(a*b^2*e*x
+ a*b^2*d)*arctanh(c*x)^2 + 3*(a^2*b*e*x + a^2*b*d)*arctanh(c*x), x)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (ex + d)(b\operatorname{artanh}(cx) + a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*arctanh(c*x) + a)^3, x)
```

maple [C]   time = 1.25, size = 12404, normalized size = 50.84

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arctanh(c*x))^3,x)
```

```
[Out] result too large to display
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^3ex^2+\frac{3}{4}\left(2x^2\operatorname{artanh}(cx)+c\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)\right)a^2be+a^3dx+\frac{3\left(2cx\operatorname{artanh}(cx)+\log\left(-c^2x^2\right.\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*e*x^2 + 3/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + l
og(c*x - 1)/c^3))*a^2*b*e + a^3*d*x + 3/2*(2*c*x*arctanh(c*x) + log(-c^2*x^
2 + 1))*a^2*b*d/c - 1/16*((b^3*c^2*e*x^2 + 2*b^3*c^2*d*x - (2*c*d + e)*b^3)
*log(-c*x + 1)^3 - 3*(2*a*b^2*c^2*e*x^2 + 2*(2*a*b^2*c^2*d + b^3*c*e)*x + (
b^3*c^2*e*x^2 + 2*b^3*c^2*d*x + (2*c*d - e)*b^3)*log(c*x + 1))*log(-c*x + 1
)^2)/c^2 - integrate(-1/8*((b^3*c^2*e*x^2 - b^3*c*d + (c^2*d - c*e)*b^3*x)*
log(c*x + 1)^3 + 6*(a*b^2*c^2*e*x^2 - a*b^2*c*d + (c^2*d - c*e)*a*b^2*x)*lo
g(c*x + 1)^2 - 3*(2*a*b^2*c^2*e*x^2 + (b^3*c^2*e*x^2 - b^3*c*d + (c^2*d - c
*e)*b^3*x)*log(c*x + 1)^2 + 2*(2*a*b^2*c^2*d + b^3*c*e)*x - (4*a*b^2*c*d -
(2*c*d - e)*b^3 - (4*a*b^2*c^2*e + b^3*c^2*e)*x^2 - 2*(b^3*c^2*d + 2*(c^2*d
 - c*e)*a*b^2)*x)*log(c*x + 1))*log(-c*x + 1))/(c^2*x - c), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\operatorname{atanh}(cx))^3\,(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3*(d + e*x),x)
```

```
[Out] int((a + b*atanh(c*x))^3*(d + e*x), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\operatorname{atanh}(cx))^3\,(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3*(d + e*x), x)
```

3.18 $\displaystyle\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{d+ex}\,dx$

Optimal. Leaf size=272

$$-\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)\mathrm{Li}_3\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2e}+\frac{3b^2\mathrm{Li}_3\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{2e}-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2\mathrm{Li}_2}{2e}$$

```
[Out] -(a+b*arctanh(c*x))^3*ln(2/(c*x+1))/e+(a+b*arctanh(c*x))^3*ln(2*c*(e*x+d)/(
c*d+e)/(c*x+1))/e+3/2*b*(a+b*arctanh(c*x))^2*polylog(2,1-2/(c*x+1))/e-3/2*b
*(a+b*arctanh(c*x))^2*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e+3/2*b^2*(a
+b*arctanh(c*x))*polylog(3,1-2/(c*x+1))/e-3/2*b^2*(a+b*arctanh(c*x))*polylo
g(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e+3/4*b^3*polylog(4,1-2/(c*x+1))/e-3/4*b
^3*polylog(4,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e
```

Rubi [A]   time = 0.06, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {5924}

$$-\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)\mathrm{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2e}+\frac{3b^2\mathrm{PolyLog}\left(3,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{2e}-\frac{3b\left(a+b\,t\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/(d + e*x),x]
```

```
[Out] -(((a + b*ArcTanh[c*x])^3*Log[2/(1 + c*x)])/e) + ((a + b*ArcTanh[c*x])^3*Lo
g[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e + (3*b*(a + b*ArcTanh[c*x])^2*P
olyLog[2, 1 - 2/(1 + c*x)])/(2*e) - (3*b*(a + b*ArcTanh[c*x])^2*PolyLog[2,
1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e) + (3*b^2*(a + b*ArcTanh[c
*x])*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e) - (3*b^2*(a + b*ArcTanh[c*x])*PolyL
og[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e) + (3*b^3*PolyLog[4,
 1 - 2/(1 + c*x)])/(4*e) - (3*b^3*PolyLog[4, 1 - (2*c*(d + e*x))/((c*d + e)
*(1 + c*x))])/(4*e)
```

Rule 5924

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^3/((d_) + (e_.)*(x_)), x_Symbol] :>
 -Simp[((a + b*ArcTanh[c*x])^3*Log[2/(1 + c*x)])/e, x] + (Simp[((a + b*ArcT
anh[c*x])^3*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Simp[(3*b*(
a + b*ArcTanh[c*x])^2*PolyLog[2, 1 - 2/(1 + c*x)])/(2*e), x] - Simp[(3*b*(a
 + b*ArcTanh[c*x])^2*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])
/(2*e), x] + Simp[(3*b^2*(a + b*ArcTanh[c*x])*PolyLog[3, 1 - 2/(1 + c*x)])/
(2*e), x] - Simp[(3*b^2*(a + b*ArcTanh[c*x])*PolyLog[3, 1 - (2*c*(d + e*x))
```

```
/((c*d + e)*(1 + c*x))])/(2*e), x] + Simp[(3*b^3*PolyLog[4, 1 - 2/(1 + c*x)
])/(4*e), x] - Simp[(3*b^3*PolyLog[4, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c
*x))])/(4*e), x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\int \frac{\left(a + b\tanh^{-1}(cx)\right)^3}{d + ex}\,dx = -\frac{\left(a + b\tanh^{-1}(cx)\right)^3 \log\left(\frac{2}{1+cx}\right)}{e} + \frac{\left(a + b\tanh^{-1}(cx)\right)^3 \log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e} + \frac{3b\left(a + b\right.}{}$$

Mathematica [F]   time = 98.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\tanh^{-1}(cx)\right)^3}{d + ex}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/(d + e*x),x]

[Out] Integrate[(a + b*ArcTanh[c*x])^3/(d + e*x), x]
```

fricas [F]   time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^3 \operatorname{artanh}(cx)^3 + 3\,ab^2 \operatorname{artanh}(cx)^2 + 3\,a^2 b\operatorname{artanh}(cx) + a^3}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(e*x+d),x, algorithm="fricas")

[Out] integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x
) + a^3)/(e*x + d), x)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{artanh}(cx) + a)^3}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(e*x+d),x, algorithm="giac")

[Out] integrate((b*arctanh(c*x) + a)^3/(e*x + d), x)
```

maple [C]   time = 0.66, size = 2367, normalized size = 8.70

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/(e*x+d),x)
```

```
[Out] 3/4*c*b^3/e*d/(c*d+e)*polylog(4,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+1/
2*I*b^3/e*arctanh(c*x)^3*Pi*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*
x+1)^2/(-c^2*x^2+1)))/(1+(c*x+1)^2/(-c^2*x^2+1)))^3-3/2*c*a*b^2/e*d/(c*d+e)
*polylog(3,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+c*b^3/e*d/(c*d+e)*arcta
nh(c*x)^3*ln(1-(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+3/2*c*b^3/e*d/(c*d+
e)*arctanh(c*x)^2*polylog(2,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-3/2*c*
b^3/e*d/(c*d+e)*arctanh(c*x)*polylog(3,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d
+e))+3/2*I*a*b^2/e*arctanh(c*x)^2*Pi*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c
*d*(1+(c*x+1)^2/(-c^2*x^2+1)))/(1+(c*x+1)^2/(-c^2*x^2+1)))^3-1/2*I*b^3/e*ar
ctanh(c*x)^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(((c*x+1)^2/(-c^2
*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1)))/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-
1/2*I*b^3/e*arctanh(c*x)^3*Pi*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(
c*x+1)^2/(-c^2*x^2+1)))/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I*(((c*x+1)^2/(-
c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1))))+3*c*a*b^2/e*d/(c*d+e)*arct
anh(c*x)^2*ln(1-(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+3*c*a*b^2/e*d/(c*d
+e)*arctanh(c*x)*polylog(2,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-3/2*I*a
*b^2/e*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(((c*x+1
)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1)))/(1+(c*x+1)^2/(-c^2*x^
2+1)))^2-3/2*I*a*b^2/e*arctanh(c*x)^2*Pi*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)
*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1)))/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I*(((
c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1))))+1/2*I*b^3/e*arc
tanh(c*x)^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(((c*x+1)^2/(-c^2*
x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1)))/(1+(c*x+1)^2/(-c^2*x^2+1)))*csg
n(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1))))-3/4*b^3/
e*polylog(4,-(c*x+1)^2/(-c^2*x^2+1))-3*a*b^2/e*arctanh(c*x)*polylog(2,-(c*x
+1)^2/(-c^2*x^2+1))+3*a*b^2/(c*d+e)*arctanh(c*x)^2*ln(1-(c*d+e)*(c*x+1)^2/(
-c^2*x^2+1)/(-c*d+e))+3*a*b^2/(c*d+e)*arctanh(c*x)*polylog(2,(c*d+e)*(c*x+1
)^2/(-c^2*x^2+1)/(-c*d+e))+3*a^2*b*ln(c*e*x+c*d)/e*arctanh(c*x)+3/2*a^2*b/e
*ln(c*e*x+c*d)*ln((c*e*x-e)/(-c*d-e))-3/2*a^2*b/e*ln(c*e*x+c*d)*ln((c*e*x+e
)/(-c*d+e))+3*a*b^2*ln(c*e*x+c*d)/e*arctanh(c*x)^2-3*a*b^2/e*arctanh(c*x)^2
*ln(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1)))+3/4*b^3/(c
*d+e)*polylog(4,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+a^3*ln(c*e*x+c*d)/
e+3/2*b^3/e*arctanh(c*x)*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+b^3/(c*d+e)*arc
tanh(c*x)^3*ln(1-(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+3/2*b^3/(c*d+e)*a
rctanh(c*x)^2*polylog(2,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-3/2*b^3/(c
*d+e)*arctanh(c*x)*polylog(3,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+b^3*l
n(c*e*x+c*d)/e*arctanh(c*x)^3-b^3/e*arctanh(c*x)^3*ln(((c*x+1)^2/(-c^2*x^2+
```

```
1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1)))-3/2*b^3/e*arctanh(c*x)^2*polylog(2,
-(c*x+1)^2/(-c^2*x^2+1))+3/2*a^2*b/e*dilog((c*e*x-e)/(-c*d-e))-3/2*a^2*b/e*
dilog((c*e*x+e)/(-c*d+e))+3/2*a*b^2/e*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-3/
2*a*b^2/(c*d+e)*polylog(3,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+3/2*I*a*
b^2/e*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(((c*x+1)
^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1)))/(1+(c*x+1)^2/(-c^2*x^2
+1)))*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1))))
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3 \log{(ex+d)}}{e} + \int \frac{b^3\left(\log{(cx+1)} - \log{(-cx+1)}\right)^3}{8\,(ex+d)} + \frac{3\,ab^2\left(\log{(cx+1)} - \log{(-cx+1)}\right)^2}{4\,(ex+d)} + \frac{3\,a^2 b\left(\log{(cx+1)} - \right.}{2\,(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x))^3/(e*x+d),x, algorithm="maxima")

[Out] a^3*log(e*x + d)/e + integrate(1/8*b^3*(log(c*x + 1) - log(-c*x + 1))^3/(e*
x + d) + 3/4*a*b^2*(log(c*x + 1) - log(-c*x + 1))^2/(e*x + d) + 3/2*a^2*b*(
log(c*x + 1) - log(-c*x + 1))/(e*x + d), x)

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{atanh}\,(c\,x))^3}{d + e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x))^3/(d + e*x),x)

[Out] int((a + b*atanh(c*x))^3/(d + e*x), x)

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atanh}\,(cx))^3}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x))**3/(e*x+d),x)

[Out] Integral((a + b*atanh(c*x))**3/(d + e*x), x)

3.19 $\quad \displaystyle\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{(d+ex)^2}\,dx$

Optimal. Leaf size=517

$$-\frac{3b^2c\mathrm{Li}_2\left(1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2-e^2}+\frac{3b^2c\left(a+b\tanh^{-1}(cx)\right)\mathrm{Li}_2\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{c^2d^2-e^2}+\frac{3b^2c\mathrm{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\right.}{2e(cd+e)}$$

```
[Out] -(a+b*arctanh(c*x))^3/e/(e*x+d)+3/2*b*c*(a+b*arctanh(c*x))^2*ln(2/(-c*x+1))
/e/(c*d+e)-3/2*b*c*(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/(c*d-e)/e+3*b*c*(a+b*
arctanh(c*x))^2*ln(2/(c*x+1))/(c^2*d^2-e^2)-3*b*c*(a+b*arctanh(c*x))^2*ln(2
*c*(e*x+d)/(c*d+e)/(c*x+1))/(c^2*d^2-e^2)+3/2*b^2*c*(a+b*arctanh(c*x))*poly
log(2,1-2/(-c*x+1))/e/(c*d+e)+3/2*b^2*c*(a+b*arctanh(c*x))*polylog(2,1-2/(c
*x+1))/(c*d-e)/e-3*b^2*c*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/(c^2*d^2
-e^2)+3*b^2*c*(a+b*arctanh(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/(
c^2*d^2-e^2)-3/4*b^3*c*polylog(3,1-2/(-c*x+1))/e/(c*d+e)+3/4*b^3*c*polylog(
3,1-2/(c*x+1))/(c*d-e)/e-3/2*b^3*c*polylog(3,1-2/(c*x+1))/(c^2*d^2-e^2)+3/2
*b^3*c*polylog(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/(c^2*d^2-e^2)
```

Rubi [A]   time = 0.52, antiderivative size = 517, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5928, 5918, 5948, 6058, 6610, 6056, 5922}

$$-\frac{3b^2c\mathrm{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2-e^2}+\frac{3b^2c\left(a+b\tanh^{-1}(cx)\right)\mathrm{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{c^2d^2-e^2}+\frac{3b^2c\mathrm{Poly}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/(d + e*x)^2,x]
```

```
[Out] -((a + b*ArcTanh[c*x])^3/(e*(d + e*x))) + (3*b*c*(a + b*ArcTanh[c*x])^2*Log
[2/(1 - c*x)])/(2*e*(c*d + e)) - (3*b*c*(a + b*ArcTanh[c*x])^2*Log[2/(1 + c
*x)])/(2*(c*d - e)*e) + (3*b*c*(a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c^
2*d^2 - e^2) - (3*b*c*(a + b*ArcTanh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)
*(1 + c*x))])/(c^2*d^2 - e^2) + (3*b^2*c*(a + b*ArcTanh[c*x])*PolyLog[2, 1
- 2/(1 - c*x)])/(2*e*(c*d + e)) + (3*b^2*c*(a + b*ArcTanh[c*x])*PolyLog[2,
1 - 2/(1 + c*x)])/(2*(c*d - e)*e) - (3*b^2*c*(a + b*ArcTanh[c*x])*PolyLog[2
, 1 - 2/(1 + c*x)])/(c^2*d^2 - e^2) + (3*b^2*c*(a + b*ArcTanh[c*x])*PolyLog
[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(c^2*d^2 - e^2) - (3*b^3*c*
PolyLog[3, 1 - 2/(1 - c*x)])/(4*e*(c*d + e)) + (3*b^3*c*PolyLog[3, 1 - 2/(1
 + c*x)])/(4*(c*d - e)*e) - (3*b^3*c*PolyLog[3, 1 - 2/(1 + c*x)])/(2*(c^2*d
^2 - e^2)) + (3*b^3*c*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))]
)/(2*(c^2*d^2 - e^2))
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
]
```

Rule 5922

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 -Simp[((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e, x] + (Simp[((a + b*ArcT
anh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Simp[(b*(a
+ b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e, x] - Simp[(b*(a + b*ArcTa
nh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Sim
p[(b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e), x] - Simp[(b^2*PolyLog[3, 1 - (2
*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e), x]) /; FreeQ[{a, b, c, d, e},
x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
```

```
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a + b\tanh^{-1}(cx)\right)^3}{(d + ex)^2}\,dx = -\frac{\left(a + b\tanh^{-1}(cx)\right)^3}{e(d + ex)} + \frac{(3bc)\int\left(-\frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{2(cd+e)(-1+cx)} + \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{2(cd-e)(1+cx)} + \frac{e^2\left(a+b\tanh^{-1}(c}{(-cd+e)(cd+e)(a}\right)}{e}$$

$$= -\frac{\left(a + b\tanh^{-1}(cx)\right)^3}{e(d + ex)} + \frac{\left(3bc^2\right)\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1+cx}\,dx}{2(cd - e)e} - \frac{\left(3bc^2\right)\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{-1+cx}\,dx}{2e(cd + e)} +$$

$$= -\frac{\left(a + b\tanh^{-1}(cx)\right)^3}{e(d + ex)} + \frac{3bc\left(a + b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1-cx}\right)}{2e(cd + e)} - \frac{3bc\left(a + b\tanh^{-1}(cx}{2(cd - e)}$$

$$= -\frac{\left(a + b\tanh^{-1}(cx)\right)^3}{e(d + ex)} + \frac{3bc\left(a + b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1-cx}\right)}{2e(cd + e)} - \frac{3bc\left(a + b\tanh^{-1}(cx}{2(cd - e)}$$

$$= -\frac{\left(a + b\tanh^{-1}(cx)\right)^3}{e(d + ex)} + \frac{3bc\left(a + b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1-cx}\right)}{2e(cd + e)} - \frac{3bc\left(a + b\tanh^{-1}(cx}{2(cd - e)}$$

Mathematica [C]   time = 13.85, size = 813, normalized size = 1.57

$$-\frac{a^3}{e(d + ex)} - \frac{3b\tanh^{-1}(cx)a^2}{e(d + ex)} - \frac{3bc\log(1 - cx)a^2}{2e(cd + e)} + \frac{3bc\log(cx + 1)a^2}{2cde - 2e^2} - \frac{3bc\log(d + ex)a^2}{c^2d^2 - e^2} + \frac{3b^2\left(-\frac{e^{-\tanh^{-1}\left(\frac{cd}{e}\right)}\tanh^{-1}(}{\sqrt{1-\frac{c^2d^2}{e^2}}}e\right)}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/(d + e*x)^2,x]

[Out] -(a^3/(e*(d + e*x))) - (3*a^2*b*ArcTanh[c*x])/(e*(d + e*x)) - (3*a^2*b*c*Lo
g[1 - c*x])/(2*e*(c*d + e)) + (3*a^2*b*c*Log[1 + c*x])/(2*c*d*e - 2*e^2) -
```

```
(3*a^2*b*c*Log[d + e*x])/(c^2*d^2 - e^2) + (3*a*b^2*(-(ArcTanh[c*x]^2/(Sqrt
[1 - (c^2*d^2)/e^2]*e*E^ArcTanh[(c*d)/e])) + (x*ArcTanh[c*x]^2)/(d + e*x) +
 (c*d*(I*Pi*Log[1 + E^(2*ArcTanh[c*x])] - 2*ArcTanh[c*x]*Log[1 - E^(-2*(Arc
Tanh[(c*d)/e] + ArcTanh[c*x]))] - I*Pi*(ArcTanh[c*x] - Log[1 - c^2*x^2]/2)
 - 2*ArcTanh[(c*d)/e]*(ArcTanh[c*x] + Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcT
anh[c*x]))] - Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]]) + PolyLog[2, E^
(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]))/(c^2*d^2 - e^2)))/d + (b^3*((x*Ar
cTanh[c*x]^3)/(d + e*x) + (3*(3*c*d*ArcTanh[c*x]^3 - e*ArcTanh[c*x]^3 + (2*
Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^3)/E^ArcTanh[(c*d)/e] + (3*I)*c*d*Pi
*ArcTanh[c*x]*Log[(E^(-ArcTanh[c*x]) + E^ArcTanh[c*x])/2] - 3*c*d*ArcTanh[c
*x]^2*Log[1 - E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] - 3*c*d*ArcTanh[c*x]^2*L
og[1 + E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] - 6*c*d*ArcTanh[(c*d)/e]*ArcTan
h[c*x]*Log[(I/2)*E^(-ArcTanh[(c*d)/e] - ArcTanh[c*x])*(-1 + E^(2*(ArcTanh[(
c*d)/e] + ArcTanh[c*x])))] - 3*c*d*ArcTanh[c*x]^2*Log[(e*(-1 + E^(2*ArcTanh
[c*x])) + c*d*(1 + E^(2*ArcTanh[c*x])))/(2*E^ArcTanh[c*x])] + 3*c*d*ArcTanh
[c*x]^2*Log[(c*(d + e*x))/Sqrt[1 - c^2*x^2]] + ((3*I)/2)*c*d*Pi*ArcTanh[c*x
]*Log[1 - c^2*x^2] + 6*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x]*Log[I*Sinh[ArcTanh
[(c*d)/e] + ArcTanh[c*x]]] - 6*c*d*ArcTanh[c*x]*PolyLog[2, -E^(ArcTanh[(c*d
)/e] + ArcTanh[c*x])] - 6*c*d*ArcTanh[c*x]*PolyLog[2, E^(ArcTanh[(c*d)/e] +
 ArcTanh[c*x])] + 6*c*d*PolyLog[3, -E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] +
6*c*d*PolyLog[3, E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])]))/(3*c^2*d^2 - 3*e^2)
))/d
```

fricas $[\mathbf{F}]$ time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\,\text{artanh}\,(cx)^3 + 3\,ab^2\,\text{artanh}\,(cx)^2 + 3\,a^2b\,\text{artanh}\,(cx) + a^3}{e^2x^2 + 2\,dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x
) + a^3)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

giac $[\mathbf{F}]$ time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{artanh}\,(cx) + a)^3}{(ex + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3/(e*x + d)^2, x)
```

maple [C]   time = 0.89, size = 3497, normalized size = 6.76

<div align="center">output too large to display</div>

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*arctanh(c*x))^3/(e*x+d)^2,x)
```

```
[Out] -c*a^3/(c*e*x+c*d)/e+3/4*I*c^2*b^3/e/(c*d+e)/(c*d-e)*arctanh(c*x)^2*Pi*d*cs
gn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2
*x^2+1)))^2-3/2*I*c^2*b^3/e/(c*d+e)/(c*d-e)*arctanh(c*x)^2*Pi*d*csgn(I*(c*x
+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2-3/4*I*c^2*b^3/e/(c*
d+e)/(c*d-e)*arctanh(c*x)^2*Pi*d*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*
(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-3/4*I*c^2*b^3/e/(c*d+e)
/(c*d-e)*arctanh(c*x)^2*Pi*d*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c
*x+1)^2/(c^2*x^2-1))-3/4*I*c*b^3/(c*d+e)/(c*d-e)*arctanh(c*x)^2*Pi*csgn(I*(
c*x+1)^2/(c^2*x^2-1))^3-3/2*I*c*b^3/(c*d+e)/(c*d-e)*arctanh(c*x)^2*Pi*csgn(
I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1)))/(1+(c*x+1)^
2/(-c^2*x^2+1)))^3-3/2*I*c*b^3/(c*d+e)/(c*d-e)*arctanh(c*x)^2*Pi*csgn(I/(1+
(c*x+1)^2/(-c^2*x^2+1)))^3+3/2*I*c*b^3/(c*d+e)/(c*d-e)*arctanh(c*x)^2*Pi*cs
gn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-3/4*I*c*b^3/(c*d+e)/(c*d-e)*arctanh(c*x)
^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3-c*b^3/(c*e
*x+c*d)/e*arctanh(c*x)^3+c*b^3/e*arctanh(c*x)^3/(c*d-e)+3/4*I*c*b^3/(c*d+e)
/(c*d-e)*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1
)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))-3
/4*I*c^2*b^3/e/(c*d+e)/(c*d-e)*arctanh(c*x)^2*Pi*d*csgn(I*(c*x+1)^2/(c^2*x^
2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3-3/4*I*c^2*b^3/e/(c*d+e)/(c*d-e)*arctanh(
c*x)^2*Pi*d*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-3/2*I*c^2*b^3/e/(c*d+e)/(c*d-e)
*arctanh(c*x)^2*Pi*d*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+3/2*I*c^2*b^3/e/(
c*d+e)/(c*d-e)*arctanh(c*x)^2*Pi*d*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3-3/2
*I*c*b^3/(c*d+e)/(c*d-e)*arctanh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)
))*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1))))*cs
gn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1)))/(1+(c*x+
1)^2/(-c^2*x^2+1)))+3/4*I*c^2*b^3/e/(c*d+e)/(c*d-e)*arctanh(c*x)^2*Pi*d*csg
n(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1
)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))-3*c*b^3/e*arctanh(c*x)^2/(2*c*d
+2*e)*ln(c*x-1)+3*c*b^3/e*arctanh(c*x)^2/(2*c*d-2*e)*ln(c*x+1)-3*c*b^3/e*ar
ctanh(c*x)^2/(c*d-e)*ln((c*x+1)/(-c^2*x^2+1)^(1/2))+3*c*b^3*arctanh(c*x)^2/
(c*d+e)/(c*d-e)*ln(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+
1)))-3*c*b^3*arctanh(c*x)^2/(c*d+e)/(c*d-e)*ln(c*e*x+c*d)-3*c*a*b^2/(c*e*x+
c*d)/e*arctanh(c*x)^2+3/2*c*a*b^2/e/(c*d+e)*dilog(1/2+1/2*c*x)+3/2*c*b^3*e/
(c*d+e)^2/(c*d-e)*polylog(3,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-3*c*b^
3/(c*d+e)/(c*d-e)*arctanh(c*x)^2*ln(2)+3/2*c^2*b^3/(c*d+e)^2/(c*d-e)*d*poly
log(3,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-3*c*a^2*b/(c*d+e)/(c*d-e)*ln
(c*e*x+c*d)-3*c*a^2*b/e/(2*c*d+2*e)*ln(c*x-1)+3*c*a^2*b/e/(2*c*d-2*e)*ln(c*
```

```
x+1)-3*c*a*b^2/(c*d+e)/(c*d-e)*dilog((c*e*x-e)/(-c*d-e))+3*c*a*b^2/(c*d+e)/
(c*d-e)*dilog((c*e*x+e)/(-c*d+e))-3/4*c*a*b^2/e/(c*d-e)*ln(c*x+1)^2-3/2*c*a
*b^2/e/(c*d-e)*dilog(1/2+1/2*c*x)-3/4*c*a*b^2/e/(c*d+e)*ln(c*x-1)^2-3*c*a^2
*b/(c*e*x+c*d)/e*arctanh(c*x)-6*c*a*b^2/e*arctanh(c*x)/(2*c*d+2*e)*ln(c*x-1
)+6*c*a*b^2/e*arctanh(c*x)/(2*c*d-2*e)*ln(c*x+1)-3*c*a*b^2/(c*d+e)/(c*d-e)*
ln(c*e*x+c*d)*ln((c*e*x-e)/(-c*d-e))+3*c*a*b^2/(c*d+e)/(c*d-e)*ln(c*e*x+c*d
)*ln((c*e*x+e)/(-c*d+e))-3*c*b^3*e/(c*d+e)^2/(c*d-e)*arctanh(c*x)^2*ln(1-(c
*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-3*c*b^3*e/(c*d+e)^2/(c*d-e)*arctanh(
c*x)*polylog(2,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-3*c^2*b^3/(c*d+e)^2
/(c*d-e)*d*arctanh(c*x)^2*ln(1-(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-3*c
^2*b^3/(c*d+e)^2/(c*d-e)*d*arctanh(c*x)*polylog(2,(c*d+e)*(c*x+1)^2/(-c^2*x
^2+1)/(-c*d+e))-3/2*c*a*b^2/e/(c*d-e)*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-3/2*
I*c*b^3/(c*d+e)/(c*d-e)*arctanh(c*x)^2*Pi+3/2*c*a*b^2/e/(c*d-e)*ln(-1/2*c*x
+1/2)*ln(c*x+1)+3/2*c*a*b^2/e/(c*d+e)*ln(c*x-1)*ln(1/2+1/2*c*x)-6*c*a*b^2*a
rctanh(c*x)/(c*d+e)/(c*d-e)*ln(c*e*x+c*d)+3/2*I*c*b^3/(c*d+e)/(c*d-e)*arcta
nh(c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(((c*x+1)^2/(-c^2*x^
2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x^2+1)))/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+3/4
*I*c*b^3/(c*d+e)/(c*d-e)*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*cs
gn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2-3/4*I*c*b^3/(c*d+e
)/(c*d-e)*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*
x+1)^2/(c^2*x^2-1))-3/4*I*c*b^3/(c*d+e)/(c*d-e)*arctanh(c*x)^2*Pi*csgn(I/(1
+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x
^2+1)))^2-3/2*I*c*b^3/(c*d+e)/(c*d-e)*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)/(-c^
2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2+3/2*I*c*b^3/(c*d+e)/(c*d-e)
*arctanh(c*x)^2*Pi*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-
c^2*x^2+1))))*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*(1+(c*x+1)^2/(-c^2*x
^2+1)))/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+3/2*I*c^2*b^3/e/(c*d+e)/(c*d-e)*arcta
nh(c*x)^2*Pi*d
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\frac{3}{2}\left(c\left(\frac{\log{(cx+1)}}{cde-e^2}-\frac{\log{(cx-1)}}{cde+e^2}-\frac{2\log{(ex+d)}}{c^2d^2-e^2}\right)-\frac{2\operatorname{artanh}{(cx)}}{e^2x+de}\right)a^2b-\frac{a^3}{e^2x+de}-\frac{\left(\left(c^2de-ce^2\right)b^3x-\left(cde-e^2\right)b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(e*x+d)^2,x, algorithm="maxima")

[Out] 3/2*(c*(log(c*x + 1)/(c*d*e - e^2) - log(c*x - 1)/(c*d*e + e^2) - 2*log(e*x
 + d)/(c^2*d^2 - e^2)) - 2*arctanh(c*x)/(e^2*x + d*e))*a^2*b - a^3/(e^2*x +
 d*e) - 1/8*(((c^2*d*e - c*e^2)*b^3*x - (c*d*e - e^2)*b^3)*log(-c*x + 1)^3
+ 3*(2*(c^2*d^2 - e^2)*a*b^2 - ((c^2*d*e + c*e^2)*b^3*x + (c*d*e + e^2)*b^3
)*log(c*x + 1))*log(-c*x + 1)^2)/(c^2*d^3*e - d*e^3 + (c^2*d^2*e^2 - e^4)*x
) - integrate(1/8*(((c^2*d*e - c*e^2)*b^3*x - (c*d*e - e^2)*b^3)*log(c*x +
1)^3 + 6*((c^2*d*e - c*e^2)*a*b^2*x - (c*d*e - e^2)*a*b^2)*log(c*x + 1)^2 +
```

```
 3*(4*(c^2*d*e - c*e^2)*a*b^2*x + 4*(c^2*d^2 - c*d*e)*a*b^2 - ((c^2*d*e - c
*e^2)*b^3*x - (c*d*e - e^2)*b^3)*log(c*x + 1)^2 - 2*(b^3*c^2*e^2*x^2 + b^3*
c*d*e - 2*(c*d*e - e^2)*a*b^2 + (2*(c^2*d*e - c*e^2)*a*b^2 + (c^2*d*e + c*e
^2)*b^3)*x)*log(c*x + 1))*log(-c*x + 1))/(c*d^3*e - d^2*e^2 - (c^2*d*e^3 -
c*e^4)*x^3 - (2*c^2*d^2*e^2 - 3*c*d*e^3 + e^4)*x^2 - (c^2*d^3*e - 3*c*d^2*e
^2 + 2*d*e^3)*x), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{atanh}\,(c\,x))^3}{(d + e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3/(d + e*x)^2,x)
```

```
[Out] int((a + b*atanh(c*x))^3/(d + e*x)^2, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atanh}\,(cx))^3}{(d + ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/(d + e*x)**2, x)
```

3.20 $\displaystyle\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{(d+ex)^3}\,dx$

Optimal. Leaf size=953

$$-\frac{3c^2\mathrm{Li}_2\left(1-\frac{2}{1-cx}\right)b^3}{4(cd-e)(cd+e)^2}-\frac{3c^2\mathrm{Li}_2\left(1-\frac{2}{cx+1}\right)b^3}{4(cd-e)^2(cd+e)}+\frac{3c^2e\mathrm{Li}_2\left(1-\frac{2}{cx+1}\right)b^3}{2(cd-e)^2(cd+e)^2}-\frac{3c^2e\mathrm{Li}_2\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)b^3}{2(cd-e)^2(cd+e)^2}-\frac{3c^2\mathrm{Li}_3\left(1-\frac{2}{1-cx}\right)}{8e(cd+e)^2}$$

```
[Out] 3/2*b*c*(a+b*arctanh(c*x))^2/(c^2*d^2-e^2)/(e*x+d)-1/2*(a+b*arctanh(c*x))^3
/e/(e*x+d)^2-3/2*b^2*c^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/(c*d-e)/(c*d+e)^
2+3/4*b*c^2*(a+b*arctanh(c*x))^2*ln(2/(-c*x+1))/e/(c*d+e)^2+3/2*b^2*c^2*(a+
b*arctanh(c*x))*ln(2/(c*x+1))/(c*d-e)^2/(c*d+e)-3*b^2*c^2*e*(a+b*arctanh(c*
x))*ln(2/(c*x+1))/(c^2*d^2-e^2)^2-3/4*b*c^2*(a+b*arctanh(c*x))^2*ln(2/(c*x+
1))/(c*d-e)^2/e+3*b*c^3*d*(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/(c^2*d^2-e^2)^
2+3*b^2*c^2*e*(a+b*arctanh(c*x))*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/(c^2*d^2-e
^2)^2-3*b*c^3*d*(a+b*arctanh(c*x))^2*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/(c^2*d
^2-e^2)^2-3/4*b^3*c^2*polylog(2,1-2/(-c*x+1))/(c*d-e)/(c*d+e)^2+3/4*b^2*c^2
*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))/e/(c*d+e)^2-3/4*b^3*c^2*polylog
(2,1-2/(c*x+1))/(c*d-e)^2/(c*d+e)+3/2*b^3*c^2*e*polylog(2,1-2/(c*x+1))/(c^2
*d^2-e^2)^2+3/4*b^2*c^2*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/(c*d-e)^2
/e-3*b^2*c^3*d*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/(c^2*d^2-e^2)^2-3/
2*b^3*c^2*e*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/(c^2*d^2-e^2)^2+3*b^2*
c^3*d*(a+b*arctanh(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/(c^2*d^2-
e^2)^2-3/8*b^3*c^2*polylog(3,1-2/(-c*x+1))/e/(c*d+e)^2+3/8*b^3*c^2*polylog(
3,1-2/(c*x+1))/(c*d-e)^2/e-3/2*b^3*c^3*d*polylog(3,1-2/(c*x+1))/(c^2*d^2-e^
2)^2+3/2*b^3*c^3*d*polylog(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/(c^2*d^2-e^2)^2
```

Rubi [A]   time = 1.03, antiderivative size = 953, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {5928, 5918, 5948, 6058, 6610, 6056, 2402, 2315, 5920, 2447, 5922}

$$-\frac{3c^2\mathrm{PolyLog}\left(2,1-\frac{2}{1-cx}\right)b^3}{4(cd-e)(cd+e)^2}-\frac{3c^2\mathrm{PolyLog}\left(2,1-\frac{2}{cx+1}\right)b^3}{4(cd-e)^2(cd+e)}+\frac{3c^2e\mathrm{PolyLog}\left(2,1-\frac{2}{cx+1}\right)b^3}{2(cd-e)^2(cd+e)^2}-\frac{3c^2e\mathrm{PolyLog}\left(2,1-\right.}{2(cd-e)^2(cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/(d + e*x)^3,x]
```

```
[Out] (3*b*c*(a + b*ArcTanh[c*x])^2)/(2*(c^2*d^2 - e^2)*(d + e*x)) - (a + b*ArcTa
nh[c*x])^3/(2*e*(d + e*x)^2) - (3*b^2*c^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c
*x)])/(2*(c*d - e)*(c*d + e)^2) + (3*b*c^2*(a + b*ArcTanh[c*x])^2*Log[2/(1
```

```
- c*x)])/(4*e*(c*d + e)^2) - (3*b^2*c^2*e*(a + b*ArcTanh[c*x])*Log[2/(1 + c
*x)])/((c*d - e)^2*(c*d + e)^2) + (3*b^2*c^2*(a + b*ArcTanh[c*x])*Log[2/(1
+ c*x)])/(2*(c*d - e)^2*(c*d + e)) - (3*b*c^2*(a + b*ArcTanh[c*x])^2*Log[2/
(1 + c*x)])/(4*(c*d - e)^2*e) + (3*b*c^3*d*(a + b*ArcTanh[c*x])^2*Log[2/(1
+ c*x)])/((c*d - e)^2*(c*d + e)^2) + (3*b^2*c^2*e*(a + b*ArcTanh[c*x])*Log[
(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/((c*d - e)^2*(c*d + e)^2) - (3*b*c^
3*d*(a + b*ArcTanh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/((c*
d - e)^2*(c*d + e)^2) - (3*b^3*c^2*PolyLog[2, 1 - 2/(1 - c*x)])/(4*(c*d - e
)*(c*d + e)^2) + (3*b^2*c^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)
])/(4*e*(c*d + e)^2) + (3*b^3*c^2*e*PolyLog[2, 1 - 2/(1 + c*x)])/(2*(c*d -
e)^2*(c*d + e)^2) - (3*b^3*c^2*PolyLog[2, 1 - 2/(1 + c*x)])/(4*(c*d - e)^2*
(c*d + e)) + (3*b^2*c^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(
4*(c*d - e)^2*e) - (3*b^2*c^3*d*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 +
c*x)])/((c*d - e)^2*(c*d + e)^2) - (3*b^3*c^2*e*PolyLog[2, 1 - (2*c*(d + e*
x))/((c*d + e)*(1 + c*x))])/(2*(c*d - e)^2*(c*d + e)^2) + (3*b^2*c^3*d*(a +
 b*ArcTanh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/((c
*d - e)^2*(c*d + e)^2) - (3*b^3*c^2*PolyLog[3, 1 - 2/(1 - c*x)])/(8*e*(c*d
+ e)^2) + (3*b^3*c^2*PolyLog[3, 1 - 2/(1 + c*x)])/(8*(c*d - e)^2*e) - (3*b^
3*c^3*d*PolyLog[3, 1 - 2/(1 + c*x)])/(2*(c*d - e)^2*(c*d + e)^2) + (3*b^3*c
^3*d*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*(c*d - e)^2*
(c*d + e)^2)
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
```

]

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5922

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 -Simp[((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e, x] + (Simp[((a + b*ArcT
anh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Simp[(b*(a
+ b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e, x] - Simp[(b*(a + b*ArcTa
nh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Sim
p[(b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e), x] - Simp[(b^2*PolyLog[3, 1 - (2
*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e), x]) /; FreeQ[{a, b, c, d, e},
x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a + b\tanh^{-1}(cx)\right)^3}{(d + ex)^3}\, dx = -\frac{\left(a + b\tanh^{-1}(cx)\right)^3}{2e(d + ex)^2} + \frac{(3bc)\int\left(-\frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{2(cd+e)^2(-1+cx)} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{2(cd-e)^2(1+cx)} + \frac{e^2\left(a+b\tanh^{-1}\right)}{(-cd+e)(cd+e)}\right)}{2e}$$

$$= -\frac{\left(a + b\tanh^{-1}(cx)\right)^3}{2e(d + ex)^2} + \frac{\left(3bc^3\right)\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1+cx}\, dx}{4(cd - e)^2 e} - \frac{\left(3bc^3\right)\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{-1+cx}\, dx}{4e(cd + e)^2} -$$

$$= \frac{3bc\left(a + b\tanh^{-1}(cx)\right)^2}{2(cd - e)(cd + e)(d + ex)} - \frac{\left(a + b\tanh^{-1}(cx)\right)^3}{2e(d + ex)^2} + \frac{3bc^2\left(a + b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1-}\right)}{4e(cd + e)^2}$$

$$= \frac{3bc\left(a + b\tanh^{-1}(cx)\right)^2}{2(cd - e)(cd + e)(d + ex)} - \frac{\left(a + b\tanh^{-1}(cx)\right)^3}{2e(d + ex)^2} + \frac{3bc^2\left(a + b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1-}\right)}{4e(cd + e)^2}$$

$$= \frac{3bc\left(a + b\tanh^{-1}(cx)\right)^2}{2(cd - e)(cd + e)(d + ex)} - \frac{\left(a + b\tanh^{-1}(cx)\right)^3}{2e(d + ex)^2} - \frac{3b^2c^2\left(a + b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-}\right)}{2(cd - e)(cd + e)^2}$$

$$= \frac{3bc\left(a + b\tanh^{-1}(cx)\right)^2}{2(cd - e)(cd + e)(d + ex)} - \frac{\left(a + b\tanh^{-1}(cx)\right)^3}{2e(d + ex)^2} - \frac{3b^2c^2\left(a + b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-}\right)}{2(cd - e)(cd + e)^2}$$

$$= \frac{3bc\left(a + b\tanh^{-1}(cx)\right)^2}{2(cd - e)(cd + e)(d + ex)} - \frac{\left(a + b\tanh^{-1}(cx)\right)^3}{2e(d + ex)^2} - \frac{3b^2c^2\left(a + b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-}\right)}{2(cd - e)(cd + e)^2}$$

Mathematica [F]   time = 85.89, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \tanh^{-1}(cx)\right)^3}{(d + ex)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/(d + e*x)^3,x]
```

```
[Out] Integrate[(a + b*ArcTanh[c*x])^3/(d + e*x)^3, x]
```

fricas [F]   time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3 \operatorname{artanh}(cx)^3 + 3\,ab^2 \operatorname{artanh}(cx)^2 + 3\,a^2b \operatorname{artanh}(cx) + a^3}{e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x
) + a^3)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(cx) + a)^3}{(ex + d)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3/(e*x + d)^3, x)
```

maple [C]   time = 3.86, size = 53538, normalized size = 56.18

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/(e*x+d)^3,x)
```

```
[Out] result too large to display
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -3/4*((4*c^2*d*log(e*x + d)/(c^4*d^4 - 2*c^2*d^2*e^2 + e^4) - c*log(c*x + 1
)/(c^2*d^2*e - 2*c*d*e^2 + e^3) + c*log(c*x - 1)/(c^2*d^2*e + 2*c*d*e^2 + e
^3) - 2/(c^2*d^3 - d*e^2 + (c^2*d^2*e - e^3)*x))*c + 2*arctanh(c*x)/(e^3*x^
2 + 2*d*e^2*x + d^2*e))*a^2*b - 1/2*a^3/(e^3*x^2 + 2*d*e^2*x + d^2*e) - 1/1
6*(((c^4*d^2*e^2 - 2*c^3*d*e^3 + c^2*e^4)*b^3*x^2 + 2*(c^4*d^3*e - 2*c^3*d^
2*e^2 + c^2*d*e^3)*b^3*x - (2*c^3*d^3*e - 3*c^2*d^2*e^2 + e^4)*b^3)*log(-c*
x + 1)^3 - 3*(2*(c^3*d^2*e^2 - c*e^4)*b^3*x - 2*(c^4*d^4 - 2*c^2*d^2*e^2 +
e^4)*a*b^2 + 2*(c^3*d^3*e - c*d*e^3)*b^3 + ((c^4*d^2*e^2 + 2*c^3*d*e^3 + c^
2*e^4)*b^3*x^2 + 2*(c^4*d^3*e + 2*c^3*d^2*e^2 + c^2*d*e^3)*b^3*x + (2*c^3*d
^3*e + 3*c^2*d^2*e^2 - e^4)*b^3)*log(c*x + 1))*log(-c*x + 1)^2)/(c^4*d^6*e
- 2*c^2*d^4*e^3 + d^2*e^5 + (c^4*d^4*e^3 - 2*c^2*d^2*e^5 + e^7)*x^2 + 2*(c^
4*d^5*e^2 - 2*c^2*d^3*e^4 + d*e^6)*x) - integrate(1/8*(((c^4*d^3*e - c^3*d^
2*e^2 - c^2*d*e^3 + c*e^4)*b^3*x - (c^3*d^3*e - c^2*d^2*e^2 - c*d*e^3 + e^4
)*b^3)*log(c*x + 1)^3 + 6*((c^4*d^3*e - c^3*d^2*e^2 - c^2*d*e^3 + c*e^4)*a*
b^2*x - (c^3*d^3*e - c^2*d^2*e^2 - c*d*e^3 + e^4)*a*b^2)*log(c*x + 1)^2 - 3
*(2*(c^3*d*e^3 - c^2*e^4)*b^3*x^2 - 2*(c^4*d^4 - c^3*d^3*e - c^2*d^2*e^2 +
c*d*e^3)*a*b^2 + 2*(c^3*d^3*e - c^2*d^2*e^2)*b^3 + ((c^4*d^3*e - c^3*d^2*e^
2 - c^2*d*e^3 + c*e^4)*b^3*x - (c^3*d^3*e - c^2*d^2*e^2 - c*d*e^3 + e^4)*b^
3)*log(c*x + 1)^2 - 2*((c^4*d^3*e - c^3*d^2*e^2 - c^2*d*e^3 + c*e^4)*a*b^2
- 2*(c^3*d^2*e^2 - c^2*d*e^3)*b^3)*x + ((c^4*d*e^3 + c^3*e^4)*b^3*x^3 + 3*(
c^4*d^2*e^2 + c^3*d*e^3)*b^3*x^2 - 4*(c^3*d^3*e - c^2*d^2*e^2 - c*d*e^3 + e
^4)*a*b^2 + (2*c^3*d^3*e + c^2*d^2*e^2 - c*d*e^3)*b^3 + (4*(c^4*d^3*e - c^3
*d^2*e^2 - c^2*d*e^3 + c*e^4)*a*b^2 + (2*c^4*d^3*e + 4*c^3*d^2*e^2 + c^2*d*
e^3 - c*e^4)*b^3)*x)*log(c*x + 1))*log(-c*x + 1))/(c^3*d^6*e - c^2*d^5*e^2
- c*d^4*e^3 + d^3*e^4 - (c^4*d^3*e^4 - c^3*d^2*e^5 - c^2*d*e^6 + c*e^7)*x^4
- (3*c^4*d^4*e^3 - 4*c^3*d^3*e^4 - 2*c^2*d^2*e^5 + 4*c*d*e^6 - e^7)*x^3 -
3*(c^4*d^5*e^2 - 2*c^3*d^4*e^3 + 2*c*d^2*e^5 - d*e^6)*x^2 - (c^4*d^6*e - 4*
c^3*d^5*e^2 + 2*c^2*d^4*e^3 + 4*c*d^3*e^4 - 3*d^2*e^5)*x), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\operatorname{atanh}(c\,x))^3}{(d + e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3/(d + e*x)^3,x)
```

```
[Out] int((a + b*atanh(c*x))^3/(d + e*x)^3, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{atanh}(cx))^3}{(d + ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*atanh(c*x))**3/(e*x+d)**3,x)

[Out] Integral((a + b*atanh(c*x))**3/(d + e*x)**3, x)
```

3.21 $\int \dfrac{a+b\tanh^{-1}(cx)}{1+2cx}\,dx$

Optimal. Leaf size=67

$$\frac{\left(a-b\tanh^{-1}\left(\frac{1}{2}\right)\right)\log\left(-\frac{2cx+1}{2d}\right)}{2c}-\frac{b\mathrm{Li}_2(-2cx-1)}{4c}+\frac{b\mathrm{Li}_2\left(\frac{1}{3}(2cx+1)\right)}{4c}$$

[Out] 1/2*(a-b*arctanh(1/2))*ln(1/2*(-2*c*x-1)/d)/c-1/4*b*polylog(2,-2*c*x-1)/c+1/4*b*polylog(2,2/3*c*x+1/3)/c

Rubi [A]   time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.63, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5920, 2402, 2315, 2447}

$$\frac{b\mathrm{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{4c}-\frac{b\mathrm{PolyLog}\left(2,1-\frac{2(2cx+1)}{3(cx+1)}\right)}{4c}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{2c}+\frac{\log\left(\frac{2(2cx+1)}{3(cx+1)}\right)\left(a+b\tanh^{-}\right)}{2c}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*x])/(1 + 2*c*x),x]

[Out] -((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(2*c) + ((a + b*ArcTanh[c*x])*Log[(2*(1 + 2*c*x))/(3*(1 + c*x))])/(2*c) + (b*PolyLog[2, 1 - 2/(1 + c*x)])/(4*c) - (b*PolyLog[2, 1 - (2*(1 + 2*c*x))/(3*(1 + c*x))])/(4*c)

Rule 2315

Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]

Rule 2402

Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]

Rule 2447

Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\int \frac{a + b \tanh^{-1}(cx)}{1 + 2cx} \, dx = -\frac{\left(a + b \tanh^{-1}(cx)\right) \log\left(\frac{2}{1+cx}\right)}{2c} + \frac{\left(a + b \tanh^{-1}(cx)\right) \log\left(\frac{2(1+2cx)}{3(1+cx)}\right)}{2c} + \frac{1}{2} b \int \frac{\log\left(\frac{2}{1+cx}\right)}{1 - c^2 x^2}$$

$$= -\frac{\left(a + b \tanh^{-1}(cx)\right) \log\left(\frac{2}{1+cx}\right)}{2c} + \frac{\left(a + b \tanh^{-1}(cx)\right) \log\left(\frac{2(1+2cx)}{3(1+cx)}\right)}{2c} - \frac{b \operatorname{Li}_2\left(1 - \frac{2(1+2cx)}{3(1+cx)}\right)}{4c}$$

$$= -\frac{\left(a + b \tanh^{-1}(cx)\right) \log\left(\frac{2}{1+cx}\right)}{2c} + \frac{\left(a + b \tanh^{-1}(cx)\right) \log\left(\frac{2(1+2cx)}{3(1+cx)}\right)}{2c} + \frac{b \operatorname{Li}_2\left(1 - \frac{2}{1+cx}\right)}{4c}$$

Mathematica [C]   time = 0.28, size = 240, normalized size = 3.58

$$a \log(2cx + 1) - \frac{1}{2} ib \left(-\log\left(\frac{2}{\sqrt{1-c^2x^2}}\right)\left(\pi - 2i \tanh^{-1}(cx)\right) - i\operatorname{Li}_2\left(-e^{2\tanh^{-1}(cx)}\right) - i\operatorname{Li}_2\left(e^{-2\left(\tanh^{-1}(cx)+\tanh^{-1}\left(\frac{1}{2}\right)\right)}\right) - $$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(1 + 2*c*x),x]

[Out] (a*Log[1 + 2*c*x] + b*ArcTanh[c*x]*(Log[1 - c^2*x^2]/2 + Log[I*Sinh[ArcTanh
[1/2] + ArcTanh[c*x]]]) - (I/2)*b*((-1/4*I)*(Pi - (2*I)*ArcTanh[c*x])^2 + I
*(ArcTanh[1/2] + ArcTanh[c*x])^2 + (Pi - (2*I)*ArcTanh[c*x])*Log[1 + E^(2*A
rcTanh[c*x])] + (2*I)*(ArcTanh[1/2] + ArcTanh[c*x])*Log[1 - E^(-2*(ArcTanh[
1/2] + ArcTanh[c*x]))] - (Pi - (2*I)*ArcTanh[c*x])*Log[2/Sqrt[1 - c^2*x^2]]
 - (2*I)*(ArcTanh[1/2] + ArcTanh[c*x])*Log[(2*I)*Sinh[ArcTanh[1/2] + ArcTan
h[c*x]]] - I*PolyLog[2, -E^(2*ArcTanh[c*x])] - I*PolyLog[2, E^(-2*(ArcTanh[
1/2] + ArcTanh[c*x]))])))/(2*c)
```

fricas [F]   time = 0.94, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b \operatorname{artanh}(cx) + a}{2 \, cx + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(2*c*x+1),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(2*c*x + 1), x)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{artanh}(cx) + a}{2\,cx + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(2*c*x+1),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/(2*c*x + 1), x)
```

maple [B]   time = 0.04, size = 118, normalized size = 1.76

$$\frac{a\ln(2cx+1)}{2c} + \frac{b\ln(2cx+1)\operatorname{arctanh}(cx)}{2c} + \frac{b\ln\left(\frac{2}{3} - \frac{2cx}{3}\right)\ln(2cx+1)}{4c} - \frac{b\ln\left(\frac{2}{3} - \frac{2cx}{3}\right)\ln\left(\frac{2cx}{3} + \frac{1}{3}\right)}{4c} - \frac{b\operatorname{dilog}\left(\frac{2cx}{3}\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(2*c*x+1),x)
```

```
[Out] 1/2/c*a*ln(2*c*x+1)+1/2/c*b*ln(2*c*x+1)*arctanh(c*x)+1/4/c*b*ln(2/3-2/3*c*x
)*ln(2*c*x+1)-1/4/c*b*ln(2/3-2/3*c*x)*ln(2/3*c*x+1/3)-1/4/c*b*dilog(2/3*c*x
+1/3)-1/4/c*b*dilog(2*c*x+2)-1/4/c*b*ln(2*c*x+1)*ln(2*c*x+2)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}b\int \frac{\log(cx+1) - \log(-cx+1)}{2\,cx+1}\,dx + \frac{a\log(2\,cx+1)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(2*c*x+1),x, algorithm="maxima")
```

```
[Out] 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/(2*c*x + 1), x) + 1/2*a*log(
2*c*x + 1)/c
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\operatorname{atanh}(c\,x)}{2\,c\,x + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(2*c*x + 1),x)
```

```
[Out] int((a + b*atanh(c*x))/(2*c*x + 1), x)
```

sympy [F]   time $= 0.00$, size $= 0$, normalized size $= 0.00$

$$\int \frac{a + b\operatorname{atanh}(cx)}{2cx + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(2*c*x+1),x)
```

```
[Out] Integral((a + b*atanh(c*x))/(2*c*x + 1), x)
```

3.22 $\int \dfrac{\tanh^{-1}(x)}{1-\sqrt{2}\,x}\,dx$

Optimal. Leaf size=88

$$-\frac{\mathrm{Li}_2\left(-\frac{\sqrt{2}-2x}{2-\sqrt{2}}\right)}{2\sqrt{2}}+\frac{\mathrm{Li}_2\left(\frac{\sqrt{2}-2x}{2+\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{1}{\sqrt{2}}\right)\log\left(1-\sqrt{2}\,x\right)}{\sqrt{2}}$$

[Out] -1/2*arctanh(1/2*2^(1/2))*ln(1-x*2^(1/2))*2^(1/2)-1/4*polylog(2,(2*x-2^(1/2
))/(2-2^(1/2)))*2^(1/2)+1/4*polylog(2,(-2*x+2^(1/2))/(2+2^(1/2)))*2^(1/2)

Rubi [A]   time = 0.07, antiderivative size = 108, normalized size of antiderivative =
1.23, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$
= 0.267, Rules used = {5920, 2402, 2315, 2447}

$$-\frac{\mathrm{PolyLog}\left(2,1-\frac{2}{x+1}\right)}{2\sqrt{2}}+\frac{\mathrm{PolyLog}\left(2,\frac{2\left(1+\sqrt{2}\right)\left(1-\sqrt{2}x\right)}{x+1}+1\right)}{2\sqrt{2}}+\frac{\log\left(\frac{2}{x+1}\right)\tanh^{-1}(x)}{\sqrt{2}}-\frac{\log\left(-\frac{2\left(1+\sqrt{2}\right)\left(1-\sqrt{2}x\right)}{x+1}\right)\tanh^{-1}}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[x]/(1 - Sqrt[2]*x),x]

[Out] (ArcTanh[x]*Log[2/(1 + x)])/Sqrt[2] - (ArcTanh[x]*Log[(-2*(1 + Sqrt[2])*(1
- Sqrt[2]*x))/(1 + x)])/Sqrt[2] - PolyLog[2, 1 - 2/(1 + x)]/(2*Sqrt[2]) + P
olyLog[2, 1 + (2*(1 + Sqrt[2])*(1 - Sqrt[2]*x))/(1 + x)]/(2*Sqrt[2])

Rule 2315

Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]

Rule 2402

Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]

Rule 2447

Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(x)}{1 - \sqrt{2}\,x}\,dx = \frac{\tanh^{-1}(x)\log\left(\frac{2}{1+x}\right)}{\sqrt{2}} - \frac{\tanh^{-1}(x)\log\left(-\frac{2\left(1+\sqrt{2}\right)\left(1-\sqrt{2}\,x\right)}{1+x}\right)}{\sqrt{2}} - \frac{\int \frac{\log\left(\frac{2}{1+x}\right)}{1-x^2}\,dx}{\sqrt{2}} + \frac{\int \frac{\log\left(\frac{2\left(1-\sqrt{2}\,x\right)}{\left(1-\sqrt{2}\right)(1+x)}\right)}{1-x^2}\,dx}{\sqrt{2}}$$

$$= \frac{\tanh^{-1}(x)\log\left(\frac{2}{1+x}\right)}{\sqrt{2}} - \frac{\tanh^{-1}(x)\log\left(-\frac{2\left(1+\sqrt{2}\right)\left(1-\sqrt{2}\,x\right)}{1+x}\right)}{\sqrt{2}} + \frac{\text{Li}_2\left(1 + \frac{2\left(1+\sqrt{2}\right)\left(1-\sqrt{2}\,x\right)}{1+x}\right)}{2\sqrt{2}} - \text{Subst}$$

$$= \frac{\tanh^{-1}(x)\log\left(\frac{2}{1+x}\right)}{\sqrt{2}} - \frac{\tanh^{-1}(x)\log\left(-\frac{2\left(1+\sqrt{2}\right)\left(1-\sqrt{2}\,x\right)}{1+x}\right)}{\sqrt{2}} - \frac{\text{Li}_2\left(1 - \frac{2}{1+x}\right)}{2\sqrt{2}} + \frac{\text{Li}_2\left(1 + \frac{2\left(1+\sqrt{2}\right)(1}{1+x}\right)}{2\sqrt{2}}$$

Mathematica [C]   time = 0.10, size = 272, normalized size = 3.09

$$4\text{Li}_2\left(e^{2\tanh^{-1}\left(\frac{1}{\sqrt{2}}\right)-2\tanh^{-1}(x)}\right) + 4\text{Li}_2\left(-e^{2\tanh^{-1}(x)}\right) - 4i\pi\log\left(\frac{2}{\sqrt{1-x^2}}\right) - 8\log\left(\frac{2}{\sqrt{1-x^2}}\right)\tanh^{-1}(x) - 4\log\left(1-x^2\right)\text{t}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[x]/(1 - Sqrt[2]*x),x]

[Out] (Pi^2 - 4*ArcTanh[1/Sqrt[2]]^2 - (4*I)*Pi*ArcTanh[x] + 8*ArcTanh[1/Sqrt[2]]
*ArcTanh[x] - 8*ArcTanh[x]^2 + 8*ArcTanh[1/Sqrt[2]]*Log[1 - E^(2*ArcTanh[1/
Sqrt[2]] - 2*ArcTanh[x])] - 8*ArcTanh[x]*Log[1 - E^(2*ArcTanh[1/Sqrt[2]] -
2*ArcTanh[x])] + (4*I)*Pi*Log[1 + E^(2*ArcTanh[x])] + 8*ArcTanh[x]*Log[1 +
E^(2*ArcTanh[x])] - (4*I)*Pi*Log[2/Sqrt[1 - x^2]] - 8*ArcTanh[x]*Log[2/Sqrt
[1 - x^2]] - 4*ArcTanh[x]*Log[1 - x^2] - 8*ArcTanh[x]*Log[(-I)*Sinh[ArcTanh
[1/Sqrt[2]] - ArcTanh[x]]] - 8*ArcTanh[1/Sqrt[2]]*Log[(-2*I)*Sinh[ArcTanh[1
/Sqrt[2]] - ArcTanh[x]]] + 8*ArcTanh[x]*Log[(-2*I)*Sinh[ArcTanh[1/Sqrt[2]]
- ArcTanh[x]]] + 4*PolyLog[2, E^(2*ArcTanh[1/Sqrt[2]] - 2*ArcTanh[x])] + 4*
PolyLog[2, -E^(2*ArcTanh[x])])/(8*Sqrt[2])
```

fricas [F]　time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(\sqrt{2}\,x+1\right)\operatorname{artanh}(x)}{2\,x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(x)/(1-x*2^(1/2)),x, algorithm="fricas")

[Out] integral(-(sqrt(2)*x + 1)*arctanh(x)/(2*x^2 - 1), x)

giac [F]　time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{artanh}(x)}{\sqrt{2}\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(x)/(1-x*2^(1/2)),x, algorithm="giac")

[Out] integrate(-arctanh(x)/(sqrt(2)*x - 1), x)

maple [A]　time = 0.04, size = 127, normalized size = 1.44

$$-\frac{\ln\left(x\sqrt{2}-1\right)\sqrt{2}\,\operatorname{arctanh}(x)}{2}-\frac{\sqrt{2}\,\ln\left(x\sqrt{2}-1\right)\ln\left(\frac{\sqrt{2}-x\sqrt{2}}{\sqrt{2}-1}\right)}{4}+\frac{\sqrt{2}\,\ln\left(x\sqrt{2}-1\right)\ln\left(\frac{\sqrt{2}+x\sqrt{2}}{1+\sqrt{2}}\right)}{4}-\frac{\sqrt{2}\,\operatorname{dilog}\left(\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(x)/(1-x*2^(1/2)),x)

[Out] -1/2*ln(x*2^(1/2)-1)*2^(1/2)*arctanh(x)-1/4*2^(1/2)*ln(x*2^(1/2)-1)*ln((2^(
1/2)-x*2^(1/2))/(2^(1/2)-1))+1/4*2^(1/2)*ln(x*2^(1/2)-1)*ln((2^(1/2)+x*2^(1
/2))/(1+2^(1/2)))-1/4*2^(1/2)*dilog((2^(1/2)-x*2^(1/2))/(2^(1/2)-1))+1/4*2^
(1/2)*dilog((2^(1/2)+x*2^(1/2))/(1+2^(1/2)))

maxima [B]　time = 0.42, size = 144, normalized size = 1.64

$$\frac{1}{4}\sqrt{2}\left(\log\left(x+1\right)-\log\left(x-1\right)\right)\log\left(\sqrt{2}\,x-1\right)-\frac{1}{2}\sqrt{2}\,\operatorname{artanh}(x)\log\left(\sqrt{2}\,x-1\right)-\frac{1}{4}\sqrt{2}\left(\log\left(x+1\right)\log\left(-\frac{\sqrt{2}\,x-}{\sqrt{2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(x)/(1-x*2^(1/2)),x, algorithm="maxima")

```
[Out] 1/4*sqrt(2)*(log(x + 1) - log(x - 1))*log(sqrt(2)*x - 1) - 1/2*sqrt(2)*arct
anh(x)*log(sqrt(2)*x - 1) - 1/4*sqrt(2)*(log(x + 1)*log(-(sqrt(2)*x + sqrt(
2))/(sqrt(2) + 1) + 1) + dilog((sqrt(2)*x + sqrt(2))/(sqrt(2) + 1))) + 1/4*
sqrt(2)*(log(x - 1)*log((sqrt(2)*x - sqrt(2))/(sqrt(2) - 1) + 1) + dilog(-(
sqrt(2)*x - sqrt(2))/(sqrt(2) - 1)))
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\text{atanh}(x)}{\sqrt{2}\,x - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(x)/(2^(1/2)*x - 1),x)
```

```
[Out] -int(atanh(x)/(2^(1/2)*x - 1), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\text{atanh}(x)}{\sqrt{2}\,x - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x)/(1-x*2**(1/2)),x)
```

```
[Out] -Integral(atanh(x)/(sqrt(2)*x - 1), x)
```

3.23 $\int (d + ex)^3 \left(a + b\tanh^{-1}\left(cx^2\right)\right)\, dx$

Optimal. Leaf size=182

$$\frac{(d + ex)^4 \left(a + b\tanh^{-1}\left(cx^2\right)\right)}{4e} + \frac{bd\left(cd^2 - e^2\right)\tan^{-1}\left(\sqrt{c}\,x\right)}{c^{3/2}} - \frac{bd\left(cd^2 + e^2\right)\tanh^{-1}\left(\sqrt{c}\,x\right)}{c^{3/2}} + \frac{b\left(c^2d^4 + 6cd^2e^2 + e^4\right)}{8c^2e}$$

```
[Out] 2*b*d*e^2*x/c+1/4*b*e^3*x^2/c+b*d*(c*d^2-e^2)*arctan(x*c^(1/2))/c^(3/2)+1/4
*(e*x+d)^4*(a+b*arctanh(c*x^2))/e-b*d*(c*d^2+e^2)*arctanh(x*c^(1/2))/c^(3/2
)+1/8*b*(c^2*d^4+6*c*d^2*e^2+e^4)*ln(-c*x^2+1)/c^2/e-1/8*b*(c^2*d^4-6*c*d^2
*e^2+e^4)*ln(c*x^2+1)/c^2/e
```

Rubi [A] time = 0.22, antiderivative size = 220, normalized size of antiderivative = 1.21, number of steps used = 19, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6742, 6091, 298, 203, 206, 6097, 260, 321, 212, 275}

$$\frac{a(d + ex)^4}{4e} + \frac{3bd^2e\log\left(1 - c^2x^4\right)}{4c} - \frac{bde^2\tan^{-1}\left(\sqrt{c}\,x\right)}{c^{3/2}} - \frac{bde^2\tanh^{-1}\left(\sqrt{c}\,x\right)}{c^{3/2}} - \frac{be^3\tanh^{-1}\left(cx^2\right)}{4c^2} + \frac{3}{2}bd^2ex^2\tanh^{-1}\left(c\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (2*b*d*e^2*x)/c + (b*e^3*x^2)/(4*c) + (a*(d + e*x)^4)/(4*e) + (b*d^3*ArcTan
[Sqrt[c]*x])/Sqrt[c] - (b*d*e^2*ArcTan[Sqrt[c]*x])/c^(3/2) - (b*d^3*ArcTanh
[Sqrt[c]*x])/Sqrt[c] - (b*d*e^2*ArcTanh[Sqrt[c]*x])/c^(3/2) - (b*e^3*ArcTan
h[c*x^2])/(4*c^2) + b*d^3*x*ArcTanh[c*x^2] + (3*b*d^2*e*x^2*ArcTanh[c*x^2])
/2 + b*d*e^2*x^3*ArcTanh[c*x^2] + (b*e^3*x^4*ArcTanh[c*x^2])/4 + (3*b*d^2*e
*Log[1 - c^2*x^4])/(4*c)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]

Rule 321

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 6091

Int[ArcTanh[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTanh[c*x^n], x] - Dist
[c*n, Int[x^n/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]

Rule 6097

Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rule 6742

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]

Rubi steps

$$\int (d + ex)^3 \left(a + b \tanh^{-1}\left(cx^2\right)\right) dx = \int \left(a(d + ex)^3 + b(d + ex)^3 \tanh^{-1}\left(cx^2\right)\right) dx$$

$$= \frac{a(d + ex)^4}{4e} + b \int (d + ex)^3 \tanh^{-1}\left(cx^2\right) dx$$

$$= \frac{a(d + ex)^4}{4e} + b \int \left(d^3 \tanh^{-1}\left(cx^2\right) + 3d^2ex \tanh^{-1}\left(cx^2\right) + 3de^2x^2 \tanh^{-1}\right.$$

$$= \frac{a(d + ex)^4}{4e} + \left(bd^3\right) \int \tanh^{-1}\left(cx^2\right) dx + \left(3bd^2e\right) \int x \tanh^{-1}\left(cx^2\right) dx +$$

$$= \frac{a(d + ex)^4}{4e} + bd^3x \tanh^{-1}\left(cx^2\right) + \frac{3}{2}bd^2ex^2 \tanh^{-1}\left(cx^2\right) + bde^2x^3 \tanh^{-1}$$

$$= \frac{2bde^2x}{c} + \frac{a(d + ex)^4}{4e} + bd^3x \tanh^{-1}\left(cx^2\right) + \frac{3}{2}bd^2ex^2 \tanh^{-1}\left(cx^2\right) + bde^2$$

$$= \frac{2bde^2x}{c} + \frac{be^3x^2}{4c} + \frac{a(d + ex)^4}{4e} + \frac{bd^3 \tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} - \frac{bd^3 \tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} +$$

$$= \frac{2bde^2x}{c} + \frac{be^3x^2}{4c} + \frac{a(d + ex)^4}{4e} + \frac{bd^3 \tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} - \frac{bde^2 \tan^{-1}\left(\sqrt{c}\,x\right)}{c^{3/2}} -$$

Mathematica [A]   time = 0.28, size = 254, normalized size = 1.40

$$\frac{1}{8} \left(\frac{2ex^2 \left(6acd^2 + be^2\right)}{c} + \frac{8dx \left(acd^2 + 2be^2\right)}{c} + 8ade^2x^3 + 2ae^3x^4 + \frac{8bd \left(cd^2 - e^2\right) \tan^{-1}\left(\sqrt{c}\,x\right)}{c^{3/2}} + \frac{6bd^2e \log \left(1 - \right.}{c} \right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*ArcTanh[c*x^2]),x]

[Out] ((8*d*(a*c*d^2 + 2*b*e^2)*x)/c + (2*e*(6*a*c*d^2 + b*e^2)*x^2)/c + 8*a*d*e^
2*x^3 + 2*a*e^3*x^4 + (8*b*d*(c*d^2 - e^2)*ArcTan[Sqrt[c]*x])/c^(3/2) + 2*b
*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)*ArcTanh[c*x^2] + (b*(4*c^(3/
2)*d^3 + 4*Sqrt[c]*d*e^2 + e^3)*Log[1 - Sqrt[c]*x])/c^2 + (b*(-4*c^2*d^3 -
4*c*d*e^2 + Sqrt[c]*e^3)*Log[1 + Sqrt[c]*x])/c^(5/2) - (b*e^3*Log[1 + c*x^2
])/c^2 + (6*b*d^2*e*Log[1 - c^2*x^4])/c)/8
```

fricas [A]   time = 0.95, size = 519, normalized size = 2.85

$$\left[\frac{2\,ac^2e^3x^4 + 8\,ac^2de^2x^3 + 2\left(6\,ac^2d^2e + bce^3\right)x^2 + 8\left(bcd^3 - bde^2\right)\sqrt{c}\,\arctan\left(\sqrt{c}\,x\right) + 4\left(bcd^3 + bde^2\right)\sqrt{c}\,\log\left(\frac{cx}{} \right.}{} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arctanh(c*x^2)),x, algorithm="fricas")

[Out] [1/8*(2*a*c^2*e^3*x^4 + 8*a*c^2*d*e^2*x^3 + 2*(6*a*c^2*d^2*e + b*c*e^3)*x^2
 + 8*(b*c*d^3 - b*d*e^2)*sqrt(c)*arctan(sqrt(c)*x) + 4*(b*c*d^3 + b*d*e^2)*
sqrt(c)*log((c*x^2 - 2*sqrt(c)*x + 1)/(c*x^2 - 1)) + 8*(a*c^2*d^3 + 2*b*c*d
*e^2)*x + (6*b*c*d^2*e - b*e^3)*log(c*x^2 + 1) + (6*b*c*d^2*e + b*e^3)*log(
c*x^2 - 1) + (b*c^2*e^3*x^4 + 4*b*c^2*d*e^2*x^3 + 6*b*c^2*d^2*e*x^2 + 4*b*c
^2*d^3*x)*log(-(c*x^2 + 1)/(c*x^2 - 1)))/c^2, 1/8*(2*a*c^2*e^3*x^4 + 8*a*c^
2*d*e^2*x^3 + 2*(6*a*c^2*d^2*e + b*c*e^3)*x^2 + 8*(b*c*d^3 + b*d*e^2)*sqrt(
-c)*arctan(sqrt(-c)*x) + 4*(b*c*d^3 - b*d*e^2)*sqrt(-c)*log((c*x^2 + 2*sqrt
(-c)*x - 1)/(c*x^2 + 1)) + 8*(a*c^2*d^3 + 2*b*c*d*e^2)*x + (6*b*c*d^2*e - b
*e^3)*log(c*x^2 + 1) + (6*b*c*d^2*e + b*e^3)*log(c*x^2 - 1) + (b*c^2*e^3*x^
4 + 4*b*c^2*d*e^2*x^3 + 6*b*c^2*d^2*e*x^2 + 4*b*c^2*d^3*x)*log(-(c*x^2 + 1)
/(c*x^2 - 1)))/c^2]
```

giac [A]   time = 66.41, size = 290, normalized size = 1.59

$$\frac{\left(bcd^3 - bde^2\right)\arctan\left(\sqrt{c}\,x\right)}{c^{\frac{3}{2}}} + \frac{\left(bcd^3 + bde^2\right)\arctan\left(\frac{cx}{\sqrt{-c}}\right)}{\sqrt{-c}\,c} + \frac{bc^2x^4e^3\log\left(-\frac{cx^2+1}{cx^2-1}\right) + 4\,bc^2dx^3e^2\log\left(-\frac{cx^2+1}{cx^2-1}\right) + 6\,bc}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arctanh(c*x^2)),x, algorithm="giac")

[Out] (b*c*d^3 - b*d*e^2)*arctan(sqrt(c)*x)/c^(3/2) + (b*c*d^3 + b*d*e^2)*arctan(
c*x/sqrt(-c))/(sqrt(-c)*c) + 1/8*(b*c^2*x^4*e^3*log(-(c*x^2 + 1)/(c*x^2 - 1
)) + 4*b*c^2*d*x^3*e^2*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 6*b*c^2*d^2*x^2*e*lo
g(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c^2*x^4*e^3 + 8*a*c^2*d*x^3*e^2 + 12*a*c^
2*d^2*x^2*e + 4*b*c^2*d^3*x*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 8*a*c^2*d^3*x +
 6*b*c*d^2*e*log(c^2*x^4 - 1) + 2*b*c*x^2*e^3 + 16*b*c*d*x*e^2 - b*e^3*log(
c*x^2 + 1) + b*e^3*log(c*x^2 - 1))/c^2
```

maple [A]   time = 0.04, size = 306, normalized size = 1.68

$$\frac{a\,x^4e^3}{4} + ad\,e^2x^3 + \frac{3a\,d^2e\,x^2}{2} + ax\,d^3 + \frac{a\,d^4}{4e} + \frac{b\,e^3\arctan\left(c\,x^2\right)x^4}{4} + b\,e^2\arctan\left(c\,x^2\right)x^3d + \frac{3be\arctan\left(c\,x^2\right)x^2d^2}{2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*arctanh(c*x^2)),x)

[Out] 1/4*a*x^4*e^3+a*d*e^2*x^3+3/2*a*d^2*e*x^2+a*x*d^3+1/4*a/e*d^4+1/4*b*e^3*arc
tanh(c*x^2)*x^4+b*e^2*arctanh(c*x^2)*x^3*d+3/2*b*e*arctanh(c*x^2)*x^2*d^2+b
```

```
*arctanh(c*x^2)*x*d^3+1/4*b/e*arctanh(c*x^2)*d^4+1/4*b*e^3*x^2/c+2*b*d*e^2*
x/c-1/8*b/e*ln(c*x^2+1)*d^4+3/4*b*e/c*ln(c*x^2+1)*d^2-1/8*b*e^3/c^2*ln(c*x^
2+1)+b/c^(1/2)*arctan(x*c^(1/2))*d^3-b*e^2/c^(3/2)*arctan(x*c^(1/2))*d+1/8*
b/e*ln(c*x^2-1)*d^4+3/4*b*e/c*ln(c*x^2-1)*d^2+1/8*b*e^3/c^2*ln(c*x^2-1)-b/c
^(1/2)*arctanh(x*c^(1/2))*d^3-b*e^2/c^(3/2)*arctanh(x*c^(1/2))*d
```

maxima $[A]$　　time $= 0.42$, size $= 240$, normalized size $= 1.32$

$$\frac{1}{4}\,ae^3x^4+ade^2x^3+\frac{3}{2}\,ad^2ex^2+\frac{1}{2}\left(c\left(\frac{2\arctan\left(\sqrt{c}\,x\right)}{c^{\frac{3}{2}}}+\frac{\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)+2\,x\,\mathrm{artanh}\left(cx^2\right)\right)bd^3+\frac{1}{2}\left(2\,x^3\,\mathrm{artanh}\left(cx^2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^3*(a+b*arctanh(c*x^2)),x, algorithm="maxima")`

[Out] `1/4*a*e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + 1/2*(c*(2*arctan(sqrt(c)*x)`
`/c^(3/2) + log((c*x - sqrt(c))/(c*x + sqrt(c)))/c^(3/2)) + 2*x*arctanh(c*x^`
`2))*b*d^3 + 1/2*(2*x^3*arctanh(c*x^2) + c*(4*x/c^2 - 2*arctan(sqrt(c)*x)/c^`
`(5/2) + log((c*x - sqrt(c))/(c*x + sqrt(c)))/c^(5/2)))*b*d*e^2 + 1/8*(2*x^4`
`*arctanh(c*x^2) + c*(2*x^2/c^2 - log(c*x^2 + 1)/c^3 + log(c*x^2 - 1)/c^3))*`
`b*e^3 + a*d^3*x + 3/4*(2*c*x^2*arctanh(c*x^2) + log(-c^2*x^4 + 1))*b*d^2*e/`
`c`

mupad $[B]$　　time $= 2.04$, size $= 823$, normalized size $= 4.52$

$$\ln\left(cx^2+1\right)\left(\frac{bd^3x}{2}+\frac{3bd^2ex^2}{4}+\frac{bde^2x^3}{2}+\frac{be^3x^4}{8}\right)-\ln\left(1-cx^2\right)\left(\frac{bd^3x}{2}+\frac{3bd^2ex^2}{4}+\frac{bde^2x^3}{2}+\frac{be^3x^4}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*atanh(c*x^2))*(d + e*x)^3,x)`

[Out] `log(c*x^2 + 1)*((b*e^3*x^4)/8 + (b*d^3*x)/2 + (3*b*d^2*e*x^2)/4 + (b*d*e^2*`
`x^3)/2) - log(1 - c*x^2)*((b*e^3*x^4)/8 + (b*d^3*x)/2 + (3*b*d^2*e*x^2)/4 +`
` (b*d*e^2*x^3)/2) + (a*e^3*x^4)/4 - (log(8*c^5*d^6 - c^2*e^6 - 4*d*e^5*(-c^`
`5)^(1/2) + e^6*x*(-c^5)^(1/2) + 8*c^3*d^2*e^4 + 4*c^4*d^3*e^3*x - 4*c^3*d*e`
`^5*x + 4*c*d^3*e^3*(-c^5)^(1/2) - 8*c^3*d^6*x*(-c^5)^(1/2) - 8*c*d^2*e^4*x*`
`(-c^5)^(1/2))*(b*c^2*e^3 - 4*b*c*d^3*(-c^5)^(1/2) + 4*b*d*e^2*(-c^5)^(1/2)`
` - 6*b*c^3*d^2*e))/(8*c^4) - (log(8*c^5*d^6 - c^2*e^6 + 4*d*e^5*(-c^5)^(1/2)`
` - e^6*x*(-c^5)^(1/2) + 8*c^3*d^2*e^4 + 4*c^4*d^3*e^3*x - 4*c^3*d*e^5*x - 4`
`*c*d^3*e^3*(-c^5)^(1/2) + 8*c^3*d^6*x*(-c^5)^(1/2) + 8*c*d^2*e^4*x*(-c^5)^(`
`1/2))*(b*c^2*e^3 + 4*b*c*d^3*(-c^5)^(1/2) - 4*b*d*e^2*(-c^5)^(1/2) - 6*b*c^`
`3*d^2*e))/(8*c^4) + (x*(2*a*c^2*d^3 + 4*b*c*d*e^2))/(2*c^2) + (log(64*c^2*d`
`^12*(c^5)^(7/2) + 128*d^8*e^4*(c^5)^(7/2) - 64*c^20*d^12*x - c^14*e^12*x +`
```

```
c*e^12*(c^5)^(5/2) - 32*c^16*d^4*e^8*x - 128*c^18*d^8*e^4*x + 32*c^3*d^4*e^
8*(c^5)^(5/2))*(b*c^2*e^3 + 4*b*c*d^3*(c^5)^(1/2) + 4*b*d*e^2*(c^5)^(1/2) +
 6*b*c^3*d^2*e))/(8*c^4) + (log(8*c^10*d^6 + c^7*e^6 + 8*c^8*d^2*e^4 - 4*d*
e^5*(c^5)^(3/2) + e^6*x*(c^5)^(3/2) - 4*c^9*d^3*e^3*x - 4*c*d^3*e^3*(c^5)^(
3/2) + 8*c^3*d^6*x*(c^5)^(3/2) - 4*c^8*d*e^5*x + 8*c*d^2*e^4*x*(c^5)^(3/2))
*(b*c^2*e^3 - 4*b*c*d^3*(c^5)^(1/2) - 4*b*d*e^2*(c^5)^(1/2) + 6*b*c^3*d^2*e
))/(8*c^4) + (x^2*(b*c*e^3 + 6*a*c^2*d^2*e))/(4*c^2) + a*d*e^2*x^3
```

sympy **[A]**   time = 18.88, size = 622, normalized size = 3.42

$$\begin{cases} ad^3x + \frac{3ad^2ex^2}{2} + ade^2x^3 + \frac{ae^3x^4}{4} + \frac{bcd^3\left(\frac{1}{c}\right)^{\frac{3}{2}}\log\left(x+i\sqrt{\frac{1}{c}}\right)}{4} - \frac{ibcd^3\left(\frac{1}{c}\right)^{\frac{3}{2}}\log\left(x+i\sqrt{\frac{1}{c}}\right)}{4} + bd^3x\operatorname{atanh}\left(cx^2\right) - \frac{bd^3\sqrt{\frac{1}{c}}\log\left(x-i\sqrt{\frac{1}{c}}\right)}{2} \\ a\left(d^3x + \frac{3d^2ex^2}{2} + de^2x^3 + \frac{e^3x^4}{4}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*atanh(c*x**2)),x)
```

```
[Out] Piecewise((a*d**3*x + 3*a*d**2*e*x**2/2 + a*d*e**2*x**3 + a*e**3*x**4/4 + b
*c*d**3*(1/c)**(3/2)*log(x + I*sqrt(1/c))/4 - I*b*c*d**3*(1/c)**(3/2)*log(x
 + I*sqrt(1/c))/4 + b*d**3*x*atanh(c*x**2) - b*d**3*sqrt(1/c)*log(x - I*sqr
t(1/c))/2 - I*b*d**3*sqrt(1/c)*log(x - I*sqrt(1/c))/2 - 3*b*d**3*sqrt(1/c)*
log(x + I*sqrt(1/c))/4 + 3*I*b*d**3*sqrt(1/c)*log(x + I*sqrt(1/c))/4 + b*d*
*3*sqrt(1/c)*log(x - sqrt(1/c)) + b*d**3*sqrt(1/c)*atanh(c*x**2) + 3*b*d**2
*e*x**2*atanh(c*x**2)/2 + b*d*e**2*x**3*atanh(c*x**2) + b*e**3*x**4*atanh(c
*x**2)/4 + 3*b*d**2*e*log(x - I*sqrt(1/c))/(2*c) + 3*b*d**2*e*log(x + I*sqr
t(1/c))/(2*c) - 3*b*d**2*e*atanh(c*x**2)/(2*c) + 2*b*d*e**2*x/c - b*d*e**2*
sqrt(1/c)*log(x - I*sqrt(1/c))/(2*c) + I*b*d*e**2*sqrt(1/c)*log(x - I*sqrt(
1/c))/(2*c) - b*d*e**2*sqrt(1/c)*log(x + I*sqrt(1/c))/(2*c) - I*b*d*e**2*sq
rt(1/c)*log(x + I*sqrt(1/c))/(2*c) + b*d*e**2*sqrt(1/c)*log(x - sqrt(1/c))/
c + b*d*e**2*sqrt(1/c)*atanh(c*x**2)/c + b*e**3*x**2/(4*c) - b*e**3*atanh(c
*x**2)/(4*c**2), Ne(c, 0)), (a*(d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 + e*
*3*x**4/4), True))
```

3.24 $\int (d + ex)^2 \left(a + b \tanh^{-1} \left(cx^2 \right) \right) dx$

Optimal. Leaf size=158

$$\frac{(d + ex)^3 \left(a + b \tanh^{-1} \left(cx^2 \right) \right)}{3e} + \frac{b \left(3cd^2 - e^2 \right) \tan^{-1} \left(\sqrt{c}\, x \right)}{3c^{3/2}} - \frac{b \left(3cd^2 + e^2 \right) \tanh^{-1} \left(\sqrt{c}\, x \right)}{3c^{3/2}} + \frac{bd \left(cd^2 + 3e^2 \right) \log \left(1 - \right.}{6ce}$$

```
[Out]  2/3*b*e^2*x/c+1/3*b*(3*c*d^2-e^2)*arctan(x*c^(1/2))/c^(3/2)+1/3*(e*x+d)^3*(
a+b*arctanh(c*x^2))/e-1/3*b*(3*c*d^2+e^2)*arctanh(x*c^(1/2))/c^(3/2)+1/6*b*
d*(c*d^2+3*e^2)*ln(-c*x^2+1)/c/e-1/6*b*d*(c*d^2-3*e^2)*ln(c*x^2+1)/c/e
```

Rubi [A]   time = 0.21, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6273, 12, 1831, 1248, 633, 31, 1280, 1167, 205, 208}

$$\frac{(d + ex)^3 \left(a + b \tanh^{-1} \left(cx^2 \right) \right)}{3e} + \frac{b \left(3cd^2 - e^2 \right) \tan^{-1} \left(\sqrt{c}\, x \right)}{3c^{3/2}} - \frac{b \left(3cd^2 + e^2 \right) \tanh^{-1} \left(\sqrt{c}\, x \right)}{3c^{3/2}} + \frac{bd \left(cd^2 + 3e^2 \right) \log \left(1 - \right.}{6ce}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)^2*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out]  (2*b*e^2*x)/(3*c) + (b*(3*c*d^2 - e^2)*ArcTan[Sqrt[c]*x])/(3*c^(3/2)) - (b*
(3*c*d^2 + e^2)*ArcTanh[Sqrt[c]*x])/(3*c^(3/2)) + ((d + e*x)^3*(a + b*ArcTa
nh[c*x^2]))/(3*e) + (b*d*(c*d^2 + 3*e^2)*Log[1 - c*x^2])/(6*c*e) - (b*d*(c*
d^2 - 3*e^2)*Log[1 + c*x^2])/(6*c*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
```

```
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rubi steps

$$\int (d + ex)^2 \left(a + b \tanh^{-1}\left(cx^2\right)\right) dx = \frac{(d + ex)^3 \left(a + b \tanh^{-1}\left(cx^2\right)\right)}{3e} - \frac{b \int \frac{2cx(d+ex)^3}{1-c^2x^4} dx}{3e}$$

$$= \frac{(d + ex)^3 \left(a + b \tanh^{-1}\left(cx^2\right)\right)}{3e} - \frac{(2bc) \int \frac{x(d+ex)^3}{1-c^2x^4} dx}{3e}$$

$$= \frac{(d + ex)^3 \left(a + b \tanh^{-1}\left(cx^2\right)\right)}{3e} - \frac{(2bc) \int \left(\frac{x\left(d^3+3de^2x^2\right)}{1-c^2x^4} + \frac{x^2\left(3d^2e+e^3x^2\right)}{1-c^2x^4}\right) dx}{3e}$$

$$= \frac{(d + ex)^3 \left(a + b \tanh^{-1}\left(cx^2\right)\right)}{3e} - \frac{(2bc) \int \frac{x\left(d^3+3de^2x^2\right)}{1-c^2x^4} dx}{3e} - \frac{(2bc) \int \frac{x^2\left(3d^2e+e^3x^2\right)}{1-c^2x^4}}{3e}$$

$$= \frac{2be^2x}{3c} + \frac{(d + ex)^3 \left(a + b \tanh^{-1}\left(cx^2\right)\right)}{3e} - \frac{(2b) \int \frac{e^3+3c^2d^2ex^2}{1-c^2x^4} dx}{3ce} - \frac{(bc) \text{Sub}}{}$$

$$= \frac{2be^2x}{3c} + \frac{(d + ex)^3 \left(a + b \tanh^{-1}\left(cx^2\right)\right)}{3e} + \frac{\left(bcd\left(cd^2 - 3e^2\right)\right) \text{Subst}\left(\int \frac{1}{-c-c}\right)}{6e}$$

$$= \frac{2be^2x}{3c} + \frac{b\left(3cd^2 - e^2\right) \tan^{-1}\left(\sqrt{c}\, x\right)}{3c^{3/2}} - \frac{b\left(3cd^2 + e^2\right) \tanh^{-1}\left(\sqrt{c}\, x\right)}{3c^{3/2}} + \frac{(d +}{}$$

Mathematica [A]   time = 0.17, size = 170, normalized size = 1.08

$$\frac{1}{6}\left(6ad^2x + 6adex^2 + 2ae^2x^3 + \frac{b\left(3cd^2 + e^2\right) \log\left(1 - \sqrt{c}\, x\right)}{c^{3/2}} - \frac{b\left(3cd^2 + e^2\right) \log\left(\sqrt{c}\, x + 1\right)}{c^{3/2}} + \frac{2b\left(3cd^2 - e^2\right) \tan^{-}}{c^{3/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcTanh[c*x^2]),x]

[Out] (6*a*d^2*x + (4*b*e^2*x)/c + 6*a*d*e*x^2 + 2*a*e^2*x^3 + (2*b*(3*c*d^2 - e^
2)*ArcTan[Sqrt[c]*x])/c^(3/2) + 2*b*x*(3*d^2 + 3*d*e*x + e^2*x^2)*ArcTanh[c
*x^2] + (b*(3*c*d^2 + e^2)*Log[1 - Sqrt[c]*x])/c^(3/2) - (b*(3*c*d^2 + e^2)
*Log[1 + Sqrt[c]*x])/c^(3/2) + (3*b*d*e*Log[1 - c^2*x^4])/c)/6
```

fricas [A]   time = 0.68, size = 401, normalized size = 2.54

$$\left| \frac{2\,ac^2e^2x^3 + 6\,ac^2dex^2 + 3\,bcde\log\left(cx^2+1\right) + 3\,bcde\log\left(cx^2-1\right) + 2\left(3\,bcd^2 - be^2\right)\sqrt{c}\,\arctan\left(\sqrt{c}\,x\right) + \left(3\,bcd^2}{6\,c^2} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a*c^2*e^2*x^3 + 6*a*c^2*d*e*x^2 + 3*b*c*d*e*log(c*x^2 + 1) + 3*b*c*
d*e*log(c*x^2 - 1) + 2*(3*b*c*d^2 - b*e^2)*sqrt(c)*arctan(sqrt(c)*x) + (3*b
*c*d^2 + b*e^2)*sqrt(c)*log((c*x^2 - 2*sqrt(c)*x + 1)/(c*x^2 - 1)) + 2*(3*a
*c^2*d^2 + 2*b*c*e^2)*x + (b*c^2*e^2*x^3 + 3*b*c^2*d*e*x^2 + 3*b*c^2*d^2*x)
*log(-(c*x^2 + 1)/(c*x^2 - 1)))/c^2, 1/6*(2*a*c^2*e^2*x^3 + 6*a*c^2*d*e*x^2
 + 3*b*c*d*e*log(c*x^2 + 1) + 3*b*c*d*e*log(c*x^2 - 1) + 2*(3*b*c*d^2 + b*e
^2)*sqrt(-c)*arctan(sqrt(-c)*x) + (3*b*c*d^2 - b*e^2)*sqrt(-c)*log((c*x^2 +
 2*sqrt(-c)*x - 1)/(c*x^2 + 1)) + 2*(3*a*c^2*d^2 + 2*b*c*e^2)*x + (b*c^2*e^
2*x^3 + 3*b*c^2*d*e*x^2 + 3*b*c^2*d^2*x)*log(-(c*x^2 + 1)/(c*x^2 - 1)))/c^2
]
```

giac [A]   time = 1.04, size = 194, normalized size = 1.23

$$\frac{\left(3\,bcd^2 - be^2\right)\arctan\left(\sqrt{c}\,x\right)}{3\,c^{\frac{3}{2}}} + \frac{\left(3\,bcd^2 + be^2\right)\arctan\left(\frac{cx}{\sqrt{-c}}\right)}{3\,\sqrt{-c}\,c} + \frac{bcx^3e^2\log\left(-\frac{cx^2+1}{cx^2-1}\right) + 3\,bcdx^2e\log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2\,acx^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/3*(3*b*c*d^2 - b*e^2)*arctan(sqrt(c)*x)/c^(3/2) + 1/3*(3*b*c*d^2 + b*e^2)
*arctan(c*x/sqrt(-c))/(sqrt(-c)*c) + 1/6*(b*c*x^3*e^2*log(-(c*x^2 + 1)/(c*x
^2 - 1)) + 3*b*c*d*x^2*e*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c*x^3*e^2 + 6*
a*c*d*x^2*e + 3*b*c*d^2*x*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 6*a*c*d^2*x + 3*b
*d*e*log(c^2*x^4 - 1) + 4*b*x*e^2)/c
```

maple [A]   time = 0.03, size = 223, normalized size = 1.41

$$\frac{a\,x^3e^2}{3} + adex^2 + axd^2 + \frac{a\,d^3}{3e} + \frac{b\,e^2\arctan\left(cx^2\right)x^3}{3} + be\arctan\left(cx^2\right)x^2d + b\arctan\left(cx^2\right)xd^2 + \frac{b\arctan\left(cx^2\right)}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a+b*arctanh(c*x^2)),x)
```

```
[Out] 1/3*a*x^3*e^2+a*d*e*x^2+a*x*d^2+1/3*a/e*d^3+1/3*b*e^2*arctanh(c*x^2)*x^3+b*
e*arctanh(c*x^2)*x^2*d+b*arctanh(c*x^2)*x*d^2+1/3*b/e*arctanh(c*x^2)*d^3+2/
3*b*e^2*x/c-1/6*b/e*ln(c*x^2+1)*d^3+1/2*b*e/c*ln(c*x^2+1)*d+b/c^(1/2)*arcta
n(x*c^(1/2))*d^2-1/3*b*e^2/c^(3/2)*arctan(x*c^(1/2))+1/6*b/e*ln(c*x^2-1)*d^
3+1/2*b*e/c*ln(c*x^2-1)*d-b/c^(1/2)*arctanh(x*c^(1/2))*d^2-1/3*b*e^2/c^(3/2
)*arctanh(x*c^(1/2))
```

maxima [A]   time = 0.41, size = 171, normalized size = 1.08

$$\frac{1}{3}\,ae^2x^3+adex^2+\frac{1}{2}\left(c\left(\frac{2\,\arctan\left(\sqrt{c}\,x\right)}{c^{\frac{3}{2}}}+\frac{\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)+2\,x\,\text{artanh}\left(cx^2\right)\right)bd^2+\frac{1}{6}\left(2\,x^3\,\text{artanh}\left(cx^2\right)+c\left(\frac{4\,x}{c^2}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 + a*d*e*x^2 + 1/2*(c*(2*arctan(sqrt(c)*x)/c^(3/2) + log((c*x
- sqrt(c))/(c*x + sqrt(c)))/c^(3/2)) + 2*x*arctanh(c*x^2))*b*d^2 + 1/6*(2*x
^3*arctanh(c*x^2) + c*(4*x/c^2 - 2*arctan(sqrt(c)*x)/c^(5/2) + log((c*x - s
qrt(c))/(c*x + sqrt(c)))/c^(5/2)))*b*e^2 + a*d^2*x + 1/2*(2*c*x^2*arctanh(c
*x^2) + log(-c^2*x^4 + 1))*b*d*e/c
```

mupad [B]   time = 1.39, size = 309, normalized size = 1.96

$$\ln\left(cx^2+1\right)\left(\frac{bd^2x}{2}+\frac{bdex^2}{2}+\frac{be^2x^3}{6}\right)-\ln\left(1-cx^2\right)\left(\frac{bd^2x}{2}+\frac{bdex^2}{2}+\frac{be^2x^3}{6}\right)+\frac{x\left(3ac^2d^2+2bce^2\right)}{3c^2}+\frac{ae^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))*(d + e*x)^2,x)
```

```
[Out] log(c*x^2 + 1)*((b*e^2*x^3)/6 + (b*d^2*x)/2 + (b*d*e*x^2)/2) - log(1 - c*x^
2)*((b*e^2*x^3)/6 + (b*d^2*x)/2 + (b*d*e*x^2)/2) + (x*(3*a*c^2*d^2 + 2*b*c*
e^2))/(3*c^2) + (a*e^2*x^3)/3 - (log(c + x*(c^3)^(1/2))*(b*e^2*(c^3)^(1/2)
+ 3*b*c*d^2*(c^3)^(1/2) - 3*b*c^2*d*e))/(6*c^3) + (log(c - x*(c^3)^(1/2))*(
b*e^2*(c^3)^(1/2) + 3*b*c*d^2*(c^3)^(1/2) + 3*b*c^2*d*e))/(6*c^3) + (log(c
+ x*(-c^3)^(1/2))*(b*e^2*(-c^3)^(1/2) + 3*b*c^2*d*e - 3*b*c*d^2*(-c^3)^(1/2
)))/(6*c^3) + (log(c - x*(-c^3)^(1/2))*(3*b*c^2*d*e - b*e^2*(-c^3)^(1/2) +
3*b*c*d^2*(-c^3)^(1/2)))/(6*c^3) + a*d*e*x^2
```

sympy [A]   time = 15.81, size = 2907, normalized size = 18.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**2*(a+b*atanh(c*x**2)),x)

[Out] Piecewise((12*a*c**2*d**2*x*sqrt(1/c)/(-12*c**3*(1/c)**(3/2) - 12*I*c**3*(1
/c)**(3/2) + 24*c**2*sqrt(1/c) + 24*I*c**2*sqrt(1/c)) + 12*I*a*c**2*d**2*x*
sqrt(1/c)/(-12*c**3*(1/c)**(3/2) - 12*I*c**3*(1/c)**(3/2) + 24*c**2*sqrt(1/
c) + 24*I*c**2*sqrt(1/c)) + 12*a*c**2*d*e*x**2*sqrt(1/c)/(-12*c**3*(1/c)**(
3/2) - 12*I*c**3*(1/c)**(3/2) + 24*c**2*sqrt(1/c) + 24*I*c**2*sqrt(1/c)) +
12*I*a*c**2*d*e*x**2*sqrt(1/c)/(-12*c**3*(1/c)**(3/2) - 12*I*c**3*(1/c)**(3
/2) + 24*c**2*sqrt(1/c) + 24*I*c**2*sqrt(1/c)) + 4*a*c**2*e**2*x**3*sqrt(1/
c)/(-12*c**3*(1/c)**(3/2) - 12*I*c**3*(1/c)**(3/2) + 24*c**2*sqrt(1/c) + 24
*I*c**2*sqrt(1/c)) + 4*I*a*c**2*e**2*x**3*sqrt(1/c)/(-12*c**3*(1/c)**(3/2)
- 12*I*c**3*(1/c)**(3/2) + 24*c**2*sqrt(1/c) + 24*I*c**2*sqrt(1/c)) + 6*I*b
*c**3*d**2*log(x - I*sqrt(1/c))/(-12*c**5*(1/c)**(3/2) - 12*I*c**5*(1/c)**(
3/2) + 24*c**4*sqrt(1/c) + 24*I*c**4*sqrt(1/c)) - 6*I*b*c**3*d**2*log(x - s
qrt(1/c))/(-12*c**5*(1/c)**(3/2) - 12*I*c**5*(1/c)**(3/2) + 24*c**4*sqrt(1/
c) + 24*I*c**4*sqrt(1/c)) + 12*b*c**2*d**2*x*sqrt(1/c)*atanh(c*x**2)/(-12*c
3*(1/c)(3/2) - 12*I*c**3*(1/c)**(3/2) + 24*c**2*sqrt(1/c) + 24*I*c**2*s
qrt(1/c)) + 12*I*b*c**2*d**2*x*sqrt(1/c)*atanh(c*x**2)/(-12*c**3*(1/c)**(3/
2) - 12*I*c**3*(1/c)**(3/2) + 24*c**2*sqrt(1/c) + 24*I*c**2*sqrt(1/c)) - 18
*I*b*c**2*d**2*log(x - I*sqrt(1/c))/(-12*c**4*(1/c)**(3/2) - 12*I*c**4*(1/c
)**(3/2) + 24*c**3*sqrt(1/c) + 24*I*c**3*sqrt(1/c)) + 18*I*b*c**2*d**2*log(
x - sqrt(1/c))/(-12*c**4*(1/c)**(3/2) - 12*I*c**4*(1/c)**(3/2) + 24*c**3*sq
rt(1/c) + 24*I*c**3*sqrt(1/c)) + 12*I*b*c**2*d**2*atanh(c*x**2)/(-12*c**4*(
1/c)**(3/2) - 12*I*c**4*(1/c)**(3/2) + 24*c**3*sqrt(1/c) + 24*I*c**3*sqrt(1
/c)) + 12*b*c**2*d*e*x**2*sqrt(1/c)*atanh(c*x**2)/(-12*c**3*(1/c)**(3/2) -
12*I*c**3*(1/c)**(3/2) + 24*c**2*sqrt(1/c) + 24*I*c**2*sqrt(1/c)) + 12*I*b*
c**2*d*e*x**2*sqrt(1/c)*atanh(c*x**2)/(-12*c**3*(1/c)**(3/2) - 12*I*c**3*(1
/c)**(3/2) + 24*c**2*sqrt(1/c) + 24*I*c**2*sqrt(1/c)) - 3*b*c**2*d*e*(1/c)*
*(3/2)*log(x - I*sqrt(1/c))/(-12*c**3*(1/c)**(3/2) - 12*I*c**3*(1/c)**(3/2)
 + 24*c**2*sqrt(1/c) + 24*I*c**2*sqrt(1/c)) - 3*I*b*c**2*d*e*(1/c)**(3/2)*l
og(x - I*sqrt(1/c))/(-12*c**3*(1/c)**(3/2) - 12*I*c**3*(1/c)**(3/2) + 24*c*
*2*sqrt(1/c) + 24*I*c**2*sqrt(1/c)) - 12*b*c**2*d*e*(1/c)**(3/2)*log(x + I*
sqrt(1/c))/(-12*c**3*(1/c)**(3/2) - 12*I*c**3*(1/c)**(3/2) + 24*c**2*sqrt(1
/c) + 24*I*c**2*sqrt(1/c)) - 12*I*b*c**2*d*e*(1/c)**(3/2)*log(x + I*sqrt(1/
c))/(-12*c**3*(1/c)**(3/2) - 12*I*c**3*(1/c)**(3/2) + 24*c**2*sqrt(1/c) + 2
4*I*c**2*sqrt(1/c)) + 4*b*c**2*e**2*x**3*sqrt(1/c)*atanh(c*x**2)/(-12*c**3*
(1/c)**(3/2) - 12*I*c**3*(1/c)**(3/2) + 24*c**2*sqrt(1/c) + 24*I*c**2*sqrt(
1/c)) + 4*I*b*c**2*e**2*x**3*sqrt(1/c)*atanh(c*x**2)/(-12*c**3*(1/c)**(3/2)
 - 12*I*c**3*(1/c)**(3/2) + 24*c**2*sqrt(1/c) + 24*I*c**2*sqrt(1/c)) - 2*I*
b*c**2*e**2*log(x - sqrt(1/c))/(-12*c**5*(1/c)**(3/2) - 12*I*c**5*(1/c)**(3
/2) + 24*c**4*sqrt(1/c) + 24*I*c**4*sqrt(1/c)) - 12*b*c*d**2*log(x + I*sqrt
(1/c))/(-12*c**3*(1/c)**(3/2) - 12*I*c**3*(1/c)**(3/2) + 24*c**2*sqrt(1/c)
+ 24*I*c**2*sqrt(1/c)) + 12*b*c*d**2*log(x - sqrt(1/c))/(-12*c**3*(1/c)**(3
/2) - 12*I*c**3*(1/c)**(3/2) + 24*c**2*sqrt(1/c) + 24*I*c**2*sqrt(1/c)) + 1
2*b*c*d**2*atanh(c*x**2)/(-12*c**3*(1/c)**(3/2) - 12*I*c**3*(1/c)**(3/2) +

```
24*c**2*sqrt(1/c) + 24*I*c**2*sqrt(1/c)) + 15*b*c*d*e*sqrt(1/c)*log(x - I*s
qrt(1/c))/(-12*c**3*(1/c)**(3/2) - 12*I*c**3*(1/c)**(3/2) + 24*c**2*sqrt(1/
c) + 24*I*c**2*sqrt(1/c)) + 15*I*b*c*d*e*sqrt(1/c)*log(x - I*sqrt(1/c))/(-1
2*c**3*(1/c)**(3/2) - 12*I*c**3*(1/c)**(3/2) + 24*c**2*sqrt(1/c) + 24*I*c**
2*sqrt(1/c)) + 24*b*c*d*e*sqrt(1/c)*log(x + I*sqrt(1/c))/(-12*c**3*(1/c)**(
3/2) - 12*I*c**3*(1/c)**(3/2) + 24*c**2*sqrt(1/c) + 24*I*c**2*sqrt(1/c)) +
24*I*b*c*d*e*sqrt(1/c)*log(x + I*sqrt(1/c))/(-12*c**3*(1/c)**(3/2) - 12*I*c
**3*(1/c)**(3/2) + 24*c**2*sqrt(1/c) + 24*I*c**2*sqrt(1/c)) - 12*b*c*d*e*sq
rt(1/c)*atanh(c*x**2)/(-12*c**3*(1/c)**(3/2) - 12*I*c**3*(1/c)**(3/2) + 24*
c**2*sqrt(1/c) + 24*I*c**2*sqrt(1/c)) - 12*I*b*c*d*e*sqrt(1/c)*atanh(c*x**2
)/(-12*c**3*(1/c)**(3/2) - 12*I*c**3*(1/c)**(3/2) + 24*c**2*sqrt(1/c) + 24*
I*c**2*sqrt(1/c)) + 8*b*c*e**2*x*sqrt(1/c)/(-12*c**3*(1/c)**(3/2) - 12*I*c*
*3*(1/c)**(3/2) + 24*c**2*sqrt(1/c) + 24*I*c**2*sqrt(1/c)) + 8*I*b*c*e**2*x
*sqrt(1/c)/(-12*c**3*(1/c)**(3/2) - 12*I*c**3*(1/c)**(3/2) + 24*c**2*sqrt(1
/c) + 24*I*c**2*sqrt(1/c)) - 4*I*b*c*e**2*log(x + I*sqrt(1/c))/(-12*c**4*(1
/c)**(3/2) - 12*I*c**4*(1/c)**(3/2) + 24*c**3*sqrt(1/c) + 24*I*c**3*sqrt(1/
c)) + 6*I*b*c*e**2*log(x - sqrt(1/c))/(-12*c**4*(1/c)**(3/2) - 12*I*c**4*(1
/c)**(3/2) + 24*c**3*sqrt(1/c) + 24*I*c**3*sqrt(1/c)) + 4*I*b*c*e**2*atanh(
c*x**2)/(-12*c**4*(1/c)**(3/2) - 12*I*c**4*(1/c)**(3/2) + 24*c**3*sqrt(1/c)
 + 24*I*c**3*sqrt(1/c)) - 4*b*e**2*log(x - I*sqrt(1/c))/(-12*c**3*(1/c)**(3
/2) - 12*I*c**3*(1/c)**(3/2) + 24*c**2*sqrt(1/c) + 24*I*c**2*sqrt(1/c)) + 4
*b*e**2*log(x - sqrt(1/c))/(-12*c**3*(1/c)**(3/2) - 12*I*c**3*(1/c)**(3/2)
+ 24*c**2*sqrt(1/c) + 24*I*c**2*sqrt(1/c)) + 4*b*e**2*atanh(c*x**2)/(-12*c*
*3*(1/c)**(3/2) - 12*I*c**3*(1/c)**(3/2) + 24*c**2*sqrt(1/c) + 24*I*c**2*sq
rt(1/c)), Ne(c, 0)), (a*(d**2*x + d*e*x**2 + e**2*x**3/3), True))
```

3.25 $\int (d + ex)\left(a + b\tanh^{-1}\left(cx^2\right)\right)\,dx$

Optimal. Leaf size=117

$$\frac{(d + ex)^2\left(a + b\tanh^{-1}\left(cx^2\right)\right)}{2e} + \frac{b\left(cd^2 + e^2\right)\log\left(1 - cx^2\right)}{4ce} - \frac{b\left(cd^2 - e^2\right)\log\left(cx^2 + 1\right)}{4ce} + \frac{bd\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} - \frac{bd\tanh^{-1}}{\sqrt{c}}$$

```
[Out] 1/2*(e*x+d)^2*(a+b*arctanh(c*x^2))/e+1/4*b*(c*d^2+e^2)*ln(-c*x^2+1)/c/e-1/4
*b*(c*d^2-e^2)*ln(c*x^2+1)/c/e+b*d*arctan(x*c^(1/2))/c^(1/2)-b*d*arctanh(x*
c^(1/2))/c^(1/2)
```

Rubi [A]   time = 0.09, antiderivative size = 94, normalized size of antiderivative = 0.80, number of steps used = 10, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6742, 6091, 298, 203, 206, 6097, 260}

$$\frac{a(d + ex)^2}{2e} + \frac{be\log\left(1 - c^2x^4\right)}{4c} + bdx\tanh^{-1}\left(cx^2\right) + \frac{bd\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} - \frac{bd\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} + \frac{1}{2}bex^2\tanh^{-1}\left(cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (a*(d + e*x)^2)/(2*e) + (b*d*ArcTan[Sqrt[c]*x])/Sqrt[c] - (b*d*ArcTanh[Sqrt
[c]*x])/Sqrt[c] + b*d*x*ArcTanh[c*x^2] + (b*e*x^2*ArcTanh[c*x^2])/2 + (b*e*
Log[1 - c^2*x^4])/(4*c)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6091

```
Int[ArcTanh[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTanh[c*x^n], x] - Dist
[c*n, Int[x^n/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (d + ex)\left(a + b\tanh^{-1}\left(cx^2\right)\right)\,dx = \int \left(a(d + ex) + b(d + ex)\tanh^{-1}\left(cx^2\right)\right)\,dx$$

$$= \frac{a(d + ex)^2}{2e} + b\int (d + ex)\tanh^{-1}\left(cx^2\right)\,dx$$

$$= \frac{a(d + ex)^2}{2e} + b\int \left(d\tanh^{-1}\left(cx^2\right) + ex\tanh^{-1}\left(cx^2\right)\right)\,dx$$

$$= \frac{a(d + ex)^2}{2e} + (bd)\int \tanh^{-1}\left(cx^2\right)\,dx + (be)\int x\tanh^{-1}\left(cx^2\right)\,dx$$

$$= \frac{a(d + ex)^2}{2e} + bdx\tanh^{-1}\left(cx^2\right) + \frac{1}{2}bex^2\tanh^{-1}\left(cx^2\right) - (2bcd)\int \frac{x^2}{1 - c^2x^4}\,($$

$$= \frac{a(d + ex)^2}{2e} + bdx\tanh^{-1}\left(cx^2\right) + \frac{1}{2}bex^2\tanh^{-1}\left(cx^2\right) + \frac{be\log\left(1 - c^2x^4\right)}{4c} -$$

$$= \frac{a(d + ex)^2}{2e} + \frac{bd\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} - \frac{bd\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} + bdx\tanh^{-1}\left(cx^2\right) + \frac{1}{2}b$$

Mathematica [A]   time = 0.06, size = 104, normalized size = 0.89

$$adx + \frac{1}{2}aex^2 + \frac{be\log\left(1 - c^2x^4\right)}{4c} + bdx\tanh^{-1}\left(cx^2\right) + \frac{bd\left(\log\left(1 - \sqrt{c}\,x\right) - \log\left(\sqrt{c}\,x + 1\right) + 2\tan^{-1}\left(\sqrt{c}\,x\right)\right)}{2\sqrt{c}} + \frac{1}{2}bex^2 \, t$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)*(a + b*ArcTanh[c*x^2]),x]

[Out] a*d*x + (a*e*x^2)/2 + b*d*x*ArcTanh[c*x^2] + (b*e*x^2*ArcTanh[c*x^2])/2 + (
b*d*(2*ArcTan[Sqrt[c]*x] + Log[1 - Sqrt[c]*x] - Log[1 + Sqrt[c]*x]))/(2*Sqr
t[c]) + (b*e*Log[1 - c^2*x^4])/(4*c)

fricas [A]   time = 1.34, size = 249, normalized size = 2.13

$$\left[\frac{2\,acex^2 + 4\,acdx + 4\,b\sqrt{c}\,d\arctan\left(\sqrt{c}\,x\right) + 2\,b\sqrt{c}\,d\log\left(\frac{cx^2 - 2\sqrt{c}\,x + 1}{cx^2 - 1}\right) + be\log\left(cx^2 + 1\right) + be\log\left(cx^2 - 1\right) + \left(bce\right.}{4\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*(a+b*arctanh(c*x^2)),x, algorithm="fricas")

[Out] [1/4*(2*a*c*e*x^2 + 4*a*c*d*x + 4*b*sqrt(c)*d*arctan(sqrt(c)*x) + 2*b*sqrt(
c)*d*log((c*x^2 - 2*sqrt(c)*x + 1)/(c*x^2 - 1)) + b*e*log(c*x^2 + 1) + b*e*
log(c*x^2 - 1) + (b*c*e*x^2 + 2*b*c*d*x)*log(-(c*x^2 + 1)/(c*x^2 - 1)))/c,
1/4*(2*a*c*e*x^2 + 4*a*c*d*x + 4*b*sqrt(-c)*d*arctan(sqrt(-c)*x) - 2*b*sqrt
(-c)*d*log((c*x^2 - 2*sqrt(-c)*x - 1)/(c*x^2 + 1)) + b*e*log(c*x^2 + 1) + b
*e*log(c*x^2 - 1) + (b*c*e*x^2 + 2*b*c*d*x)*log(-(c*x^2 + 1)/(c*x^2 - 1)))/
c]

giac [A]   time = 0.36, size = 143, normalized size = 1.22

$$\frac{bd\sqrt{|c|}\arctan\left(x\sqrt{|c|}\right)}{c} - \frac{bcd\log\left(\left|x + \frac{1}{\sqrt{|c|}}\right|\right)}{2\,|c|^{\frac{3}{2}}} + \frac{bd\sqrt{|c|}\log\left(\left|x - \frac{1}{\sqrt{|c|}}\right|\right)}{2\,c} + \frac{bcx^2e\log\left(-\frac{cx^2 + 1}{cx^2 - 1}\right) + 2\,acx^2e + 2\,bcdx\log\left(}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*(a+b*arctanh(c*x^2)),x, algorithm="giac")

[Out] b*d*sqrt(abs(c))*arctan(x*sqrt(abs(c)))/c - 1/2*b*c*d*log(abs(x + 1/sqrt(ab
s(c))))/abs(c)^(3/2) + 1/2*b*d*sqrt(abs(c))*log(abs(x - 1/sqrt(abs(c))))/c
+ 1/4*(b*c*x^2*e*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c*x^2*e + 2*b*c*d*x*lo
g(-(c*x^2 + 1)/(c*x^2 - 1)) + 4*a*c*d*x + b*e*log(c^2*x^4 - 1))/c

maple [A]   time = 0.03, size = 91, normalized size = 0.78

$$\frac{a\,x^2 e}{2} + adx + \frac{b\arctanh\left(c\,x^2\right)x^2 e}{2} + b\arctanh\left(c\,x^2\right)dx + \frac{be\ln\left(c\,x^2+1\right)}{4c} + \frac{bd\arctan\left(x\sqrt{c}\right)}{\sqrt{c}} + \frac{be\ln\left(c\,x^2-1\right)}{4c} - \frac{bd\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)*(a+b*arctanh(c*x^2)),x)

[Out] 1/2*a*x^2*e+a*d*x+1/2*b*arctanh(c*x^2)*x^2*e+b*arctanh(c*x^2)*d*x+1/4*b*e/c
*ln(c*x^2+1)+b*d*arctan(x*c^(1/2))/c^(1/2)+1/4*b*e/c*ln(c*x^2-1)-b*d*arctan
h(x*c^(1/2))/c^(1/2)

maxima [A]   time = 0.42, size = 95, normalized size = 0.81

$$\frac{1}{2}aex^2 + \frac{1}{2}\left(c\left(\frac{2\arctan\left(\sqrt{c}\,x\right)}{c^{\frac{3}{2}}} + \frac{\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right) + 2\,x\,artanh\left(cx^2\right)\right)bd + adx + \frac{\left(2\,cx^2\,artanh\left(cx^2\right) + \log\left(-c^2x^4 +\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*(a+b*arctanh(c*x^2)),x, algorithm="maxima")

[Out] 1/2*a*e*x^2 + 1/2*(c*(2*arctan(sqrt(c)*x)/c^(3/2) + log((c*x - sqrt(c))/(c*
x + sqrt(c)))/c^(3/2)) + 2*x*arctanh(c*x^2))*b*d + a*d*x + 1/4*(2*c*x^2*arc
tanh(c*x^2) + log(-c^2*x^4 + 1))*b*e/c

mupad [B]   time = 1.31, size = 242, normalized size = 2.07

$$adx + \frac{a\,e\,x^2}{2} + \frac{b\,d\,x\,\ln\left(c\,x^2+1\right)}{2} - \frac{b\,d\,x\,\ln\left(1-c\,x^2\right)}{2} + \frac{be\,\ln\left(c+x\,\sqrt{c^3}\right)}{4c} + \frac{be\,\ln\left(c-x\,\sqrt{c^3}\right)}{4c} + \frac{be\,x^2\,\ln\left(c\,x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x^2))*(d + e*x),x)

[Out] a*d*x + (a*e*x^2)/2 + (b*d*x*log(c*x^2 + 1))/2 - (b*d*x*log(1 - c*x^2))/2 +
 (b*e*log(c + x*(c^3)^(1/2)))/(4*c) + (b*e*log(c - x*(c^3)^(1/2)))/(4*c) +
(b*e*x^2*log(c*x^2 + 1))/4 - (b*e*x^2*log(1 - c*x^2))/4 + (b*e*log(c + x*(-
c^3)^(1/2)))/(4*c) + (b*e*log(c - x*(-c^3)^(1/2)))/(4*c) - (b*d*log(c + x*(
c^3)^(1/2))*(c^3)^(1/2))/(2*c^2) + (b*d*log(c - x*(c^3)^(1/2))*(c^3)^(1/2))
/(2*c^2) - (b*d*log(c + x*(-c^3)^(1/2))*(-c^3)^(1/2))/(2*c^2) + (b*d*log(c
- x*(-c^3)^(1/2))*(-c^3)^(1/2))/(2*c^2)

sympy [A]   time = 10.48, size = 294, normalized size = 2.51

$$
\begin{cases}
adx + \dfrac{aex^2}{2} + \dfrac{bcd\left(\frac{1}{c}\right)^{\frac{3}{2}}\log\left(x+i\sqrt{\frac{1}{c}}\right)}{4} - \dfrac{ibcd\left(\frac{1}{c}\right)^{\frac{3}{2}}\log\left(x+i\sqrt{\frac{1}{c}}\right)}{4} + bdx\operatorname{atanh}\left(cx^2\right) - \dfrac{bd\sqrt{\frac{1}{c}}\log\left(x-i\sqrt{\frac{1}{c}}\right)}{2} - \dfrac{ibd\sqrt{\frac{1}{c}}\log\left(x-i\sqrt{\frac{1}{c}}\right)}{2} - \\[2em]
a\left(dx + \dfrac{ex^2}{2}\right)
\end{cases}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*atanh(c*x**2)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x**2/2 + b*c*d*(1/c)**(3/2)*log(x + I*sqrt(1/c))/4 -
 I*b*c*d*(1/c)**(3/2)*log(x + I*sqrt(1/c))/4 + b*d*x*atanh(c*x**2) - b*d*sq
rt(1/c)*log(x - I*sqrt(1/c))/2 - I*b*d*sqrt(1/c)*log(x - I*sqrt(1/c))/2 - 3
*b*d*sqrt(1/c)*log(x + I*sqrt(1/c))/4 + 3*I*b*d*sqrt(1/c)*log(x + I*sqrt(1/
c))/4 + b*d*sqrt(1/c)*log(x - sqrt(1/c)) + b*d*sqrt(1/c)*atanh(c*x**2) + b*
e*x**2*atanh(c*x**2)/2 + b*e*log(x - I*sqrt(1/c))/(2*c) + b*e*log(x + I*sqr
t(1/c))/(2*c) - b*e*atanh(c*x**2)/(2*c), Ne(c, 0)), (a*(d*x + e*x**2/2), Tr
ue))
```

3.26 $\quad \displaystyle\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{d+ex}\,dx$

Optimal. Leaf size=325

$$\frac{\log(d+ex)\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{e}-\frac{b\mathrm{Li}_2\left(\frac{\sqrt{-c}\,(d+ex)}{\sqrt{-c}\,d-e}\right)}{2e}+\frac{b\mathrm{Li}_2\left(\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-e}\right)}{2e}-\frac{b\mathrm{Li}_2\left(\frac{\sqrt{-c}\,(d+ex)}{\sqrt{-c}\,d+e}\right)}{2e}+\frac{b\mathrm{Li}_2\left(\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+e}\right)}{2e}-\frac{b\log(c}{}$$

```
[Out] (a+b*arctanh(c*x^2))*ln(e*x+d)/e-1/2*b*ln(e*x+d)*ln(e*(1-x*(-c)^(1/2))/(e+d
*(-c)^(1/2)))/e-1/2*b*ln(e*x+d)*ln(-e*(1+x*(-c)^(1/2))/(-e+d*(-c)^(1/2)))/e
+1/2*b*ln(e*x+d)*ln(e*(1-x*c^(1/2))/(e+d*c^(1/2)))/e+1/2*b*ln(e*x+d)*ln(-e*
(1+x*c^(1/2))/(-e+d*c^(1/2)))/e-1/2*b*polylog(2,(e*x+d)*(-c)^(1/2)/(-e+d*(-
c)^(1/2)))/e-1/2*b*polylog(2,(e*x+d)*(-c)^(1/2)/(e+d*(-c)^(1/2)))/e+1/2*b*p
olylog(2,(e*x+d)*c^(1/2)/(-e+d*c^(1/2)))/e+1/2*b*polylog(2,(e*x+d)*c^(1/2)/
(e+d*c^(1/2)))/e
```

Rubi [F]   time $= 0.06$, antiderivative size $= 0$, normalized size of antiderivative $= 0.00$, number of steps used $= 0$, number of rules used $= 0$, integrand size $= 0$, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.000, Rules used $= \{\}$

$$\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{d+ex}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^2])/(d + e*x),x]
```

```
[Out] (a*Log[d + e*x])/e + b*Defer[Int][ArcTanh[c*x^2]/(d + e*x), x]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{d+ex}\,dx = \int\left(\frac{a}{d+ex}+\frac{b\tanh^{-1}\left(cx^2\right)}{d+ex}\right)dx$$

$$= \frac{a\log(d+ex)}{e}+b\int\frac{\tanh^{-1}\left(cx^2\right)}{d+ex}\,dx$$

Mathematica [C]   time $= 17.37$, size $= 285$, normalized size $= 0.88$

$$\frac{a\log(d+ex)}{e}+\frac{b\left(\mathrm{Li}_2\left(\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-e}\right)-\mathrm{Li}_2\left(\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-ie}\right)-\mathrm{Li}_2\left(\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+ie}\right)+\mathrm{Li}_2\left(\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+e}\right)+2\tanh^{-1}\left(cx^2\right)\log(d+ex)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/(d + e*x),x]
```

```
[Out] (a*Log[d + e*x])/e + (b*(2*ArcTanh[c*x^2]*Log[d + e*x] - Log[(e*(I - Sqrt[c
]*x))/(Sqrt[c]*d + I*e)]*Log[d + e*x] - Log[-((e*(I + Sqrt[c]*x))/(Sqrt[c]*
d - I*e))]*Log[d + e*x] + Log[-((e*(1 + Sqrt[c]*x))/(Sqrt[c]*d - e))]*Log[d
 + e*x] + Log[d + e*x]*Log[(e - Sqrt[c]*e*x)/(Sqrt[c]*d + e)] + PolyLog[2,
(Sqrt[c]*(d + e*x))/(Sqrt[c]*d - e)] - PolyLog[2, (Sqrt[c]*(d + e*x))/(Sqrt
[c]*d - I*e)] - PolyLog[2, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + I*e)] + PolyLog
[2, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + e)]))/(2*e)
```

fricas [F]   time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{artanh}\left(cx^2\right) + a}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x^2) + a)/(e*x + d), x)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{artanh}\left(cx^2\right) + a}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)/(e*x + d), x)
```

maple [A]   time = 0.07, size = 362, normalized size = 1.11

$$\frac{a\ln\left(ex+d\right)}{e} + \frac{b\ln\left(ex+d\right)\text{arctanh}\left(c\,x^2\right)}{e} + \frac{b\ln\left(ex+d\right)\ln\left(\frac{e\sqrt{c}-(ex+d)c+cd}{e\sqrt{c}+cd}\right)}{2e} + \frac{b\ln\left(ex+d\right)\ln\left(\frac{e\sqrt{c}+(ex+d)c-cd}{e\sqrt{c}-cd}\right)}{2e} + \frac{b\,\text{di}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/(e*x+d),x)
```

```
[Out] a*ln(e*x+d)/e+b*ln(e*x+d)/e*arctanh(c*x^2)+1/2*b/e*ln(e*x+d)*ln((e*c^(1/2)-
(e*x+d)*c+c*d)/(e*c^(1/2)+c*d))+1/2*b/e*ln(e*x+d)*ln((e*c^(1/2)+(e*x+d)*c-c
*d)/(e*c^(1/2)-c*d))+1/2*b/e*dilog((e*c^(1/2)-(e*x+d)*c+c*d)/(e*c^(1/2)+c*d
```

```
))+1/2*b/e*dilog((e*c^(1/2)+(e*x+d)*c-c*d)/(e*c^(1/2)-c*d))-1/2*b/e*ln(e*x+
d)*ln((e*(-c)^(1/2)-(e*x+d)*c+c*d)/(e*(-c)^(1/2)+c*d))-1/2*b/e*ln(e*x+d)*ln
((e*(-c)^(1/2)+(e*x+d)*c-c*d)/(e*(-c)^(1/2)-c*d))-1/2*b/e*dilog((e*(-c)^(1/
2)-(e*x+d)*c+c*d)/(e*(-c)^(1/2)+c*d))-1/2*b/e*dilog((e*(-c)^(1/2)+(e*x+d)*c
-c*d)/(e*(-c)^(1/2)-c*d))
```

maxima $[\mathbf{F}]$ time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,b\int \frac{\log\left(cx^2+1\right)-\log\left(-cx^2+1\right)}{ex+d}\,dx + \frac{a\log\left(ex+d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^2))/(e*x+d),x, algorithm="maxima")

[Out] 1/2*b*integrate((log(c*x^2 + 1) - log(-c*x^2 + 1))/(e*x + d), x) + a*log(e*
x + d)/e

mupad $[\mathbf{F}]$ time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{atanh}\left(c\,x^2\right)}{d + e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x^2))/(d + e*x),x)

[Out] int((a + b*atanh(c*x^2))/(d + e*x), x)

sympy $[\mathbf{F(-1)}]$ time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x**2))/(e*x+d),x)

[Out] Timed out

3.27 $\int \dfrac{a+b\tanh^{-1}\left(cx^2\right)}{(d+ex)^2}\,dx$

Optimal. Leaf size=166

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{e(d+ex)}+\frac{2bcde\log(d+ex)}{c^2d^4-e^4}-\frac{bcd\log\left(1-cx^2\right)}{2e\left(cd^2-e^2\right)}+\frac{bcd\log\left(cx^2+1\right)}{2e\left(cd^2+e^2\right)}+\frac{b\sqrt{c}\,\tan^{-1}\left(\sqrt{c}\,x\right)}{cd^2+e^2}-\frac{b\sqrt{c}\,\tanh^{-1}\left(\right.}{cd^2-e^2}$$

[Out] (-a-b*arctanh(c*x^2))/e/(e*x+d)+2*b*c*d*e*ln(e*x+d)/(c^2*d^4-e^4)-1/2*b*c*d
*ln(-c*x^2+1)/e/(c*d^2-e^2)+1/2*b*c*d*ln(c*x^2+1)/e/(c*d^2+e^2)+b*arctan(x*
c^(1/2))*c^(1/2)/(c*d^2+e^2)-b*arctanh(x*c^(1/2))*c^(1/2)/(c*d^2-e^2)

Rubi [A] time = 0.28, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 18, $\dfrac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6273, 12, 6725, 635, 207, 260, 203}

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{e(d+ex)}+\frac{2bcde\log(d+ex)}{c^2d^4-e^4}-\frac{bcd\log\left(1-cx^2\right)}{2e\left(cd^2-e^2\right)}+\frac{bcd\log\left(cx^2+1\right)}{2e\left(cd^2+e^2\right)}+\frac{b\sqrt{c}\,\tan^{-1}\left(\sqrt{c}\,x\right)}{cd^2+e^2}-\frac{b\sqrt{c}\,\tanh^{-1}\left(\right.}{cd^2-e^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*x^2])/(d + e*x)^2,x]

[Out] (b*Sqrt[c]*ArcTan[Sqrt[c]*x])/(c*d^2 + e^2) - (b*Sqrt[c]*ArcTanh[Sqrt[c]*x]
)/(c*d^2 - e^2) - (a + b*ArcTanh[c*x^2])/(e*(d + e*x)) + (2*b*c*d*e*Log[d +
 e*x])/(c^2*d^4 - e^4) - (b*c*d*Log[1 - c*x^2])/(2*e*(c*d^2 - e^2)) + (b*c*
d*Log[1 + c*x^2])/(2*e*(c*d^2 + e^2))

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{a + b\tanh^{-1}\left(cx^2\right)}{(d+ex)^2}\,dx = -\frac{a + b\tanh^{-1}\left(cx^2\right)}{e(d+ex)} + \frac{b\int \frac{2cx}{(d+ex)\left(1-c^2x^4\right)}\,dx}{e}$$

$$= -\frac{a + b\tanh^{-1}\left(cx^2\right)}{e(d+ex)} + \frac{(2bc)\int \frac{x}{(d+ex)\left(1-c^2x^4\right)}\,dx}{e}$$

$$= -\frac{a + b\tanh^{-1}\left(cx^2\right)}{e(d+ex)} + \frac{(2bc)\int \left(-\frac{de^3}{\left(-c^2d^4+e^4\right)(d+ex)} + \frac{e-cdx}{2\left(cd^2-e^2\right)\left(-1+cx^2\right)} + \frac{e+cdx}{2\left(cd^2+e^2\right)\left(1+cx^2\right)}\right)\,dx}{e}$$

$$= -\frac{a + b\tanh^{-1}\left(cx^2\right)}{e(d+ex)} + \frac{2bcde\log(d+ex)}{c^2d^4-e^4} + \frac{(bc)\int \frac{e-cdx}{-1+cx^2}\,dx}{e\left(cd^2-e^2\right)} + \frac{(bc)\int \frac{e+cdx}{1+cx^2}\,dx}{e\left(cd^2+e^2\right)}$$

$$= -\frac{a + b\tanh^{-1}\left(cx^2\right)}{e(d+ex)} + \frac{2bcde\log(d+ex)}{c^2d^4-e^4} + \frac{(bc)\int \frac{1}{-1+cx^2}\,dx}{cd^2-e^2} - \frac{\left(bc^2d\right)\int \frac{x}{-1+cx^2}\,dx}{e\left(cd^2-e^2\right)} + ($$

$$= \frac{b\sqrt{c}\,\tan^{-1}\left(\sqrt{c}\,x\right)}{cd^2+e^2} - \frac{b\sqrt{c}\,\tanh^{-1}\left(\sqrt{c}\,x\right)}{cd^2-e^2} - \frac{a + b\tanh^{-1}\left(cx^2\right)}{e(d+ex)} + \frac{2bcde\log(d+ex)}{c^2d^4-e^4} -$$

Mathematica [A]   time = 0.36, size = 261, normalized size = 1.57

$$\frac{1}{2}\left(-\frac{2a}{e(d+ex)} - \frac{bcde\log\left(1-c^2x^4\right)}{c^2d^4-e^4} + \frac{4bcde\log(d+ex)}{c^2d^4-e^4} + \frac{bc^2d^3\log\left(cx^2+1\right)}{c^2d^4e-e^5} + \frac{b\sqrt{c}\left(c^{3/2}d^3-cd^2e-e^3\right)\log\left(1\right)}{e^5-c^2d^4e}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/(d + e*x)^2,x]
```

```
[Out] ((-2*a)/(e*(d + e*x)) + (2*b*Sqrt[c]*ArcTan[Sqrt[c]*x])/(c*d^2 + e^2) - (2*
b*ArcTanh[c*x^2])/(e*(d + e*x)) + (b*Sqrt[c]*(c^(3/2)*d^3 - c*d^2*e - e^3)*
Log[1 - Sqrt[c]*x])/(-(c^2*d^4*e) + e^5) + (b*Sqrt[c]*(c^(3/2)*d^3 + c*d^2*
e + e^3)*Log[1 + Sqrt[c]*x])/(-(c^2*d^4*e) + e^5) + (4*b*c*d*e*Log[d + e*x]
)/(c^2*d^4 - e^4) + (b*c^2*d^3*Log[1 + c*x^2])/(c^2*d^4*e - e^5) - (b*c*d*e
*Log[1 - c^2*x^4])/(c^2*d^4 - e^4))/2
```

fricas [B]   time = 8.20, size = 634, normalized size = 3.82

$$\left[-\frac{2\,ac^2d^4 - 2\,ae^4 - 2\left(bcd^3e - bde^3 + \left(bcd^2e^2 - be^4\right)x\right)\sqrt{c}\,\arctan\left(\sqrt{c}\,x\right) + \left(bcd^3e + bde^3 + \left(bcd^2e^2 + be^4\right)x\right)\sqrt{c}\,\text{lc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*c^2*d^4 - 2*a*e^4 - 2*(b*c*d^3*e - b*d*e^3 + (b*c*d^2*e^2 - b*e^
4)*x)*sqrt(c)*arctan(sqrt(c)*x) + (b*c*d^3*e + b*d*e^3 + (b*c*d^2*e^2 + b*e
^4)*x)*sqrt(c)*log((c*x^2 + 2*sqrt(c)*x + 1)/(c*x^2 - 1)) - (b*c^2*d^4 - b*
c*d^2*e^2 + (b*c^2*d^3*e - b*c*d*e^3)*x)*log(c*x^2 + 1) + (b*c^2*d^4 + b*c*
d^2*e^2 + (b*c^2*d^3*e + b*c*d*e^3)*x)*log(c*x^2 - 1) - 4*(b*c*d*e^3*x + b*
c*d^2*e^2)*log(e*x + d) + (b*c^2*d^4 - b*e^4)*log(-(c*x^2 + 1)/(c*x^2 - 1))
)/(c^2*d^5*e - d*e^5 + (c^2*d^4*e^2 - e^6)*x), -1/2*(2*a*c^2*d^4 - 2*a*e^4
- 2*(b*c*d^3*e + b*d*e^3 + (b*c*d^2*e^2 + b*e^4)*x)*sqrt(-c)*arctan(sqrt(-c
)*x) - (b*c*d^3*e - b*d*e^3 + (b*c*d^2*e^2 - b*e^4)*x)*sqrt(-c)*log((c*x^2
+ 2*sqrt(-c)*x - 1)/(c*x^2 + 1)) - (b*c^2*d^4 - b*c*d^2*e^2 + (b*c^2*d^3*e
- b*c*d*e^3)*x)*log(c*x^2 + 1) + (b*c^2*d^4 + b*c*d^2*e^2 + (b*c^2*d^3*e +
b*c*d*e^3)*x)*log(c*x^2 - 1) - 4*(b*c*d*e^3*x + b*c*d^2*e^2)*log(e*x + d) +
 (b*c^2*d^4 - b*e^4)*log(-(c*x^2 + 1)/(c*x^2 - 1)))/(c^2*d^5*e - d*e^5 + (c
^2*d^4*e^2 - e^6)*x)]
```

giac [A]   time = 0.23, size = 271, normalized size = 1.63

$$
\frac{1}{2}\left(\frac{cd\log\left(c - \frac{2\,cd}{xe+d} + \frac{cd^2}{(xe+d)^2} + \frac{e^2}{(xe+d)^2}\right)}{cd^2e + e^3} - \frac{cd\log\left(c - \frac{2\,cd}{xe+d} + \frac{cd^2}{(xe+d)^2} - \frac{e^2}{(xe+d)^2}\right)}{cd^2e - e^3} - \frac{e^{(-1)}\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{xe + d} + \frac{2\,c\arctan\left(\right.}{\left(cd^2 \right.} \right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(c*d*log(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + e^2/(x*e + d)^2)/(c*
d^2*e + e^3) - c*d*log(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 - e^2/(x*e +
 d)^2)/(c*d^2*e - e^3) - e^(-1)*log(-(c*x^2 + 1)/(c*x^2 - 1))/(x*e + d) + 2
*c*arctan((c*d - c*d^2/(x*e + d) + e^2/(x*e + d))*e^(-1)/sqrt(-c))/((c*d^2
- e^2)*sqrt(-c)) + 2*sqrt(c)*arctan((c*d - c*d^2/(x*e + d) - e^2/(x*e + d))
*e^(-1)/sqrt(c))/(c*d^2 + e^2))*b - a*e^(-1)/(x*e + d)
```

maple [A]   time = 0.04, size = 181, normalized size = 1.09

$$
-\frac{a}{(ex+d)\,e} - \frac{b\,\mathrm{arctanh}\left(c\,x^2\right)}{(ex+d)\,e} + \frac{bcd\ln\left(c\,x^2 + 1\right)}{e\left(2c\,d^2 + 2e^2\right)} + \frac{2b\sqrt{c}\,\arctan\left(x\sqrt{c}\right)}{2c\,d^2 + 2e^2} + \frac{2becd\ln\left(ex+d\right)}{\left(c\,d^2 - e^2\right)\left(c\,d^2 + e^2\right)} - \frac{bcd\ln\left(c\,x^2 - 1\right)}{e\left(2c\,d^2 - 2e^2\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/(e*x+d)^2,x)
```

```
[Out] -a/(e*x+d)/e-b/(e*x+d)/e*arctanh(c*x^2)+b/e*c/(2*c*d^2+2*e^2)*d*ln(c*x^2+1)
+2*b*c^(1/2)/(2*c*d^2+2*e^2)*arctan(x*c^(1/2))+2*b*e*c*d/(c*d^2-e^2)/(c*d^2
+e^2)*ln(e*x+d)-b/e*c/(2*c*d^2-2*e^2)*d*ln(c*x^2-1)-2*b*c^(1/2)/(2*c*d^2-2*
e^2)*arctanh(x*c^(1/2))
```

maxima **[A]**   time = 0.41, size = 172, normalized size = 1.04

$$\frac{1}{2}\left(\left(\frac{4\,de\log\left(ex+d\right)}{c^2d^4-e^4}+\frac{d\log\left(cx^2+1\right)}{cd^2e+e^3}-\frac{d\log\left(cx^2-1\right)}{cd^2e-e^3}+\frac{2\,\arctan\left(\sqrt{c}\,x\right)}{\left(cd^2+e^2\right)\sqrt{c}}+\frac{\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)}{\left(cd^2-e^2\right)\sqrt{c}}\right)c-\frac{2\,\operatorname{artanh}\left(cx^2\right)}{e^2x+de}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((4*d*e*log(e*x + d)/(c^2*d^4 - e^4) + d*log(c*x^2 + 1)/(c*d^2*e + e^3)
 - d*log(c*x^2 - 1)/(c*d^2*e - e^3) + 2*arctan(sqrt(c)*x)/((c*d^2 + e^2)*sq
rt(c)) + log((c*x - sqrt(c))/(c*x + sqrt(c)))/((c*d^2 - e^2)*sqrt(c)))*c -
2*arctanh(c*x^2)/(e^2*x + d*e))*b - a/(e^2*x + d*e)
```

mupad [B]   time = 2.60, size = 727, normalized size = 4.38

$$\ln\left|\frac{16\,b^4\,c^{10}\,x}{e}\right| - \frac{\left(b\sqrt{-c}\,e+b\,c\,d\right)\left|8\,b^3\,c^9\,e - \dfrac{\left(b\sqrt{-c}\,e+b\,c\,d\right)\left|\dfrac{\left(b\sqrt{-c}\,e+b\,c\,d\right)\left(\dfrac{8\,x\left(4\,b\,c^{11}\,d^5\,e^2+52\,b\,c^9\,d\,e^6\right)}{e^2} - \dfrac{\left(b\sqrt{-c}\,e+b\,c\,d\right)\left(384\,c^8\,d\,e^6+\dfrac{8\,x\left(24\,c^{10}\,d^4\,e^5+40\,c^8\,e^9\right)}{e^2}\right)}{2\left(c\,d^2\,e+e^3\right)}\right)}{2\left(c\,d^2\,e+e^3\right)}\right|}{2\left(c\,d^2\,e+e^3\right)}}{2\left(c\,d^2\,e+e^3\right)}}{2\left(c\,d^2\,e+e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))/(d + e*x)^2,x)

[Out] (log((16*b^4*c^10*x)/e - ((b*(-c)^(1/2)*e + b*c*d)*(8*b^3*c^9*e - ((b*(-c)^
(1/2)*e + b*c*d)*(((b*(-c)^(1/2)*e + b*c*d)*((8*x*(4*b*c^11*d^5*e^2 + 52*b*
c^9*d*e^6))/e^2 - ((b*(-c)^(1/2)*e + b*c*d)*(384*c^8*d*e^6 + (8*x*(40*c^8*e
^9 + 24*c^10*d^4*e^5))/e^2 + 128*c^10*d^5*e^2))/(2*(e^3 + c*d^2*e)) + 320*b
*c^9*d^2*e^3))/(2*(e^3 + c*d^2*e)) - 64*b^2*c^10*d^2*e*x))/(2*(e^3 + c*d^2*
e)) + (8*b^3*c^11*d^3*x)/e^2))/(2*(e^3 + c*d^2*e)))*(b*(-c)^(1/2)*e + b*c*d
))/(2*(e^3 + c*d^2*e)) - a/(d*e + e^2*x) - (log(((b*(-c)^(1/2)*e - b*c*d)*(
8*b^3*c^9*e - ((b*(-c)^(1/2)*e - b*c*d)*(((b*(-c)^(1/2)*e - b*c*d)*((8*x*(4
*b*c^11*d^5*e^2 + 52*b*c^9*d*e^6))/e^2 + ((b*(-c)^(1/2)*e - b*c*d)*(384*c^8
*d*e^6 + (8*x*(40*c^8*e^9 + 24*c^10*d^4*e^5))/e^2 + 128*c^10*d^5*e^2))/(2*(
```

```
e^3 + c*d^2*e)) + 320*b*c^9*d^2*e^3))/(2*(e^3 + c*d^2*e)) + 64*b^2*c^10*d^2
*e*x))/(2*(e^3 + c*d^2*e)) + (8*b^3*c^11*d^3*x)/e^2))/(2*(e^3 + c*d^2*e)) +
 (16*b^4*c^10*x)/e)*(b*(-c)^(1/2)*e - b*c*d))/(2*(e^3 + c*d^2*e)) - (b*log(
c*x^2 + 1))/(2*e*(d + e*x)) - (b*c^(1/2)*log(c^(1/2)*x - 1))/(2*(e^2 + c^(1
/2)*d*e)) + (b*c^(1/2)*log(c^(1/2)*x + 1))/(2*(e^2 - c^(1/2)*d*e)) + (b*log
(1 - c*x^2))/(e*(2*d + 2*e*x)) - (2*b*c*d*e*log(d + e*x))/(e^4 - c^2*d^4)
```

sympy $[\mathbf{F(-1)}]$ time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x**2))/(e*x+d)**2,x)

[Out] Timed out

3.28 $\int \dfrac{a+b\tanh^{-1}\left(cx^2\right)}{(d+ex)^3}\,dx$

Optimal. Leaf size=226

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{2e(d+ex)^2}+\frac{bc^{3/2}d\tan^{-1}\left(\sqrt{c}\,x\right)}{\left(cd^2+e^2\right)^2}-\frac{bc^{3/2}d\tanh^{-1}\left(\sqrt{c}\,x\right)}{\left(cd^2-e^2\right)^2}-\frac{bcde}{\left(c^2d^4-e^4\right)(d+ex)}+\frac{bce\left(3c^2d^4+e^4\right)\log(d+}{\left(c^2d^4-e^4\right)^2}$$

```
[Out]  -b*c*d*e/(c^2*d^4-e^4)/(e*x+d)+b*c^(3/2)*d*arctan(x*c^(1/2))/(c*d^2+e^2)^2+
1/2*(-a-b*arctanh(c*x^2))/e/(e*x+d)^2-b*c^(3/2)*d*arctanh(x*c^(1/2))/(c*d^2
-e^2)^2+b*c*e*(3*c^2*d^4+e^4)*ln(e*x+d)/(c^2*d^4-e^4)^2-1/4*b*c*(c*d^2+e^2)
*ln(-c*x^2+1)/e/(c*d^2-e^2)^2+1/4*b*c*(c*d^2-e^2)*ln(c*x^2+1)/e/(c*d^2+e^2)
^2
```

Rubi $[\mathbf{F}]$ time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = $ 0.000, Rules used = {}

$$\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{(d+ex)^3}\,dx$$

Verification is Not applicable to the result.

```
[In]  Int[(a + b*ArcTanh[c*x^2])/(d + e*x)^3,x]
```

```
[Out]  -a/(2*e*(d + e*x)^2) + b*Defer[Int][ArcTanh[c*x^2]/(d + e*x)^3, x]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{(d+ex)^3}\,dx = \int\left(\frac{a}{(d+ex)^3}+\frac{b\tanh^{-1}\left(cx^2\right)}{(d+ex)^3}\right)dx$$

$$= -\frac{a}{2e(d+ex)^2}+b\int \frac{\tanh^{-1}\left(cx^2\right)}{(d+ex)^3}\,dx$$

Mathematica $[\mathbf{A}]$ time = 0.68, size = 379, normalized size = 1.68

$$\frac{1}{4}\left(-\frac{2a}{e(d+ex)^2}+\frac{4bc^{3/2}d\tan^{-1}\left(\sqrt{c}\,x\right)}{\left(cd^2+e^2\right)^2}-\frac{bce\left(3c^2d^4+e^4\right)\log\left(1-c^2x^4\right)}{\left(e^4-c^2d^4\right)^2}-\frac{4bcde}{\left(c^2d^4-e^4\right)(d+ex)}+\frac{4bce\left(3c^2d^4+\right.}{\left(e^4-\right.}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcTanh[c*x^2])/(d + e*x)^3,x]

[Out] ((-2*a)/(e*(d + e*x)^2) - (4*b*c*d*e)/((c^2*d^4 - e^4)*(d + e*x)) + (4*b*c^(3/2)*d*ArcTan[Sqrt[c]*x])/(c*d^2 + e^2)^2 - (2*b*ArcTanh[c*x^2])/(e*(d + e*x)^2) - (b*c^(3/2)*d*(c^(5/2)*d^5 - 2*c^2*d^4*e - 4*c*d^2*e^3 + 3*Sqrt[c]*d*e^4 - 2*e^5)*Log[1 - Sqrt[c]*x])/(e*(-(c^2*d^4) + e^4)^2) - (b*c^(3/2)*d*(c^(5/2)*d^5 + 2*c^2*d^4*e + 4*c*d^2*e^3 + 3*Sqrt[c]*d*e^4 + 2*e^5)*Log[1 + Sqrt[c]*x])/(e*(-(c^2*d^4) + e^4)^2) + (4*b*c*e*(3*c^2*d^4 + e^4)*Log[d + e*x])/(-(c^2*d^4) + e^4)^2 + (b*c^2*(c^2*d^6 + 3*d^2*e^4)*Log[1 + c*x^2])/(e*(-(c^2*d^4) + e^4)^2) - (b*c*e*(3*c^2*d^4 + e^4)*Log[1 - c^2*x^4])/(-(c^2*d^4) + e^4)^2)/4

fricas **[B]** time = 48.70, size = 1639, normalized size = 7.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^2))/(e*x+d)^3,x, algorithm="fricas")

[Out] [-1/4*(2*a*c^4*d^8 + 4*b*c^3*d^6*e^2 - 4*a*c^2*d^4*e^4 - 4*b*c*d^2*e^6 + 2*a*e^8 - 4*(b*c^3*d^7*e + 2*b*c^2*d^5*e^3 + b*c*d^3*e^5 + (b*c^3*d^5*e^3 + 2*b*c^2*d^3*e^5 + b*c*d*e^7)*x^2 + 2*(b*c^3*d^6*e^2 + 2*b*c^2*d^4*e^4 + b*c*d^2*e^6)*x)*sqrt(-c)*arctan(sqrt(-c)*x) - 2*(b*c^3*d^7*e - 2*b*c^2*d^5*e^3 + b*c*d^3*e^5 + (b*c^3*d^5*e^3 - 2*b*c^2*d^3*e^5 + b*c*d*e^7)*x^2 + 2*(b*c^3*d^6*e^2 - 2*b*c^2*d^4*e^4 + b*c*d^2*e^6)*x)*sqrt(-c)*log((c*x^2 + 2*sqrt(-c)*x - 1)/(c*x^2 + 1)) + 4*(b*c^3*d^5*e^3 - b*c*d*e^7)*x - (b*c^4*d^8 - 3*b*c^3*d^6*e^2 + 3*b*c^2*d^4*e^4 - b*c*d^2*e^6 + (b*c^4*d^6*e^2 - 3*b*c^3*d^4*e^4 + 3*b*c^2*d^2*e^6 - b*c*e^8)*x^2 + 2*(b*c^4*d^7*e - 3*b*c^3*d^5*e^3 + 3*b*c^2*d^3*e^5 - b*c*d*e^7)*x)*log(c*x^2 + 1) + (b*c^4*d^8 + 3*b*c^3*d^6*e^2 + 3*b*c^2*d^4*e^4 + b*c*d^2*e^6 + (b*c^4*d^6*e^2 + 3*b*c^3*d^4*e^4 + 3*b*c^2*d^2*e^6 + b*c*e^8)*x^2 + 2*(b*c^4*d^7*e + 3*b*c^3*d^5*e^3 + 3*b*c^2*d^3*e^5 + b*c*d*e^7)*x)*log(c*x^2 - 1) - 4*(3*b*c^3*d^6*e^2 + b*c*d^2*e^6 + (3*b*c^3*d^4*e^4 + b*c*e^8)*x^2 + 2*(3*b*c^3*d^5*e^3 + b*c*d*e^7)*x)*log(e*x + d) + (b*c^4*d^8 - 2*b*c^2*d^4*e^4 + b*e^8)*log(-(c*x^2 + 1)/(c*x^2 - 1)))/(c^4*d^10*e - 2*c^2*d^6*e^5 + d^2*e^9 + (c^4*d^8*e^3 - 2*c^2*d^4*e^7 + e^11)*x^2 + 2*(c^4*d^9*e^2 - 2*c^2*d^5*e^6 + d*e^10)*x), -1/4*(2*a*c^4*d^8 + 4*b*c^3*d^6*e^2 - 4*a*c^2*d^4*e^4 - 4*b*c*d^2*e^6 + 2*a*e^8 - 4*(b*c^3*d^7*e - 2*b*c^2*d^5*e^3 + b*c*d^3*e^5 + (b*c^3*d^5*e^3 - 2*b*c^2*d^3*e^5 + b*c*d*e^7)*x^2 + 2*(b*c^3*d^6*e^2 - 2*b*c^2*d^4*e^4 + b*c*d^2*e^6)*x)*sqrt(c)*arctan(sqrt(c)*x) - 2*(b*c^3*d^7*e + 2*b*c^2*d^5*e^3 + b*c*d^3*e^5 + (b*c^3*d^5*e^3 + 2*b*c^2*d^3*e^5 + b*c*d*e^7)*x^2 + 2*(b*c^3*d^6*e^2 + 2*b*c^2*d^4*e^4 + b*c*d^2*e^6)*x)*sqrt(c)*log((c*x^2 - 2*sqrt(c)*x + 1)/(c*x^2 - 1)) + 4*(b*c^3*d^5*e^3 - b*c*d*e^7)*x - (b*c^4*d^8 - 3*b*c^3*d^6*e^2 + 3*b*c^2*d^4*e^4 - b*c*d^2*e^6 + (b*c^4*d^6*e^2 - 3*b*c^3*d^4*e^4 + 3*b*c^2*d^2*e^6 - b*c*e^8)*x^2 + 2*(b*c^4*d^7*e - 3*b*c^3*d^5*e^3 + 3*b*c^2*d^3*e^5 - b*c*d

```
*e^7)*x)*log(c*x^2 + 1) + (b*c^4*d^8 + 3*b*c^3*d^6*e^2 + 3*b*c^2*d^4*e^4 +
b*c*d^2*e^6 + (b*c^4*d^6*e^2 + 3*b*c^3*d^4*e^4 + 3*b*c^2*d^2*e^6 + b*c*e^8)
*x^2 + 2*(b*c^4*d^7*e + 3*b*c^3*d^5*e^3 + 3*b*c^2*d^3*e^5 + b*c*d*e^7)*x)*l
og(c*x^2 - 1) - 4*(3*b*c^3*d^6*e^2 + b*c*d^2*e^6 + (3*b*c^3*d^4*e^4 + b*c*e
^8)*x^2 + 2*(3*b*c^3*d^5*e^3 + b*c*d*e^7)*x)*log(e*x + d) + (b*c^4*d^8 - 2*
b*c^2*d^4*e^4 + b*e^8)*log(-(c*x^2 + 1)/(c*x^2 - 1)))/(c^4*d^10*e - 2*c^2*d
^6*e^5 + d^2*e^9 + (c^4*d^8*e^3 - 2*c^2*d^4*e^7 + e^11)*x^2 + 2*(c^4*d^9*e^
2 - 2*c^2*d^5*e^6 + d*e^10)*x)]
```

giac [B]   time = 41.46, size = 507, normalized size = 2.24

$$\frac{bc^{\frac{3}{2}}d\arctan\left(\sqrt{c}\,x\right)}{c^2d^4+2\,cd^2e^2+e^4}+\frac{bc^2d\arctan\left(\frac{cx}{\sqrt{-c}}\right)}{\left(c^2d^4-2\,cd^2e^2+e^4\right)\sqrt{-c}}+\frac{\left(bc^2d^2-bce^2\right)\log\left(cx^2+1\right)}{4\left(c^2d^4e+2\,cd^2e^3+e^5\right)}-\frac{\left(bc^2d^2+bce^2\right)\log\left(-cx^2+1\right)}{4\left(c^2d^4e-2\,cd^2e^3+e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] b*c^(3/2)*d*arctan(sqrt(c)*x)/(c^2*d^4 + 2*c*d^2*e^2 + e^4) + b*c^2*d*arcta
n(c*x/sqrt(-c))/((c^2*d^4 - 2*c*d^2*e^2 + e^4)*sqrt(-c)) + 1/4*(b*c^2*d^2 -
 b*c*e^2)*log(c*x^2 + 1)/(c^2*d^4*e + 2*c*d^2*e^3 + e^5) - 1/4*(b*c^2*d^2 +
 b*c*e^2)*log(-c*x^2 + 1)/(c^2*d^4*e - 2*c*d^2*e^3 + e^5) - 1/4*(b*c^4*d^8*
log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c^4*d^8 - 12*b*c^3*d^4*x^2*e^4*log(x*e
+ d) - 24*b*c^3*d^5*x*e^3*log(x*e + d) - 12*b*c^3*d^6*e^2*log(x*e + d) + 4*
b*c^3*d^5*x*e^3 + 4*b*c^3*d^6*e^2 - 2*b*c^2*d^4*e^4*log(-(c*x^2 + 1)/(c*x^2
 - 1)) - 4*a*c^2*d^4*e^4 - 4*b*c*x^2*e^8*log(x*e + d) - 8*b*c*d*x*e^7*log(x
*e + d) - 4*b*c*d^2*e^6*log(x*e + d) - 4*b*c*d*x*e^7 - 4*b*c*d^2*e^6 + b*e^
8*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*e^8)/(c^4*d^8*x^2*e^3 + 2*c^4*d^9*x*e
^2 + c^4*d^10*e - 2*c^2*d^4*x^2*e^7 - 4*c^2*d^5*x*e^6 - 2*c^2*d^6*e^5 + x^2
*e^11 + 2*d*x*e^10 + d^2*e^9)
```

maple [A]   time = 0.04, size = 310, normalized size = 1.37

$$-\frac{a}{2\left(ex+d\right)^2e}-\frac{b\arctanh\left(cx^2\right)}{2\left(ex+d\right)^2e}+\frac{bc^2\ln\left(cx^2+1\right)d^2}{4e\left(cd^2+e^2\right)^2}-\frac{bec\ln\left(cx^2+1\right)}{4\left(cd^2+e^2\right)^2}+\frac{bc^{\frac{3}{2}}d\arctan\left(x\sqrt{c}\right)}{\left(cd^2+e^2\right)^2}-\frac{becd}{\left(cd^2-e^2\right)\left(cd^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/(e*x+d)^3,x)
```

```
[Out] -1/2*a/(e*x+d)^2/e-1/2*b/(e*x+d)^2/e*arctanh(c*x^2)+1/4*b/e*c^2/(c*d^2+e^2)
^2*ln(c*x^2+1)*d^2-1/4*b*e*c/(c*d^2+e^2)^2*ln(c*x^2+1)+b*c^(3/2)*d*arctan(x
*c^(1/2))/(c*d^2+e^2)^2-b*e*c*d/(c*d^2-e^2)/(c*d^2+e^2)/(e*x+d)+3*b*e*c^3/(
c*d^2-e^2)^2/(c*d^2+e^2)^2*ln(e*x+d)*d^4+b*e^5*c/(c*d^2-e^2)^2/(c*d^2+e^2)^
```

```
2*ln(e*x+d)-1/4*b/e*c^2/(c*d^2-e^2)^2*ln(c*x^2-1)*d^2-1/4*b*e*c/(c*d^2-e^2)
^2*ln(c*x^2-1)-b*c^(3/2)*d*arctanh(x*c^(1/2))/(c*d^2-e^2)^2
```

maxima [A]   time = 0.42, size = 311, normalized size = 1.38

$$\frac{1}{4}\left(\left(\frac{4\sqrt{c}\,d\arctan\left(\sqrt{c}\,x\right)}{c^2d^4+2\,cd^2e^2+e^4}+\frac{2\sqrt{c}\,d\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)}{c^2d^4-2\,cd^2e^2+e^4}-\frac{4\,de}{c^2d^5-de^4+\left(c^2d^4e-e^5\right)x}+\frac{\left(cd^2-e^2\right)\log\left(cx^2+1\right)}{c^2d^4e+2\,cd^2e^3+e^5}-\frac{\left(cd^2+\right.}{c^2d^4e}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^2))/(e*x+d)^3,x, algorithm="maxima")

```
[Out] 1/4*((4*sqrt(c)*d*arctan(sqrt(c)*x)/(c^2*d^4 + 2*c*d^2*e^2 + e^4) + 2*sqrt(
c)*d*log((c*x - sqrt(c))/(c*x + sqrt(c)))/(c^2*d^4 - 2*c*d^2*e^2 + e^4) - 4
*d*e/(c^2*d^5 - d*e^4 + (c^2*d^4*e - e^5)*x) + (c*d^2 - e^2)*log(c*x^2 + 1)
/(c^2*d^4*e + 2*c*d^2*e^3 + e^5) - (c*d^2 + e^2)*log(c*x^2 - 1)/(c^2*d^4*e
- 2*c*d^2*e^3 + e^5) + 4*(3*c^2*d^4*e + e^5)*log(e*x + d)/(c^4*d^8 - 2*c^2*
d^4*e^4 + e^8))*c - 2*arctanh(c*x^2)/(e^3*x^2 + 2*d*e^2*x + d^2*e))*b - 1/2
*a/(e^3*x^2 + 2*d*e^2*x + d^2*e)
```

mupad [B]   time = 5.28, size = 2016, normalized size = 8.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x^2))/(d + e*x)^3,x)

```
[Out] ((a*c^2*d^4 - a*e^4 + 2*b*c*d^2*e^2)/(2*(e^4 - c^2*d^4)) + (b*c*d*e^3*x)/(e
^4 - c^2*d^4))/(d^2*e + e^3*x^2 + 2*d*e^2*x) + (log(c*d^28*(-c^3)^(13/2) +
c^21*d^28*x + 100*c^7*e^28*x - 100*c^2*e^28*(-c^3)^(3/2) + 496*d^2*e^26*(-c
^3)^(5/2) - 7398*d^8*e^20*(-c^3)^(7/2) + 17176*d^14*e^14*(-c^3)^(9/2) - 270
3*d^20*e^8*(-c^3)^(11/2) - 20*d^26*e^2*(-c^3)^(13/2) + 496*c^8*d^2*e^26*x +
 1473*c^9*d^4*e^24*x + 3692*c^10*d^6*e^22*x + 7398*c^11*d^8*e^20*x + 11868*
c^12*d^10*e^18*x + 16015*c^13*d^12*e^16*x + 17176*c^14*d^14*e^14*x + 13192*
c^15*d^16*e^12*x + 6984*c^16*d^18*e^10*x + 2703*c^17*d^20*e^8*x + 764*c^18*
d^22*e^6*x + 78*c^19*d^24*e^4*x - 20*c^20*d^26*e^2*x + 3692*c^2*d^6*e^22*(-
c^3)^(5/2) - 16015*c^2*d^12*e^16*(-c^3)^(7/2) + 6984*c^2*d^18*e^10*(-c^3)^(
9/2) - 78*c^2*d^24*e^4*(-c^3)^(11/2) + 1473*c*d^4*e^24*(-c^3)^(5/2) - 11868
*c*d^10*e^18*(-c^3)^(7/2) + 13192*c*d^16*e^12*(-c^3)^(9/2) - 764*c*d^22*e^6
*(-c^3)^(11/2))*(b*c^2*d^2 - b*c*e^2 + 2*b*d*e*(-c^3)^(1/2)))/(4*(e^5 + 2*c
*d^2*e^3 + c^2*d^4*e)) - (log(c^21*d^28*x - c*d^28*(-c^3)^(13/2) + 100*c^7*
e^28*x + 100*c^2*e^28*(-c^3)^(3/2) - 496*d^2*e^26*(-c^3)^(5/2) + 7398*d^8*e
^20*(-c^3)^(7/2) - 17176*d^14*e^14*(-c^3)^(9/2) + 2703*d^20*e^8*(-c^3)^(11/
2) + 20*d^26*e^2*(-c^3)^(13/2) + 496*c^8*d^2*e^26*x + 1473*c^9*d^4*e^24*x +
```

```
   3692*c^10*d^6*e^22*x + 7398*c^11*d^8*e^20*x + 11868*c^12*d^10*e^18*x + 160
15*c^13*d^12*e^16*x + 17176*c^14*d^14*e^14*x + 13192*c^15*d^16*e^12*x + 698
4*c^16*d^18*e^10*x + 2703*c^17*d^20*e^8*x + 764*c^18*d^22*e^6*x + 78*c^19*d
^24*e^4*x - 20*c^20*d^26*e^2*x - 3692*c^2*d^6*e^22*(-c^3)^(5/2) + 16015*c^2
*d^12*e^16*(-c^3)^(7/2) - 6984*c^2*d^18*e^10*(-c^3)^(9/2) + 78*c^2*d^24*e^4
*(-c^3)^(11/2) - 1473*c*d^4*e^24*(-c^3)^(5/2) + 11868*c*d^10*e^18*(-c^3)^(7
/2) - 13192*c*d^16*e^12*(-c^3)^(9/2) + 764*c*d^22*e^6*(-c^3)^(11/2))*(b*c*e
^2 - b*c^2*d^2 + 2*b*d*e*(-c^3)^(1/2)))/(4*(e^5 + 2*c*d^2*e^3 + c^2*d^4*e))
 - (log(100*e^28*(c^3)^(7/2) + c^2*d^28*(c^3)^(15/2) - 3692*d^6*e^22*(c^3)^
(9/2) + 16015*d^12*e^16*(c^3)^(11/2) - 6984*d^18*e^10*(c^3)^(13/2) + 78*d^2
4*e^4*(c^3)^(15/2) + c^25*d^28*x + 100*c^11*e^28*x - 496*c^12*d^2*e^26*x +
1473*c^13*d^4*e^24*x - 3692*c^14*d^6*e^22*x + 7398*c^15*d^8*e^20*x - 11868*
c^16*d^10*e^18*x + 16015*c^17*d^12*e^16*x - 17176*c^18*d^14*e^14*x + 13192*
c^19*d^16*e^12*x - 6984*c^20*d^18*e^10*x + 2703*c^21*d^20*e^8*x - 764*c^22*
d^22*e^6*x + 78*c^23*d^24*e^4*x + 20*c^24*d^26*e^2*x - 496*c*d^2*e^26*(c^3)
^(7/2) + 7398*c*d^8*e^20*(c^3)^(9/2) - 17176*c*d^14*e^14*(c^3)^(11/2) + 270
3*c*d^20*e^8*(c^3)^(13/2) + 20*c*d^26*e^2*(c^3)^(15/2) + 1473*c^2*d^4*e^24*
(c^3)^(7/2) - 11868*c^2*d^10*e^18*(c^3)^(9/2) + 13192*c^2*d^16*e^12*(c^3)^(
11/2) - 764*c^2*d^22*e^6*(c^3)^(13/2))*(b*c^2*d^2 + b*c*e^2 + 2*b*d*e*(c^3)
^(1/2)))/(4*(e^5 - 2*c*d^2*e^3 + c^2*d^4*e)) - (log(100*e^28*(c^3)^(7/2) +
c^2*d^28*(c^3)^(15/2) - 3692*d^6*e^22*(c^3)^(9/2) + 16015*d^12*e^16*(c^3)^(
11/2) - 6984*d^18*e^10*(c^3)^(13/2) + 78*d^24*e^4*(c^3)^(15/2) - c^25*d^28*
x - 100*c^11*e^28*x + 496*c^12*d^2*e^26*x - 1473*c^13*d^4*e^24*x + 3692*c^1
4*d^6*e^22*x - 7398*c^15*d^8*e^20*x + 11868*c^16*d^10*e^18*x - 16015*c^17*d
^12*e^16*x + 17176*c^18*d^14*e^14*x - 13192*c^19*d^16*e^12*x + 6984*c^20*d^
18*e^10*x - 2703*c^21*d^20*e^8*x + 764*c^22*d^22*e^6*x - 78*c^23*d^24*e^4*x
 - 20*c^24*d^26*e^2*x - 496*c*d^2*e^26*(c^3)^(7/2) + 7398*c*d^8*e^20*(c^3)^
(9/2) - 17176*c*d^14*e^14*(c^3)^(11/2) + 2703*c*d^20*e^8*(c^3)^(13/2) + 20*
c*d^26*e^2*(c^3)^(15/2) + 1473*c^2*d^4*e^24*(c^3)^(7/2) - 11868*c^2*d^10*e^
18*(c^3)^(9/2) + 13192*c^2*d^16*e^12*(c^3)^(11/2) - 764*c^2*d^22*e^6*(c^3)^
(13/2))*(b*c^2*d^2 + b*c*e^2 - 2*b*d*e*(c^3)^(1/2)))/(4*(e^5 - 2*c*d^2*e^3
+ c^2*d^4*e)) + (log(d + e*x)*(b*c*e^5 + 3*b*c^3*d^4*e))/(e^8 + c^4*d^8 - 2
*c^2*d^4*e^4) - (b*log(c*x^2 + 1))/(4*e*(d^2 + e^2*x^2 + 2*d*e*x)) + (b*log
(1 - c*x^2))/(2*e*(2*d^2 + 2*e^2*x^2 + 4*d*e*x))
```

sympy $[\mathbf{F(-1)}]$ time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/(e*x+d)**3,x)
```

```
[Out] Timed out
```

3.29 $\quad \int (d + ex)\left(a + b\tanh^{-1}\left(cx^2\right)\right)^2 dx$

Optimal. Leaf size=1085

$$dxa^2 + \frac{2bd\tan^{-1}\left(\sqrt{c}\,x\right)a}{\sqrt{c}} - \frac{2bd\tanh^{-1}\left(\sqrt{c}\,x\right)a}{\sqrt{c}} - bdx\log\left(1 - cx^2\right)a + bdx\log\left(cx^2 + 1\right)a + \frac{ib^2d\tan^{-1}\left(\sqrt{c}\,x\right)^2}{\sqrt{c}} - \frac{b^2d}{}$$

[Out] 1/2*e*(a+b*arctanh(c*x^2))^2/c+1/2*e*x^2*(a+b*arctanh(c*x^2))^2+a^2*d*x-1/2
*I*b^2*d*polylog(2,1+(-1+I)*(1+x*c^(1/2))/(1-I*x*c^(1/2)))/c^(1/2)+2*b^2*d*
arctan(x*c^(1/2))*ln(2/(1+I*x*c^(1/2)))/c^(1/2)-2*b^2*d*arctanh(x*c^(1/2))*
ln(2/(1+x*c^(1/2)))/c^(1/2)-1/2*I*b^2*d*polylog(2,1-(1+I)*(1-x*c^(1/2))/(1-
I*x*c^(1/2)))/c^(1/2)-2*a*b*d*arctanh(x*c^(1/2))/c^(1/2)+2*b^2*d*arctanh(x*
c^(1/2))*ln(2/(1-x*c^(1/2)))/c^(1/2)-2*b^2*d*arctan(x*c^(1/2))*ln(2/(1-I*x*
c^(1/2)))/c^(1/2)-b*e*(a+b*arctanh(c*x^2))*ln(2/(-c*x^2+1))/c-a*b*d*x*ln(-c
*x^2+1)+a*b*d*x*ln(c*x^2+1)+I*b^2*d*arctan(x*c^(1/2))^2/c^(1/2)-b^2*d*arcta
n(x*c^(1/2))*ln(-c*x^2+1)/c^(1/2)+b^2*d*arctanh(x*c^(1/2))*ln(-c*x^2+1)/c^(
1/2)+b^2*d*arctan(x*c^(1/2))*ln(c*x^2+1)/c^(1/2)-b^2*d*arctanh(x*c^(1/2))*l
n(c*x^2+1)/c^(1/2)+b^2*d*arctan(x*c^(1/2))*ln((1+I)*(1-x*c^(1/2))/(1-I*x*c^
(1/2)))/c^(1/2)+b^2*d*arctanh(x*c^(1/2))*ln(-2*(1-x*(-c)^(1/2))*c^(1/2)/((-
c)^(1/2)-c^(1/2))/(1+x*c^(1/2)))/c^(1/2)+b^2*d*arctanh(x*c^(1/2))*ln(2*(1+x
*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)+c^(1/2))/(1+x*c^(1/2)))/c^(1/2)+b^2*d*arct
an(x*c^(1/2))*ln((1-I)*(1+x*c^(1/2))/(1-I*x*c^(1/2)))/c^(1/2)+I*b^2*d*polyl
og(2,1-2/(1-I*x*c^(1/2)))/c^(1/2)+I*b^2*d*polylog(2,1-2/(1+I*x*c^(1/2)))/c^
(1/2)-1/2*b^2*d*x*ln(-c*x^2+1)*ln(c*x^2+1)+2*a*b*d*arctan(x*c^(1/2))/c^(1/2
)+b^2*d*polylog(2,1-2/(1+x*c^(1/2)))/c^(1/2)-b^2*d*arctanh(x*c^(1/2))^2/c^(
1/2)+b^2*d*polylog(2,1-2/(1-x*c^(1/2)))/c^(1/2)-1/2*b^2*d*polylog(2,1-2*(1+
x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)+c^(1/2))/(1+x*c^(1/2)))/c^(1/2)+1/4*b^2*d
*x*ln(-c*x^2+1)^2+1/4*b^2*d*x*ln(c*x^2+1)^2-1/2*b^2*d*polylog(2,1+2*(1-x*(-
c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(1+x*c^(1/2)))/c^(1/2)-1/2*b^2*e*pol
ylog(2,1-2/(-c*x^2+1))/c

Rubi [A]   time = 2.44, antiderivative size = 1216, normalized size of antiderivative = 1.12, number of steps used = 104, number of rules used = 39, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.167, Rules used = {6742, 6091, 298, 203, 206, 6097, 260, 6093, 2450, 2476, 2448, 321, 2470, 12, 5984, 5918, 2402, 2315, 2556, 5992, 5920, 2447, 4928, 4856, 4920, 4854, 6099, 2454, 2389, 2296, 2295, 30, 2557, 2475, 43, 2416, 2394, 2393, 2391}

<div align="center">result too large to display</div>

Warning: Unable to verify antiderivative.

[In] Int[(d + e*x)*(a + b*ArcTanh[c*x^2])^2,x]

```
[Out] (a^2*(d + e*x)^2)/(2*e) + (2*a*b*d*ArcTan[Sqrt[c]*x])/Sqrt[c] + (I*b^2*d*Ar
cTan[Sqrt[c]*x]^2)/Sqrt[c] - (2*a*b*d*ArcTanh[Sqrt[c]*x])/Sqrt[c] - (b^2*d*
ArcTanh[Sqrt[c]*x]^2)/Sqrt[c] + 2*a*b*d*x*ArcTanh[c*x^2] + a*b*e*x^2*ArcTan
h[c*x^2] + (2*b^2*d*ArcTanh[Sqrt[c]*x]*Log[2/(1 - Sqrt[c]*x)])/Sqrt[c] - (2
*b^2*d*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*x)])/Sqrt[c] + (b^2*d*ArcTan[
Sqrt[c]*x]*Log[((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/Sqrt[c] + (2*b
^2*d*ArcTan[Sqrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)])/Sqrt[c] - (2*b^2*d*ArcTanh
[Sqrt[c]*x]*Log[2/(1 + Sqrt[c]*x)])/Sqrt[c] + (b^2*d*ArcTanh[Sqrt[c]*x]*Log
[(-2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/Sqr
t[c] + (b^2*d*ArcTanh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c
] + Sqrt[c])*(1 + Sqrt[c]*x))])/Sqrt[c] + (b^2*d*ArcTan[Sqrt[c]*x]*Log[((1
- I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/Sqrt[c] - (b^2*d*ArcTan[Sqrt[c]*x
]*Log[1 - c*x^2])/Sqrt[c] + (b^2*d*ArcTanh[Sqrt[c]*x]*Log[1 - c*x^2])/Sqrt[
c] + (b^2*d*x*Log[1 - c*x^2]^2)/4 - (b^2*e*(1 - c*x^2)*Log[1 - c*x^2]^2)/(8
*c) - (b^2*e*Log[1 - c*x^2]*Log[(1 + c*x^2)/2])/(4*c) + (b^2*d*ArcTan[Sqrt[
c]*x]*Log[1 + c*x^2])/Sqrt[c] - (b^2*d*ArcTanh[Sqrt[c]*x]*Log[1 + c*x^2])/S
qrt[c] + (b^2*e*Log[(1 - c*x^2)/2]*Log[1 + c*x^2])/(4*c) - (b^2*d*x*Log[1 -
 c*x^2]*Log[1 + c*x^2])/2 - (b^2*e*x^2*Log[1 - c*x^2]*Log[1 + c*x^2])/4 + (
b^2*d*x*Log[1 + c*x^2]^2)/4 + (b^2*e*(1 + c*x^2)*Log[1 + c*x^2]^2)/(8*c) +
(a*b*e*Log[1 - c^2*x^4])/(2*c) - (b^2*e*PolyLog[2, (1 - c*x^2)/2])/(4*c) +
(b^2*e*PolyLog[2, (1 + c*x^2)/2])/(4*c) + (b^2*d*PolyLog[2, 1 - 2/(1 - Sqrt
[c]*x)])/Sqrt[c] + (I*b^2*d*PolyLog[2, 1 - 2/(1 - I*Sqrt[c]*x)])/Sqrt[c] -
((I/2)*b^2*d*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/S
qrt[c] + (I*b^2*d*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)])/Sqrt[c] + (b^2*d*Pol
yLog[2, 1 - 2/(1 + Sqrt[c]*x)])/Sqrt[c] - (b^2*d*PolyLog[2, 1 + (2*Sqrt[c]*
(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/(2*Sqrt[c]) - (b
^2*d*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 +
 Sqrt[c]*x))])/(2*Sqrt[c]) - ((I/2)*b^2*d*PolyLog[2, 1 - ((1 - I)*(1 + Sqrt
[c]*x))/(1 - I*Sqrt[c]*x)])/Sqrt[c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2450

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[(x^n*
(a + b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2556

```
Int[Log[v_]*Log[w_], x_Symbol] :> Simp[x*Log[v]*Log[w], x] + (-Int[Simplify
Integrand[(x*Log[w]*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(x*Log[v]*D[
w, x])/w, x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w,
x]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6091

```
Int[ArcTanh[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTanh[c*x^n], x] - Dist
[c*n, Int[x^n/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rule 6093

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.), x_Symbol] :> Int[Expand
Integrand[(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 - c*x^n])/2)^p, x], x] /; Fr
```

eeQ[{a, b, c, n}, x] && IGtQ[p, 0] && IntegerQ[n]

Rule 6097

Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rule 6099

Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]

Rule 6742

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]

Rubi steps

$$\int (d + ex)\left(a + b\tanh^{-1}\left(cx^2\right)\right)^2 dx = \int \left(a^2(d + ex) + 2ab(d + ex)\tanh^{-1}\left(cx^2\right) + b^2(d + ex)\tanh^{-1}\left(cx^2\right)^2\right) dx$$

$$= \frac{a^2(d + ex)^2}{2e} + (2ab)\int (d + ex)\tanh^{-1}\left(cx^2\right) dx + b^2 \int (d + ex)\tanh^{-1}\left(cx^2\right.$$

$$= \frac{a^2(d + ex)^2}{2e} + (2ab)\int \left(d\tanh^{-1}\left(cx^2\right) + ex\tanh^{-1}\left(cx^2\right)\right) dx + b^2 \int \left(d\tan\right.$$

$$= \frac{a^2(d + ex)^2}{2e} + (2abd)\int \tanh^{-1}\left(cx^2\right) dx + \left(b^2d\right)\int \tanh^{-1}\left(cx^2\right)^2 dx + (2$$

$$= \frac{a^2(d + ex)^2}{2e} + 2abdx\tanh^{-1}\left(cx^2\right) + abex^2\tanh^{-1}\left(cx^2\right) + \left(b^2d\right)\int \left(\frac{1}{4}\log^2\right.$$

$$= \frac{a^2(d + ex)^2}{2e} + 2abdx\tanh^{-1}\left(cx^2\right) + abex^2\tanh^{-1}\left(cx^2\right) + \frac{abe\log\left(1 - c^2x^4\right.}{2c}$$

$$= \frac{a^2(d + ex)^2}{2e} + \frac{2abd\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} - \frac{2abd\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} + 2abdx\tanh^{-1}\left(c\right.$$

$$= \frac{a^2(d + ex)^2}{2e} + \frac{2abd\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} - \frac{2abd\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} + 2abdx\tanh^{-1}\left(c\right.$$

$$= \frac{a^2(d + ex)^2}{2e} + \frac{2abd\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} - \frac{2abd\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} + 2abdx\tanh^{-1}\left(c\right.$$

$$= \frac{1}{2}b^2ex^2 + \frac{a^2(d + ex)^2}{2e} + \frac{2abd\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} - \frac{2abd\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} + 2abdx$$

$$= 4b^2dx + \frac{1}{2}b^2ex^2 + \frac{a^2(d + ex)^2}{2e} + \frac{2abd\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} - \frac{2abd\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}}$$

$$= \frac{1}{2}b^2ex^2 + \frac{a^2(d + ex)^2}{2e} + \frac{2abd\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} - \frac{2b^2d\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} + \frac{ib^2d\tan}{}$$

$$= \frac{a^2(d + ex)^2}{2e} + \frac{2abd\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} + \frac{ib^2d\tan^{-1}\left(\sqrt{c}\,x\right)^2}{\sqrt{c}} - \frac{2abd\tanh^{-1}\left(\sqrt{c}\right.}{\sqrt{c}}$$

$$= \frac{a^2(d + ex)^2}{2e} + \frac{2abd\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} + \frac{ib^2d\tan^{-1}\left(\sqrt{c}\,x\right)^2}{\sqrt{c}} - \frac{2abd\tanh^{-1}\left(\sqrt{c}\right.}{\sqrt{c}}$$

$$= \frac{a^2(d + ex)^2}{2e} + \frac{2abd\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} + \frac{ib^2d\tan^{-1}\left(\sqrt{c}\,x\right)^2}{\sqrt{c}} - \frac{2abd\tanh^{-1}\left(\sqrt{c}\right.}{\sqrt{c}}$$

$$= \frac{a^2(d + ex)^2}{2e} + \frac{2abd\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}} + \frac{ib^2d\tan^{-1}\left(\sqrt{c}\,x\right)^2}{\sqrt{c}} - \frac{2abd\tanh^{-1}\left(\sqrt{c}\right.}{\sqrt{c}}$$

Mathematica [A]   time = 3.00, size = 684, normalized size = 0.63

$$2a^2cdx^2 + a^2cex^3 + abex\left(\log\left(1 - c^2x^4\right) + 2cx^2\tanh^{-1}\left(cx^2\right)\right) + 4abcdx^2\tanh^{-1}\left(cx^2\right) + 4abd\sqrt{cx^2}\left(\tan^{-1}\left(\sqrt{cx}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)*(a + b*ArcTanh[c*x^2])^2,x]

[Out] (2*a^2*c*d*x^2 + a^2*c*e*x^3 + 4*a*b*c*d*x^2*ArcTanh[c*x^2] + 4*a*b*d*Sqrt[
c*x^2]*(ArcTan[Sqrt[c*x^2]] - ArcTanh[Sqrt[c*x^2]]) + b^2*e*x*ArcTanh[c*x^2
]*((-1 + c*x^2)*ArcTanh[c*x^2] - 2*Log[1 + E^(-2*ArcTanh[c*x^2])]) + a*b*e*
x*(2*c*x^2*ArcTanh[c*x^2] + Log[1 - c^2*x^4]) + b^2*e*x*PolyLog[2, -E^(-2*A
rcTanh[c*x^2])] - b^2*d*Sqrt[c*x^2]*((2*I)*ArcTan[Sqrt[c*x^2]]^2 - 4*ArcTan
[Sqrt[c*x^2]]*ArcTanh[c*x^2] - 2*Sqrt[c*x^2]*ArcTanh[c*x^2]^2 - 2*ArcTan[Sq
rt[c*x^2]]*Log[1 + E^((4*I)*ArcTan[Sqrt[c*x^2]])] - 2*ArcTanh[c*x^2]*Log[1
- Sqrt[c*x^2]] + Log[2]*Log[1 - Sqrt[c*x^2]] - Log[1 - Sqrt[c*x^2]]^2/2 + L
og[1 - Sqrt[c*x^2]]*Log[(1/2 + I/2)*(-I + Sqrt[c*x^2])] + 2*ArcTanh[c*x^2]*
Log[1 + Sqrt[c*x^2]] - Log[2]*Log[1 + Sqrt[c*x^2]] - Log[((1 + I) - (1 - I)
*Sqrt[c*x^2])/2]*Log[1 + Sqrt[c*x^2]] - Log[(-1/2 - I/2)*(I + Sqrt[c*x^2])]
*Log[1 + Sqrt[c*x^2]] + Log[1 + Sqrt[c*x^2]]^2/2 + Log[1 - Sqrt[c*x^2]]*Log
[((1 + I) + (1 - I)*Sqrt[c*x^2])/2] + (I/2)*PolyLog[2, -E^((4*I)*ArcTan[Sqr
t[c*x^2]])] - PolyLog[2, (1 - Sqrt[c*x^2])/2] + PolyLog[2, (-1/2 - I/2)*(-1
 + Sqrt[c*x^2])] + PolyLog[2, (-1/2 + I/2)*(-1 + Sqrt[c*x^2])] + PolyLog[2,
 (1 + Sqrt[c*x^2])/2] - PolyLog[2, (1/2 - I/2)*(1 + Sqrt[c*x^2])] - PolyLog
[2, (1/2 + I/2)*(1 + Sqrt[c*x^2])]]))/(2*c*x)
```

fricas [F]   time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(a^2ex + a^2d + \left(b^2ex + b^2d\right)\operatorname{artanh}\left(cx^2\right)^2 + 2\left(abex + abd\right)\operatorname{artanh}\left(cx^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")

[Out] integral(a^2*e*x + a^2*d + (b^2*e*x + b^2*d)*arctanh(c*x^2)^2 + 2*(a*b*e*x
+ a*b*d)*arctanh(c*x^2), x)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (ex + d)\left(b\operatorname{artanh}\left(cx^2\right) + a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*arctanh(c*x^2) + a)^2, x)
```

maple [F]   time = 0.42, size = 0, normalized size = 0.00

$$\int (ex + d)\left(a + b\operatorname{arctanh}\left(c\,x^2\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arctanh(c*x^2))^2,x)
```

```
[Out] int((e*x+d)*(a+b*arctanh(c*x^2))^2,x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^2ex^2 + \left(c\left(\frac{2\arctan\left(\sqrt{c}\,x\right)}{c^{\frac{3}{2}}} + \frac{\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right) + 2x\operatorname{artanh}\left(cx^2\right)\right)abd + a^2dx + \frac{\left(2cx^2\operatorname{artanh}\left(cx^2\right) + \log\left(-c^2x^4 + 1\right)\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*e*x^2 + (c*(2*arctan(sqrt(c)*x)/c^(3/2) + log((c*x - sqrt(c))/(c*x
+ sqrt(c)))/c^(3/2)) + 2*x*arctanh(c*x^2))*a*b*d + a^2*d*x + 1/2*(2*c*x^2*a
rctanh(c*x^2) + log(-c^2*x^4 + 1))*a*b*e/c + 1/8*(b^2*e*x^2 + 2*b^2*d*x)*lo
g(-c*x^2 + 1)^2 - integrate(-1/4*((b^2*c*e*x^3 + b^2*c*d*x^2 - b^2*e*x - b^
2*d)*log(c*x^2 + 1)^2 - 2*(b^2*c*e*x^3 + 2*b^2*c*d*x^2 + (b^2*c*e*x^3 + b^2
*c*d*x^2 - b^2*e*x - b^2*d)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^2 - 1), x
)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + b\operatorname{atanh}\left(c\,x^2\right)\right)^2 (d + ex)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^2*(d + e*x),x)
```

```
[Out] int((a + b*atanh(c*x^2))^2*(d + e*x), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{atanh}\left(cx^2\right)\right)^2 (d + ex)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*atanh(c*x**2))**2,x)
[Out] Integral((a + b*atanh(c*x**2))**2*(d + e*x), x)
```

3.30 $\int \dfrac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{d+ex}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{d+ex},x\right)$$

[Out] Unintegrable((a+b*arctanh(c*x^2))^2/(e*x+d),x)

Rubi [A]   time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{d+ex}\,dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*ArcTanh[c*x^2])^2/(d + e*x),x]

[Out] (a^2*Log[d + e*x])/e + 2*a*b*Defer[Int][ArcTanh[c*x^2]/(d + e*x), x] + b^2*
Defer[Int][ArcTanh[c*x^2]^2/(d + e*x), x]

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{d+ex}\,dx = \int\left(\frac{a^2}{d+ex}+\frac{2ab\tanh^{-1}\left(cx^2\right)}{d+ex}+\frac{b^2\tanh^{-1}\left(cx^2\right)^2}{d+ex}\right)dx$$

$$= \frac{a^2\log(d+ex)}{e}+(2ab)\int\frac{\tanh^{-1}\left(cx^2\right)}{d+ex}\,dx+b^2\int\frac{\tanh^{-1}\left(cx^2\right)^2}{d+ex}\,dx$$

Mathematica [A]   time = 44.30, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{d+ex}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*ArcTanh[c*x^2])^2/(d + e*x),x]

[Out] Integrate[(a + b*ArcTanh[c*x^2])^2/(d + e*x), x]

fricas [A]   time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\operatorname{artanh}\left(cx^2\right)^2 + 2\,ab\operatorname{artanh}\left(cx^2\right) + a^2}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)/(e*x + d), x)
```

giac [A]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\operatorname{artanh}\left(cx^2\right) + a\right)^2}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2/(e*x + d), x)
```

maple [A]   time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\operatorname{arctanh}\left(c\,x^2\right)\right)^2}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/(e*x+d),x)
```

```
[Out] int((a+b*arctanh(c*x^2))^2/(e*x+d),x)
```

maxima [A]   time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\log\left(ex + d\right)}{e} + \int \frac{b^2\left(\log\left(cx^2 + 1\right) - \log\left(-cx^2 + 1\right)\right)^2}{4\left(ex + d\right)} + \frac{ab\left(\log\left(cx^2 + 1\right) - \log\left(-cx^2 + 1\right)\right)}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] a^2*log(e*x + d)/e + integrate(1/4*b^2*(log(c*x^2 + 1) - log(-c*x^2 + 1))^2
/(e*x + d) + a*b*(log(c*x^2 + 1) - log(-c*x^2 + 1))/(e*x + d), x)
```

mupad [A]   time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + b\operatorname{atanh}\left(c\,x^2\right)\right)^2}{d + e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^2/(d + e*x),x)
```

```
[Out] int((a + b*atanh(c*x^2))^2/(d + e*x), x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/(e*x+d),x)
```

```
[Out] Timed out
```

3.31 $\int \dfrac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(d+ex)^2}\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left(\frac{\left(a + b\tanh^{-1}\left(cx^2\right)\right)^2}{(d + ex)^2}, x\right)$$

[Out] Unintegrable((a+b*arctanh(c*x^2))^2/(e*x+d)^2,x)

Rubi [A]   time $= 0.37$, antiderivative size $= 0$, normalized size of antiderivative $= 0.00$, number of steps used $= 0$, number of rules used $= 0$, integrand size $= 0$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used $= \{\}$

$$\int \frac{\left(a + b\tanh^{-1}\left(cx^2\right)\right)^2}{(d + ex)^2}\,dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*ArcTanh[c*x^2])^2/(d + e*x)^2,x]

[Out] -(a^2/(e*(d + e*x))) + (2*a*b*Sqrt[c]*ArcTan[Sqrt[c]*x])/(c*d^2 + e^2) - (2
*a*b*Sqrt[c]*ArcTanh[Sqrt[c]*x])/(c*d^2 - e^2) - (2*a*b*ArcTanh[c*x^2])/(e*
(d + e*x)) + (4*a*b*c*d*e*Log[d + e*x])/(c^2*d^4 - e^4) - (a*b*c*d*Log[1 -
c*x^2])/(e*(c*d^2 - e^2)) + (a*b*c*d*Log[1 + c*x^2])/(e*(c*d^2 + e^2)) + b^
2*Defer[Int][ArcTanh[c*x^2]^2/(d + e*x)^2, x]

Rubi steps

$$\int \frac{\left(a + b\tanh^{-1}\left(cx^2\right)\right)^2}{(d+ex)^2}\,dx = \int \left(\frac{a^2}{(d+ex)^2} + \frac{2ab\tanh^{-1}\left(cx^2\right)}{(d+ex)^2} + \frac{b^2\tanh^{-1}\left(cx^2\right)^2}{(d+ex)^2}\right)dx$$

$$= -\frac{a^2}{e(d+ex)} + (2ab)\int \frac{\tanh^{-1}\left(cx^2\right)}{(d+ex)^2}\,dx + b^2\int \frac{\tanh^{-1}\left(cx^2\right)^2}{(d+ex)^2}\,dx$$

$$= -\frac{a^2}{e(d+ex)} - \frac{2ab\tanh^{-1}\left(cx^2\right)}{e(d+ex)} + b^2\int \frac{\tanh^{-1}\left(cx^2\right)^2}{(d+ex)^2}\,dx + \frac{(2ab)\int \frac{2cx}{(d+ex)\left(1-c^2x^4\right)}\,dx}{e}$$

$$= -\frac{a^2}{e(d+ex)} - \frac{2ab\tanh^{-1}\left(cx^2\right)}{e(d+ex)} + b^2\int \frac{\tanh^{-1}\left(cx^2\right)^2}{(d+ex)^2}\,dx + \frac{(4abc)\int \frac{x}{(d+ex)\left(1-c^2x^4\right)}\,dx}{e}$$

$$= -\frac{a^2}{e(d+ex)} - \frac{2ab\tanh^{-1}\left(cx^2\right)}{e(d+ex)} + b^2\int \frac{\tanh^{-1}\left(cx^2\right)^2}{(d+ex)^2}\,dx + \frac{(4abc)\int \left(-\frac{de^3}{\left(-c^2d^4+e^4\right)(d+\cdots)}\right)}{}$$

$$= -\frac{a^2}{e(d+ex)} - \frac{2ab\tanh^{-1}\left(cx^2\right)}{e(d+ex)} + \frac{4abcde\log(d+ex)}{c^2d^4-e^4} + b^2\int \frac{\tanh^{-1}\left(cx^2\right)^2}{(d+ex)^2}\,dx + \cdots$$

$$= -\frac{a^2}{e(d+ex)} - \frac{2ab\tanh^{-1}\left(cx^2\right)}{e(d+ex)} + \frac{4abcde\log(d+ex)}{c^2d^4-e^4} + b^2\int \frac{\tanh^{-1}\left(cx^2\right)^2}{(d+ex)^2}\,dx + \cdots$$

$$= -\frac{a^2}{e(d+ex)} + \frac{2ab\sqrt{c}\tan^{-1}\left(\sqrt{c}\,x\right)}{cd^2+e^2} - \frac{2ab\sqrt{c}\tanh^{-1}\left(\sqrt{c}\,x\right)}{cd^2-e^2} - \frac{2ab\tanh^{-1}\left(cx^2\right)}{e(d+ex)} + \cdots$$

Mathematica [A]   time = 42.01, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\tanh^{-1}\left(cx^2\right)\right)^2}{(d+ex)^2}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(a + b*ArcTanh[c*x^2])^2/(d + e*x)^2,x]
[Out] Integrate[(a + b*ArcTanh[c*x^2])^2/(d + e*x)^2, x]
```

fricas [A]   time = 0.91, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b^2\operatorname{artanh}\left(cx^2\right)^2 + 2\,ab\operatorname{artanh}\left(cx^2\right) + a^2}{e^2x^2 + 2\,dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)/(e^2*x^2 + 2*d
*e*x + d^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\operatorname{artanh}\left(cx^2\right)+a\right)^2}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2/(e*x + d)^2, x)
```

maple [A] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\arctanh\left(c\,x^2\right)\right)^2}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/(e*x+d)^2,x)
```

```
[Out] int((a+b*arctanh(c*x^2))^2/(e*x+d)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\left(\left(\left[\frac{4\,de\log\left(ex+d\right)}{c^2d^4-e^4}+\frac{d\log\left(cx^2+1\right)}{cd^2e+e^3}-\frac{d\log\left(cx^2-1\right)}{cd^2e-e^3}+\frac{2\,\arctan\left(\sqrt{c}\,x\right)}{\left(cd^2+e^2\right)\sqrt{c}}+\frac{\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)}{\left(cd^2-e^2\right)\sqrt{c}}\right]c-\frac{2\,\operatorname{artanh}\left(cx^2\right)}{e^2x+de}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] ((4*d*e*log(e*x + d)/(c^2*d^4 - e^4) + d*log(c*x^2 + 1)/(c*d^2*e + e^3) - d
*log(c*x^2 - 1)/(c*d^2*e - e^3) + 2*arctan(sqrt(c)*x)/((c*d^2 + e^2)*sqrt(c
)) + log((c*x - sqrt(c))/(c*x + sqrt(c)))/((c*d^2 - e^2)*sqrt(c)))*c - 2*ar
ctanh(c*x^2)/(e^2*x + d*e))*a*b - 1/4*b^2*(log(-c*x^2 + 1)^2/(e^2*x + d*e)
+ integrate(-((c*e*x^2 - e)*log(c*x^2 + 1)^2 + 2*(2*c*e*x^2 + 2*c*d*x - (c*
e*x^2 - e)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*e^3*x^4 + 2*c*d*e^2*x^3 - 2*
d*e^2*x - d^2*e + (c*d^2*e - e^3)*x^2), x)) - a^2/(e^2*x + d*e)
```

mupad [A]   time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + b\operatorname{atanh}\left(c\,x^2\right)\right)^2}{(d + e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^2/(d + e*x)^2,x)
```

```
[Out] int((a + b*atanh(c*x^2))^2/(d + e*x)^2, x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/(e*x+d)**2,x)
```

```
[Out] Timed out
```

3.32 $\int (d + ex)^2 \left(a + b\tanh^{-1}\left(cx^3\right)\right)\, dx$

Optimal. Leaf size=336

$$\frac{(d + ex)^3 \left(a + b\tanh^{-1}\left(cx^3\right)\right)}{3e} + \frac{bd^2 \log\left(1 - c^{2/3}x^2\right)}{2\sqrt[3]{c}} + \frac{\sqrt{3}\,bd^2 \tan^{-1}\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)}{2\sqrt[3]{c}} - \frac{bd^2 \log\left(c^{4/3}x^4 + c^{2/3}x^2 + 1\right)}{4\sqrt[3]{c}} + \frac{bd}{}$$

```
[Out] -b*d*e*arctanh(c^(1/3)*x)/c^(2/3)+1/3*(e*x+d)^3*(a+b*arctanh(c*x^3))/e+1/2*
b*d^2*ln(1-c^(2/3)*x^2)/c^(1/3)+1/4*b*d*e*ln(1-c^(1/3)*x+c^(2/3)*x^2)/c^(2/
3)-1/4*b*d*e*ln(1+c^(1/3)*x+c^(2/3)*x^2)/c^(2/3)+1/6*b*(c*d^3+e^3)*ln(-c*x^
3+1)/c/e-1/6*b*(c*d^3-e^3)*ln(c*x^3+1)/c/e-1/4*b*d^2*ln(1+c^(2/3)*x^2+c^(4/
3)*x^4)/c^(1/3)+1/2*b*d*e*arctan(-1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2
)/c^(2/3)+1/2*b*d*e*arctan(1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)/c^(2/
3)+1/2*b*d^2*arctan(1/3*(1+2*c^(2/3)*x^2)*3^(1/2))*3^(1/2)/c^(1/3)
```

Rubi [A] time = 0.51, antiderivative size = 332, normalized size of antiderivative = 0.99, number of steps used = 25, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {6742, 6091, 275, 292, 31, 634, 617, 204, 628, 6097, 296, 618, 206, 260}

$$\frac{a(d + ex)^3}{3e} + \frac{bd^2 \log\left(1 - c^{2/3}x^2\right)}{2\sqrt[3]{c}} - \frac{bd^2 \log\left(c^{4/3}x^4 + c^{2/3}x^2 + 1\right)}{4\sqrt[3]{c}} + \frac{\sqrt{3}\,bd^2 \tan^{-1}\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)}{2\sqrt[3]{c}} + \frac{bde \log\left(c^{2/3}x^2 - \sqrt[3]{c}\,x\right)}{4c^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (a*(d + e*x)^3)/(3*e) - (Sqrt[3]*b*d*e*ArcTan[1/Sqrt[3] - (2*c^(1/3)*x)/Sqr
t[3]])/(2*c^(2/3)) + (Sqrt[3]*b*d*e*ArcTan[1/Sqrt[3] + (2*c^(1/3)*x)/Sqrt[3
]])/(2*c^(2/3)) + (Sqrt[3]*b*d^2*ArcTan[(1 + 2*c^(2/3)*x^2)/Sqrt[3]])/(2*c^
(1/3)) - (b*d*e*ArcTanh[c^(1/3)*x])/c^(2/3) + b*d^2*x*ArcTanh[c*x^3] + b*d*
e*x^2*ArcTanh[c*x^3] + (b*e^2*x^3*ArcTanh[c*x^3])/3 + (b*d^2*Log[1 - c^(2/3
)*x^2])/(2*c^(1/3)) + (b*d*e*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(4*c^(2/3))
- (b*d*e*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/(4*c^(2/3)) - (b*d^2*Log[1 + c^(
2/3)*x^2 + c^(4/3)*x^4])/(4*c^(1/3)) + (b*e^2*Log[1 - c^2*x^6])/(6*c)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

a, 0] || LtQ[b, 0])

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6091

```
Int[ArcTanh[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTanh[c*x^n], x] - Dist
[c*n, Int[x^n/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (d + ex)^2 \left(a + b\tanh^{-1}\left(cx^3\right)\right) dx = \int \left(a(d + ex)^2 + b(d + ex)^2 \tanh^{-1}\left(cx^3\right)\right) dx$$

$$= \frac{a(d + ex)^3}{3e} + b \int (d + ex)^2 \tanh^{-1}\left(cx^3\right) dx$$

$$= \frac{a(d + ex)^3}{3e} + b \int \left(d^2 \tanh^{-1}\left(cx^3\right) + 2dex \tanh^{-1}\left(cx^3\right) + e^2x^2 \tanh^{-1}\left(cx^3\right.\right.$$

$$= \frac{a(d + ex)^3}{3e} + \left(bd^2\right) \int \tanh^{-1}\left(cx^3\right) dx + (2bde) \int x \tanh^{-1}\left(cx^3\right) dx + \left(be\right.$$

$$= \frac{a(d + ex)^3}{3e} + bd^2x \tanh^{-1}\left(cx^3\right) + bdex^2 \tanh^{-1}\left(cx^3\right) + \frac{1}{3}be^2x^3 \tanh^{-1}\left(cx\right.$$

$$= \frac{a(d + ex)^3}{3e} + bd^2x \tanh^{-1}\left(cx^3\right) + bdex^2 \tanh^{-1}\left(cx^3\right) + \frac{1}{3}be^2x^3 \tanh^{-1}\left(cx\right.$$

$$= \frac{a(d + ex)^3}{3e} - \frac{bde \tanh^{-1}\left(\sqrt[3]{c}\,x\right)}{c^{2/3}} + bd^2x \tanh^{-1}\left(cx^3\right) + bdex^2 \tanh^{-1}\left(cx^3\right)$$

$$= \frac{a(d + ex)^3}{3e} - \frac{bde \tanh^{-1}\left(\sqrt[3]{c}\,x\right)}{c^{2/3}} + bd^2x \tanh^{-1}\left(cx^3\right) + bdex^2 \tanh^{-1}\left(cx^3\right)$$

$$= \frac{a(d + ex)^3}{3e} - \frac{\sqrt{3}\,bde \tan^{-1}\left(\frac{1 - 2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{2c^{2/3}} + \frac{\sqrt{3}\,bde \tan^{-1}\left(\frac{1 + 2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{2c^{2/3}} - \frac{bde \tanh^{-1}}{c^2}$$

$$= \frac{a(d + ex)^3}{3e} - \frac{\sqrt{3}\,bde \tan^{-1}\left(\frac{1 - 2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{2c^{2/3}} + \frac{\sqrt{3}\,bde \tan^{-1}\left(\frac{1 + 2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{2c^{2/3}} + \frac{\sqrt{3}\,bd^2 \tan}{}$$

Mathematica [A]　　time = 0.28, size = 299, normalized size = 0.89

$$12acd^2x + 12acdex^2 + 4ace^2x^3 - 3b\sqrt[3]{c}\,d\left(\sqrt[3]{c}\,d - e\right)\log\left(c^{2/3}x^2 - \sqrt[3]{c}\,x + 1\right) - 3b\sqrt[3]{c}\,d\left(\sqrt[3]{c}\,d + e\right)\log\left(c^{2/3}x^2 + \sqrt[3]{c}\,x\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcTanh[c*x^3]),x]

[Out] (12*a*c*d^2*x + 12*a*c*d*e*x^2 + 4*a*c*e^2*x^3 + 6*Sqrt[3]*b*c^(1/3)*d*(c^(
1/3)*d + e)*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]] - 6*Sqrt[3]*b*c^(1/3)*d*(c^(
1/3)*d - e)*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]] + 4*b*c*x*(3*d^2 + 3*d*e*x +
e^2*x^2)*ArcTanh[c*x^3] + 6*b*c^(1/3)*d*(c^(1/3)*d + e)*Log[1 - c^(1/3)*x]
```

+ 6*b*c^(1/3)*d*(c^(1/3)*d - e)*Log[1 + c^(1/3)*x] - 3*b*c^(1/3)*d*(c^(1/3)
*d - e)*Log[1 - c^(1/3)*x + c^(2/3)*x^2] - 3*b*c^(1/3)*d*(c^(1/3)*d + e)*Lo
g[1 + c^(1/3)*x + c^(2/3)*x^2] + 2*b*e^2*Log[1 - c^2*x^6])/(12*c)

fricas [C]   time = 2.84, size = 9282, normalized size = 27.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(a+b*arctanh(c*x^3)),x, algorithm="fricas")

[Out] 1/24*(8*a*c*e^2*x^3 + 24*a*c*d*e*x^2 + 24*a*c*d^2*x - 2*(2*(1/2)^(2/3)*(b^2
*e^4/c^2 - (9*c*d^3*e + e^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*e^6/c^3 + 27*
(c*d^3 + e^3)*b^3*d^3/c^2 - 3*(9*c*d^3*e + e^4)*b^3*e^2/c^3 + (27*c^2*d^6 +
 e^6)*b^3/c^3)^(1/3) - 2*b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6/c^3 + 27*(c*d^3 +
 e^3)*b^3*d^3/c^2 - 3*(9*c*d^3*e + e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^
3/c^3)^(1/3)*(I*sqrt(3) + 1))*c*log(15*b^2*c*d^3*e^2 + b^2*e^5 + 1/4*(2*(1/
2)^(2/3)*(b^2*e^4/c^2 - (9*c*d^3*e + e^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*
e^6/c^3 + 27*(c*d^3 + e^3)*b^3*d^3/c^2 - 3*(9*c*d^3*e + e^4)*b^3*e^2/c^3 +
(27*c^2*d^6 + e^6)*b^3/c^3)^(1/3) - 2*b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6/c^3
+ 27*(c*d^3 + e^3)*b^3*d^3/c^2 - 3*(9*c*d^3*e + e^4)*b^3*e^2/c^3 + (27*c^2*
d^6 + e^6)*b^3/c^3)^(1/3)*(I*sqrt(3) + 1))^2*c^2*e - 1/2*(3*b*c^2*d^3 - 2*b
*c*e^3)*(2*(1/2)^(2/3)*(b^2*e^4/c^2 - (9*c*d^3*e + e^4)*b^2/c^2)*(-I*sqrt(3
) + 1)/(2*b^3*e^6/c^3 + 27*(c*d^3 + e^3)*b^3*d^3/c^2 - 3*(9*c*d^3*e + e^4)*
b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3) - 2*b*e^2/c + (1/2)^(1/3)*(
2*b^3*e^6/c^3 + 27*(c*d^3 + e^3)*b^3*d^3/c^2 - 3*(9*c*d^3*e + e^4)*b^3*e^2/
c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3)*(I*sqrt(3) + 1)) + 9*(b^2*c^2*d^5 +
 b^2*c*d^2*e^3)*x) - 2*(2*(1/2)^(2/3)*(b^2*e^4/c^2 + (9*c*d^3*e - e^4)*b^2/
c^2)*(-I*sqrt(3) + 1)/(2*b^3*e^6/c^3 - 27*(c*d^3 - e^3)*b^3*d^3/c^2 + 3*(9*
c*d^3*e - e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3) - 2*b*e^2/c
+ (1/2)^(1/3)*(2*b^3*e^6/c^3 - 27*(c*d^3 - e^3)*b^3*d^3/c^2 + 3*(9*c*d^3*e
- e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3)*(I*sqrt(3) + 1))*c*l
og(15*b^2*c*d^3*e^2 - b^2*e^5 - 1/4*(2*(1/2)^(2/3)*(b^2*e^4/c^2 + (9*c*d^3*
e - e^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*e^6/c^3 - 27*(c*d^3 - e^3)*b^3*d^
3/c^2 + 3*(9*c*d^3*e - e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3)
 - 2*b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6/c^3 - 27*(c*d^3 - e^3)*b^3*d^3/c^2 +
3*(9*c*d^3*e - e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3)*(I*sqrt
(3) + 1))^2*c^2*e - 1/2*(3*b*c^2*d^3 + 2*b*c*e^3)*(2*(1/2)^(2/3)*(b^2*e^4/c
^2 + (9*c*d^3*e - e^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*e^6/c^3 - 27*(c*d^3
 - e^3)*b^3*d^3/c^2 + 3*(9*c*d^3*e - e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*
b^3/c^3)^(1/3) - 2*b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6/c^3 - 27*(c*d^3 - e^3)*
b^3*d^3/c^2 + 3*(9*c*d^3*e - e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)
^(1/3)*(I*sqrt(3) + 1)) - 9*(b^2*c^2*d^5 - b^2*c*d^2*e^3)*x) + (6*b*e^2 + (
2*(1/2)^(2/3)*(b^2*e^4/c^2 + (9*c*d^3*e - e^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2
*b^3*e^6/c^3 - 27*(c*d^3 - e^3)*b^3*d^3/c^2 + 3*(9*c*d^3*e - e^4)*b^3*e^2/c

^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3) - 2*b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6
/c^3 - 27*(c*d^3 - e^3)*b^3*d^3/c^2 + 3*(9*c*d^3*e - e^4)*b^3*e^2/c^3 + (27
*c^2*d^6 + e^6)*b^3/c^3)^(1/3)*(I*sqrt(3) + 1))*c + 3*sqrt(1/3)*c*sqrt((144
*b^2*c*d^3*e - 4*b^2*e^4 - 4*(2*(1/2)^(2/3)*(b^2*e^4/c^2 + (9*c*d^3*e - e^4
)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*e^6/c^3 - 27*(c*d^3 - e^3)*b^3*d^3/c^2 +
 3*(9*c*d^3*e - e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3) - 2*b*
e^2/c + (1/2)^(1/3)*(2*b^3*e^6/c^3 - 27*(c*d^3 - e^3)*b^3*d^3/c^2 + 3*(9*c*
d^3*e - e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3)*(I*sqrt(3) + 1
))*b*c*e^2 - (2*(1/2)^(2/3)*(b^2*e^4/c^2 + (9*c*d^3*e - e^4)*b^2/c^2)*(-I*s
qrt(3) + 1)/(2*b^3*e^6/c^3 - 27*(c*d^3 - e^3)*b^3*d^3/c^2 + 3*(9*c*d^3*e -
e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3) - 2*b*e^2/c + (1/2)^(1
/3)*(2*b^3*e^6/c^3 - 27*(c*d^3 - e^3)*b^3*d^3/c^2 + 3*(9*c*d^3*e - e^4)*b^3
*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3)*(I*sqrt(3) + 1))^2*c^2)/c^2))*
log(-15*b^2*c*d^3*e^2 + b^2*e^5 + 1/4*(2*(1/2)^(2/3)*(b^2*e^4/c^2 + (9*c*d^
3*e - e^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*e^6/c^3 - 27*(c*d^3 - e^3)*b^3*
d^3/c^2 + 3*(9*c*d^3*e - e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/
3) - 2*b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6/c^3 - 27*(c*d^3 - e^3)*b^3*d^3/c^2
+ 3*(9*c*d^3*e - e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3)*(I*sq
rt(3) + 1))^2*c^2*e + 1/2*(3*b*c^2*d^3 + 2*b*c*e^3)*(2*(1/2)^(2/3)*(b^2*e^4
/c^2 + (9*c*d^3*e - e^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*e^6/c^3 - 27*(c*d
^3 - e^3)*b^3*d^3/c^2 + 3*(9*c*d^3*e - e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6
)*b^3/c^3)^(1/3) - 2*b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6/c^3 - 27*(c*d^3 - e^3
)*b^3*d^3/c^2 + 3*(9*c*d^3*e - e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^
3)^(1/3)*(I*sqrt(3) + 1)) - 18*(b^2*c^2*d^5 - b^2*c*d^2*e^3)*x + 3/4*sqrt(1
/3)*(6*b*c^2*d^3 - 2*b*c*e^3 - (2*(1/2)^(2/3)*(b^2*e^4/c^2 + (9*c*d^3*e - e
^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*e^6/c^3 - 27*(c*d^3 - e^3)*b^3*d^3/c^2
 + 3*(9*c*d^3*e - e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3) - 2*
b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6/c^3 - 27*(c*d^3 - e^3)*b^3*d^3/c^2 + 3*(9*
c*d^3*e - e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3)*(I*sqrt(3) +
 1))*c^2*e)*sqrt((144*b^2*c*d^3*e - 4*b^2*e^4 - 4*(2*(1/2)^(2/3)*(b^2*e^4/c
^2 + (9*c*d^3*e - e^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*e^6/c^3 - 27*(c*d^3
 - e^3)*b^3*d^3/c^2 + 3*(9*c*d^3*e - e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*
b^3/c^3)^(1/3) - 2*b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6/c^3 - 27*(c*d^3 - e^3)*
b^3*d^3/c^2 + 3*(9*c*d^3*e - e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)
^(1/3)*(I*sqrt(3) + 1))*b*c*e^2 - (2*(1/2)^(2/3)*(b^2*e^4/c^2 + (9*c*d^3*e
- e^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*e^6/c^3 - 27*(c*d^3 - e^3)*b^3*d^3/
c^2 + 3*(9*c*d^3*e - e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3) -
 2*b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6/c^3 - 27*(c*d^3 - e^3)*b^3*d^3/c^2 + 3*
(9*c*d^3*e - e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3)*(I*sqrt(3
) + 1))^2*c^2)/c^2)) + (6*b*e^2 + (2*(1/2)^(2/3)*(b^2*e^4/c^2 + (9*c*d^3*e
- e^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*e^6/c^3 - 27*(c*d^3 - e^3)*b^3*d^3/
c^2 + 3*(9*c*d^3*e - e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3) -
 2*b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6/c^3 - 27*(c*d^3 - e^3)*b^3*d^3/c^2 + 3*
(9*c*d^3*e - e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3)*(I*sqrt(3
) + 1))*c - 3*sqrt(1/3)*c*sqrt((144*b^2*c*d^3*e - 4*b^2*e^4 - 4*(2*(1/2)^(2

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/3)*(b^2*e^4/c^2 + (9*c*d^3*e - e^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*e^6/c
^3 - 27*(c*d^3 - e^3)*b^3*d^3/c^2 + 3*(9*c*d^3*e - e^4)*b^3*e^2/c^3 + (27*c
^2*d^6 + e^6)*b^3/c^3)^(1/3) - 2*b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6/c^3 - 27*
(c*d^3 - e^3)*b^3*d^3/c^2 + 3*(9*c*d^3*e - e^4)*b^3*e^2/c^3 + (27*c^2*d^6 +
 e^6)*b^3/c^3)^(1/3)*(I*sqrt(3) + 1))*b*c*e^2 - (2*(1/2)^(2/3)*(b^2*e^4/c^2
 + (9*c*d^3*e - e^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*e^6/c^3 - 27*(c*d^3 -
 e^3)*b^3*d^3/c^2 + 3*(9*c*d^3*e - e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^
3/c^3)^(1/3) - 2*b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6/c^3 - 27*(c*d^3 - e^3)*b^
3*d^3/c^2 + 3*(9*c*d^3*e - e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(
1/3)*(I*sqrt(3) + 1))^2*c^2)/c^2))*log(-15*b^2*c*d^3*e^2 + b^2*e^5 + 1/4*(2
*(1/2)^(2/3)*(b^2*e^4/c^2 + (9*c*d^3*e - e^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*
b^3*e^6/c^3 - 27*(c*d^3 - e^3)*b^3*d^3/c^2 + 3*(9*c*d^3*e - e^4)*b^3*e^2/c^
3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3) - 2*b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6/
c^3 - 27*(c*d^3 - e^3)*b^3*d^3/c^2 + 3*(9*c*d^3*e - e^4)*b^3*e^2/c^3 + (27*
c^2*d^6 + e^6)*b^3/c^3)^(1/3)*(I*sqrt(3) + 1))^2*c^2*e + 1/2*(3*b*c^2*d^3 +
 2*b*c*e^3)*(2*(1/2)^(2/3)*(b^2*e^4/c^2 + (9*c*d^3*e - e^4)*b^2/c^2)*(-I*sq
rt(3) + 1)/(2*b^3*e^6/c^3 - 27*(c*d^3 - e^3)*b^3*d^3/c^2 + 3*(9*c*d^3*e - e
^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3) - 2*b*e^2/c + (1/2)^(1/
3)*(2*b^3*e^6/c^3 - 27*(c*d^3 - e^3)*b^3*d^3/c^2 + 3*(9*c*d^3*e - e^4)*b^3*
e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3)*(I*sqrt(3) + 1)) - 18*(b^2*c^2*
d^5 - b^2*c*d^2*e^3)*x - 3/4*sqrt(1/3)*(6*b*c^2*d^3 - 2*b*c*e^3 - (2*(1/2)^
(2/3)*(b^2*e^4/c^2 + (9*c*d^3*e - e^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*e^6
/c^3 - 27*(c*d^3 - e^3)*b^3*d^3/c^2 + 3*(9*c*d^3*e - e^4)*b^3*e^2/c^3 + (27
*c^2*d^6 + e^6)*b^3/c^3)^(1/3) - 2*b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6/c^3 - 2
7*(c*d^3 - e^3)*b^3*d^3/c^2 + 3*(9*c*d^3*e - e^4)*b^3*e^2/c^3 + (27*c^2*d^6
 + e^6)*b^3/c^3)^(1/3)*(I*sqrt(3) + 1))*c^2*e)*sqrt((144*b^2*c*d^3*e - 4*b^
2*e^4 - 4*(2*(1/2)^(2/3)*(b^2*e^4/c^2 + (9*c*d^3*e - e^4)*b^2/c^2)*(-I*sqrt
(3) + 1)/(2*b^3*e^6/c^3 - 27*(c*d^3 - e^3)*b^3*d^3/c^2 + 3*(9*c*d^3*e - e^4
)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3) - 2*b*e^2/c + (1/2)^(1/3)
*(2*b^3*e^6/c^3 - 27*(c*d^3 - e^3)*b^3*d^3/c^2 + 3*(9*c*d^3*e - e^4)*b^3*e^
2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3)*(I*sqrt(3) + 1))*b*c*e^2 - (2*(1/
2)^(2/3)*(b^2*e^4/c^2 + (9*c*d^3*e - e^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*
e^6/c^3 - 27*(c*d^3 - e^3)*b^3*d^3/c^2 + 3*(9*c*d^3*e - e^4)*b^3*e^2/c^3 +
(27*c^2*d^6 + e^6)*b^3/c^3)^(1/3) - 2*b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6/c^3
 - 27*(c*d^3 - e^3)*b^3*d^3/c^2 + 3*(9*c*d^3*e - e^4)*b^3*e^2/c^3 + (27*c^2*
d^6 + e^6)*b^3/c^3)^(1/3)*(I*sqrt(3) + 1))^2*c^2)/c^2)) + (6*b*e^2 + (2*(1/
2)^(2/3)*(b^2*e^4/c^2 - (9*c*d^3*e + e^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*
e^6/c^3 + 27*(c*d^3 + e^3)*b^3*d^3/c^2 - 3*(9*c*d^3*e + e^4)*b^3*e^2/c^3 +
(27*c^2*d^6 + e^6)*b^3/c^3)^(1/3) - 2*b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6/c^3
+ 27*(c*d^3 + e^3)*b^3*d^3/c^2 - 3*(9*c*d^3*e + e^4)*b^3*e^2/c^3 + (27*c^2*
d^6 + e^6)*b^3/c^3)^(1/3)*(I*sqrt(3) + 1))*c + 3*sqrt(1/3)*c*sqrt(-(144*b^2
*c*d^3*e + 4*b^2*e^4 + 4*(2*(1/2)^(2/3)*(b^2*e^4/c^2 - (9*c*d^3*e + e^4)*b^
2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*e^6/c^3 + 27*(c*d^3 + e^3)*b^3*d^3/c^2 - 3*(
9*c*d^3*e + e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3) - 2*b*e^2/
c + (1/2)^(1/3)*(2*b^3*e^6/c^3 + 27*(c*d^3 + e^3)*b^3*d^3/c^2 - 3*(9*c*d^3*
```

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e + e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3)*(I*sqrt(3) + 1))*b
*c*e^2 + (2*(1/2)^(2/3)*(b^2*e^4/c^2 - (9*c*d^3*e + e^4)*b^2/c^2)*(-I*sqrt(
3) + 1)/(2*b^3*e^6/c^3 + 27*(c*d^3 + e^3)*b^3*d^3/c^2 - 3*(9*c*d^3*e + e^4)
*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3) - 2*b*e^2/c + (1/2)^(1/3)*
(2*b^3*e^6/c^3 + 27*(c*d^3 + e^3)*b^3*d^3/c^2 - 3*(9*c*d^3*e + e^4)*b^3*e^2
/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3)*(I*sqrt(3) + 1))^2*c^2)/c^2))*log(
-15*b^2*c*d^3*e^2 - b^2*e^5 - 1/4*(2*(1/2)^(2/3)*(b^2*e^4/c^2 - (9*c*d^3*e
+ e^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*e^6/c^3 + 27*(c*d^3 + e^3)*b^3*d^3/
c^2 - 3*(9*c*d^3*e + e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3) -
 2*b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6/c^3 + 27*(c*d^3 + e^3)*b^3*d^3/c^2 - 3*
(9*c*d^3*e + e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3)*(I*sqrt(3
) + 1))^2*c^2*e + 1/2*(3*b*c^2*d^3 - 2*b*c*e^3)*(2*(1/2)^(2/3)*(b^2*e^4/c^2
 - (9*c*d^3*e + e^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*e^6/c^3 + 27*(c*d^3 +
 e^3)*b^3*d^3/c^2 - 3*(9*c*d^3*e + e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^
3/c^3)^(1/3) - 2*b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6/c^3 + 27*(c*d^3 + e^3)*b^
3*d^3/c^2 - 3*(9*c*d^3*e + e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(
1/3)*(I*sqrt(3) + 1)) + 18*(b^2*c^2*d^5 + b^2*c*d^2*e^3)*x + 3/4*sqrt(1/3)*
(6*b*c^2*d^3 + 2*b*c*e^3 + (2*(1/2)^(2/3)*(b^2*e^4/c^2 - (9*c*d^3*e + e^4)*
b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*e^6/c^3 + 27*(c*d^3 + e^3)*b^3*d^3/c^2 - 3
*(9*c*d^3*e + e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3) - 2*b*e^
2/c + (1/2)^(1/3)*(2*b^3*e^6/c^3 + 27*(c*d^3 + e^3)*b^3*d^3/c^2 - 3*(9*c*d^
3*e + e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3)*(I*sqrt(3) + 1))
*c^2*e)*sqrt(-(144*b^2*c*d^3*e + 4*b^2*e^4 + 4*(2*(1/2)^(2/3)*(b^2*e^4/c^2
 - (9*c*d^3*e + e^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*e^6/c^3 + 27*(c*d^3 +
e^3)*b^3*d^3/c^2 - 3*(9*c*d^3*e + e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3
/c^3)^(1/3) - 2*b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6/c^3 + 27*(c*d^3 + e^3)*b^3
*d^3/c^2 - 3*(9*c*d^3*e + e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1
/3)*(I*sqrt(3) + 1))*b*c*e^2 + (2*(1/2)^(2/3)*(b^2*e^4/c^2 - (9*c*d^3*e + e
^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*e^6/c^3 + 27*(c*d^3 + e^3)*b^3*d^3/c^2
 - 3*(9*c*d^3*e + e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3) - 2*
b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6/c^3 + 27*(c*d^3 + e^3)*b^3*d^3/c^2 - 3*(9*
c*d^3*e + e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3)*(I*sqrt(3) +
 1))^2*c^2)/c^2)) + (6*b*e^2 + (2*(1/2)^(2/3)*(b^2*e^4/c^2 - (9*c*d^3*e + e
^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*e^6/c^3 + 27*(c*d^3 + e^3)*b^3*d^3/c^2
 - 3*(9*c*d^3*e + e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3) - 2*
b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6/c^3 + 27*(c*d^3 + e^3)*b^3*d^3/c^2 - 3*(9*
c*d^3*e + e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3)*(I*sqrt(3) +
 1))*c - 3*sqrt(1/3)*c*sqrt(-(144*b^2*c*d^3*e + 4*b^2*e^4 + 4*(2*(1/2)^(2/3
)*(b^2*e^4/c^2 - (9*c*d^3*e + e^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*e^6/c^3
 + 27*(c*d^3 + e^3)*b^3*d^3/c^2 - 3*(9*c*d^3*e + e^4)*b^3*e^2/c^3 + (27*c^2
*d^6 + e^6)*b^3/c^3)^(1/3) - 2*b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6/c^3 + 27*(c
*d^3 + e^3)*b^3*d^3/c^2 - 3*(9*c*d^3*e + e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e
^6)*b^3/c^3)^(1/3)*(I*sqrt(3) + 1))*b*c*e^2 + (2*(1/2)^(2/3)*(b^2*e^4/c^2 -
 (9*c*d^3*e + e^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*e^6/c^3 + 27*(c*d^3 + e
^3)*b^3*d^3/c^2 - 3*(9*c*d^3*e + e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/
```

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c^3)^(1/3) - 2*b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6/c^3 + 27*(c*d^3 + e^3)*b^3*
d^3/c^2 - 3*(9*c*d^3*e + e^4)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/
3)*(I*sqrt(3) + 1))^2*c^2)/c^2))*log(-15*b^2*c*d^3*e^2 - b^2*e^5 - 1/4*(2*(
1/2)^(2/3)*(b^2*e^4/c^2 - (9*c*d^3*e + e^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^
3*e^6/c^3 + 27*(c*d^3 + e^3)*b^3*d^3/c^2 - 3*(9*c*d^3*e + e^4)*b^3*e^2/c^3
+ (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3) - 2*b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6/c^
3 + 27*(c*d^3 + e^3)*b^3*d^3/c^2 - 3*(9*c*d^3*e + e^4)*b^3*e^2/c^3 + (27*c^
2*d^6 + e^6)*b^3/c^3)^(1/3)*(I*sqrt(3) + 1))^2*c^2*e + 1/2*(3*b*c^2*d^3 - 2
*b*c*e^3)*(2*(1/2)^(2/3)*(b^2*e^4/c^2 - (9*c*d^3*e + e^4)*b^2/c^2)*(-I*sqrt
(3) + 1)/(2*b^3*e^6/c^3 + 27*(c*d^3 + e^3)*b^3*d^3/c^2 - 3*(9*c*d^3*e + e^4
)*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3) - 2*b*e^2/c + (1/2)^(1/3)
*(2*b^3*e^6/c^3 + 27*(c*d^3 + e^3)*b^3*d^3/c^2 - 3*(9*c*d^3*e + e^4)*b^3*e^
2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3)*(I*sqrt(3) + 1)) + 18*(b^2*c^2*d^
5 + b^2*c*d^2*e^3)*x - 3/4*sqrt(1/3)*(6*b*c^2*d^3 + 2*b*c*e^3 + (2*(1/2)^(2
/3)*(b^2*e^4/c^2 - (9*c*d^3*e + e^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*e^6/c
^3 + 27*(c*d^3 + e^3)*b^3*d^3/c^2 - 3*(9*c*d^3*e + e^4)*b^3*e^2/c^3 + (27*c
^2*d^6 + e^6)*b^3/c^3)^(1/3) - 2*b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6/c^3 + 27*
(c*d^3 + e^3)*b^3*d^3/c^2 - 3*(9*c*d^3*e + e^4)*b^3*e^2/c^3 + (27*c^2*d^6 +
 e^6)*b^3/c^3)^(1/3)*(I*sqrt(3) + 1))*c^2*e)*sqrt(-(144*b^2*c*d^3*e + 4*b^2
*e^4 + 4*(2*(1/2)^(2/3)*(b^2*e^4/c^2 - (9*c*d^3*e + e^4)*b^2/c^2)*(-I*sqrt(
3) + 1)/(2*b^3*e^6/c^3 + 27*(c*d^3 + e^3)*b^3*d^3/c^2 - 3*(9*c*d^3*e + e^4)
*b^3*e^2/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3) - 2*b*e^2/c + (1/2)^(1/3)*
(2*b^3*e^6/c^3 + 27*(c*d^3 + e^3)*b^3*d^3/c^2 - 3*(9*c*d^3*e + e^4)*b^3*e^2
/c^3 + (27*c^2*d^6 + e^6)*b^3/c^3)^(1/3)*(I*sqrt(3) + 1))*b*c*e^2 + (2*(1/2
)^(2/3)*(b^2*e^4/c^2 - (9*c*d^3*e + e^4)*b^2/c^2)*(-I*sqrt(3) + 1)/(2*b^3*e
^6/c^3 + 27*(c*d^3 + e^3)*b^3*d^3/c^2 - 3*(9*c*d^3*e + e^4)*b^3*e^2/c^3 + (
27*c^2*d^6 + e^6)*b^3/c^3)^(1/3) - 2*b*e^2/c + (1/2)^(1/3)*(2*b^3*e^6/c^3 +
 27*(c*d^3 + e^3)*b^3*d^3/c^2 - 3*(9*c*d^3*e + e^4)*b^3*e^2/c^3 + (27*c^2*d
^6 + e^6)*b^3/c^3)^(1/3)*(I*sqrt(3) + 1))^2*c^2)/c^2)) + 4*(b*c*e^2*x^3 + 3
*b*c*d*e*x^2 + 3*b*c*d^2*x)*log(-(c*x^3 + 1)/(c*x^3 - 1)))/c
```

giac [A]   time = 4.76, size = 383, normalized size = 1.14

$$
-\frac{\sqrt{3}\left(bcd^2|c|^{\frac{2}{3}} - bcd|c|^{\frac{1}{3}}e\right)\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{2c^2} + \frac{\sqrt{3}\left(bcd^2|c|^{\frac{2}{3}} + bcd|c|^{\frac{1}{3}}e\right)\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{2c^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*(b*c*d^2*abs(c)^(2/3) - b*c*d*abs(c)^(1/3)*e)*arctan(1/3*sqrt(
3)*(2*x + 1/abs(c)^(1/3))*abs(c)^(1/3))/c^2 + 1/2*sqrt(3)*(b*c*d^2*abs(c)^(
2/3) + b*c*d*abs(c)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x - 1/abs(c)^(1/3))*abs(
c)^(1/3))/c^2 + 1/6*(b*c*x^3*e^2*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 3*b*c*d*x^
```

```
2*e*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 2*a*c*x^3*e^2 + 6*a*c*d*x^2*e + 3*b*c*d
^2*x*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 6*a*c*d^2*x + b*e^2*log(c^2*x^6 - 1))/
c - 1/4*(b*c*d^2*abs(c)^(2/3) + b*c*d*abs(c)^(1/3)*e)*log(x^2 + x/abs(c)^(1
/3) + 1/abs(c)^(2/3))/c^2 - 1/4*(b*c*d^2*abs(c)^(2/3) - b*c*d*abs(c)^(1/3)*
e)*log(x^2 - x/abs(c)^(1/3) + 1/abs(c)^(2/3))/c^2 + 1/2*(b*c*d^2*abs(c)^(2/
3) - b*c*d*abs(c)^(1/3)*e)*log(abs(x + 1/abs(c)^(1/3)))/c^2 + 1/2*(b*c*d^2*
abs(c)^(2/3) + b*c*d*abs(c)^(1/3)*e)*log(abs(x - 1/abs(c)^(1/3)))/c^2
```

maple [A]   time = 0.04, size = 500, normalized size = 1.49

$$\frac{a x^3 e^2}{3}+ade\,x^2+ax\,d^2+\frac{a\,d^3}{3e}+\frac{b e^2 \operatorname{arctanh}\left(c x^3\right) x^3}{3}+be\operatorname{arctanh}\left(c x^3\right) x^2d+b\operatorname{arctanh}\left(c x^3\right) x\,d^2+\frac{b\operatorname{arctanh}\left(c x^3\right)}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^2*(a+b*arctanh(c*x^3)),x)

[Out] 1/3*a*x^3*e^2+a*d*e*x^2+a*x*d^2+1/3*a/e*d^3+1/3*b*e^2*arctanh(c*x^3)*x^3+b*
e*arctanh(c*x^3)*x^2*d+b*arctanh(c*x^3)*x*d^2+1/3*b/e*arctanh(c*x^3)*d^3+1/
2*b/c*d^2/(1/c)^(2/3)*ln(x-(1/c)^(1/3))-1/4*b/c*d^2/(1/c)^(2/3)*ln(x^2+(1/c
)^(1/3)*x+(1/c)^(2/3))-1/2*b/c*d^2/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(
2/(1/c)^(1/3)*x+1))+1/2*b*e/c*d/(1/c)^(1/3)*ln(x-(1/c)^(1/3))-1/4*b*e/c*d/(
1/c)^(1/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))+1/2*b*e/c*d*3^(1/2)/(1/c)^(1/3
)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))+1/6*b/e*ln(c*x^3-1)*d^3+1/6*b*e^2
/c*ln(c*x^3-1)+1/2*b/c*d^2/(1/c)^(2/3)*ln(x+(1/c)^(1/3))-1/4*b/c*d^2/(1/c)^
(2/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/3))+1/2*b/c*d^2/(1/c)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))-1/2*b*e/c*d/(1/c)^(1/3)*ln(x+(1/c)^(1/
3))+1/4*b*e/c*d/(1/c)^(1/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/3))+1/2*b*e/c*d*3
^(1/2)/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))-1/6*b/e*ln(c*x^3
+1)*d^3+1/6*b*e^2/c*ln(c*x^3+1)

maxima [A]   time = 0.42, size = 295, normalized size = 0.88

$$\frac{1}{3}\,ae^2x^3+adex^2+\frac{1}{4}\left(c\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{4}{3}}x^2+c^{\frac{2}{3}}\right)}{3c^{\frac{2}{3}}}\right)}{c^{\frac{4}{3}}}-\frac{\log\left(c^{\frac{4}{3}}x^4+c^{\frac{2}{3}}x^2+1\right)}{c^{\frac{4}{3}}}+\frac{2\log\left(\frac{c^{\frac{2}{3}}x^2-1}{c^{\frac{2}{3}}}\right)}{c^{\frac{4}{3}}}\right)+4x\operatorname{artanh}\left(cx^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 + a*d*e*x^2 + 1/4*(c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(4/3)
*x^2 + c^(2/3))/c^(2/3))/c^(4/3) - log(c^(4/3)*x^4 + c^(2/3)*x^2 + 1)/c^(4/
3) + 2*log((c^(2/3)*x^2 - 1)/c^(2/3))/c^(4/3)) + 4*x*arctanh(c*x^3))*b*d^2
+ 1/4*(4*x^2*arctanh(c*x^3) + c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*x
+ c^(1/3))/c^(1/3))/c^(5/3) + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*x - c
^(1/3))/c^(1/3))/c^(5/3) - log(c^(2/3)*x^2 + c^(1/3)*x + 1)/c^(5/3) + log(c
^(2/3)*x^2 - c^(1/3)*x + 1)/c^(5/3) - 2*log((c^(1/3)*x + 1)/c^(1/3))/c^(5/3
) + 2*log((c^(1/3)*x - 1)/c^(1/3))/c^(5/3)))*b*d*e + a*d^2*x + 1/6*(2*c*x^3
*arctanh(c*x^3) + log(-c^2*x^6 + 1))*b*e^2/c
```

mupad $[\mathbf{B}]$ time $= 1.82$, size $= 1081$, normalized size $= 3.22$

$$\left(\sum_{k=1}^{3} \ln\left(x\left(162\,b^5\,c^9\,d^8\,e^2 + 6\,b^5\,c^7\,d^2\,e^8\right) + \text{root}\left(216\,c^3\,z^3 - 108\,b\,c^2\,e^2\,z^2 - 162\,b^2\,c^2\,d^3\,e\,z + 18\,b^2\,c\,e^4\,z - 27\,b^3\,c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))*(d + e*x)^2,x)
```

```
[Out] symsum(log(x*(6*b^5*c^7*d^2*e^8 + 162*b^5*c^9*d^8*e^2) + root(216*c^3*z^3 -
 108*b*c^2*e^2*z^2 - 162*b^2*c^2*d^3*e*z + 18*b^2*c*e^4*z - 27*b^3*c^2*d^6
- b^3*e^6, z, k)*(x*(486*b^4*c^10*d^8 - 90*b^4*c^8*d^2*e^6) + root(216*c^3*
z^3 - 108*b*c^2*e^2*z^2 - 162*b^2*c^2*d^3*e*z + 18*b^2*c*e^4*z - 27*b^3*c^2
*d^6 - b^3*e^6, z, k)*(root(216*c^3*z^3 - 108*b*c^2*e^2*z^2 - 162*b^2*c^2*d
^3*e*z + 18*b^2*c*e^4*z - 27*b^3*c^2*d^6 - b^3*e^6, z, k)*(3888*b^2*c^10*d^
3*e - 3888*root(216*c^3*z^3 - 108*b*c^2*e^2*z^2 - 162*b^2*c^2*d^3*e*z + 18*
b^2*c*e^4*z - 27*b^3*c^2*d^6 - b^3*e^6, z, k)*b*c^11*d^2*x + 648*b^2*c^10*d
^2*e^2*x) - 972*b^3*c^9*d^3*e^3 + 324*b^3*c^9*d^2*e^4*x)) + 243*b^5*c^9*d^9
*e + 9*b^5*c^7*d^3*e^7)*root(216*c^3*z^3 - 108*b*c^2*e^2*z^2 - 162*b^2*c^2*
d^3*e*z + 18*b^2*c*e^4*z - 27*b^3*c^2*d^6 - b^3*e^6, z, k), k, 1, 3) + syms
um(log(x*(6*b^5*c^7*d^2*e^8 + 162*b^5*c^9*d^8*e^2) + root(216*c^3*z^3 - 108
*b*c^2*e^2*z^2 + 162*b^2*c^2*d^3*e*z + 18*b^2*c*e^4*z - 27*b^3*c^2*d^6 - b^
3*e^6, z, k)*(x*(486*b^4*c^10*d^8 - 90*b^4*c^8*d^2*e^6) + root(216*c^3*z^3
- 108*b*c^2*e^2*z^2 + 162*b^2*c^2*d^3*e*z + 18*b^2*c*e^4*z - 27*b^3*c^2*d^6
 - b^3*e^6, z, k)*(root(216*c^3*z^3 - 108*b*c^2*e^2*z^2 + 162*b^2*c^2*d^3*e
*z + 18*b^2*c*e^4*z - 27*b^3*c^2*d^6 - b^3*e^6, z, k)*(3888*b^2*c^10*d^3*e
- 3888*root(216*c^3*z^3 - 108*b*c^2*e^2*z^2 + 162*b^2*c^2*d^3*e*z + 18*b^2*
c*e^4*z - 27*b^3*c^2*d^6 - b^3*e^6, z, k)*b*c^11*d^2*x + 648*b^2*c^10*d^2*e
^2*x) - 972*b^3*c^9*d^3*e^3 + 324*b^3*c^9*d^2*e^4*x)) + 243*b^5*c^9*d^9*e +
 9*b^5*c^7*d^3*e^7)*root(216*c^3*z^3 - 108*b*c^2*e^2*z^2 + 162*b^2*c^2*d^3*
e*z + 18*b^2*c*e^4*z - 27*b^3*c^2*d^6 - b^3*e^6, z, k), k, 1, 3) + log(c*x^
3 + 1)*((b*e^2*x^3)/6 + (b*d^2*x)/2 + (b*d*e*x^2)/2) - log(1 - c*x^3)*((b*e
```

```
^2*x^3)/6 + (b*d^2*x)/2 + (b*d*e*x^2)/2) + (a*e^2*x^3)/3 + a*d^2*x + a*d*e*
x^2
```

sympy $[\mathbf{F(-1)}]$　　time = 0.00, size = 0, normalized size = 0.00

<div align="center">Timed out</div>

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*atanh(c*x**3)),x)
```

```
[Out] Timed out
```

3.33 $\int (d + ex)\left(a + b\tanh^{-1}\left(cx^3\right)\right)\,dx$

Optimal. Leaf size=285

$$\frac{(d + ex)^2\left(a + b\tanh^{-1}\left(cx^3\right)\right)}{2e} + \frac{bd\log\left(1 - c^{2/3}x^2\right)}{2\sqrt[3]{c}} + \frac{\sqrt{3}\,bd\tan^{-1}\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)}{2\sqrt[3]{c}} - \frac{bd\log\left(c^{4/3}x^4 + c^{2/3}x^2 + 1\right)}{4\sqrt[3]{c}} + be\log$$

```
[Out] -1/2*b*e*arctanh(c^(1/3)*x)/c^(2/3)-1/2*b*d^2*arctanh(c*x^3)/e+1/2*(e*x+d)^
2*(a+b*arctanh(c*x^3))/e+1/2*b*d*ln(1-c^(2/3)*x^2)/c^(1/3)+1/8*b*e*ln(1-c^(
1/3)*x+c^(2/3)*x^2)/c^(2/3)-1/8*b*e*ln(1+c^(1/3)*x+c^(2/3)*x^2)/c^(2/3)-1/4
*b*d*ln(1+c^(2/3)*x^2+c^(4/3)*x^4)/c^(1/3)+1/4*b*e*arctan(-1/3*3^(1/2)+2/3*
c^(1/3)*x*3^(1/2))*3^(1/2)/c^(2/3)+1/4*b*e*arctan(1/3*3^(1/2)+2/3*c^(1/3)*x
*3^(1/2))*3^(1/2)/c^(2/3)+1/2*b*d*arctan(1/3*(1+2*c^(2/3)*x^2)*3^(1/2))*3^(
1/2)/c^(1/3)
```

Rubi [A]   time = 0.45, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {6273, 12, 1831, 275, 206, 292, 31, 634, 617, 204, 628, 296, 618}

$$\frac{(d + ex)^2\left(a + b\tanh^{-1}\left(cx^3\right)\right)}{2e} + \frac{bd\log\left(1 - c^{2/3}x^2\right)}{2\sqrt[3]{c}} - \frac{bd\log\left(c^{4/3}x^4 + c^{2/3}x^2 + 1\right)}{4\sqrt[3]{c}} + \frac{\sqrt{3}\,bd\tan^{-1}\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)}{2\sqrt[3]{c}} + be\log$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] -(Sqrt[3]*b*e*ArcTan[1/Sqrt[3] - (2*c^(1/3)*x)/Sqrt[3]])/(4*c^(2/3)) + (Sqr
t[3]*b*e*ArcTan[1/Sqrt[3] + (2*c^(1/3)*x)/Sqrt[3]])/(4*c^(2/3)) + (Sqrt[3]*
b*d*ArcTan[(1 + 2*c^(2/3)*x^2)/Sqrt[3]])/(2*c^(1/3)) - (b*e*ArcTanh[c^(1/3)
*x])/(2*c^(2/3)) - (b*d^2*ArcTanh[c*x^3])/(2*e) + ((d + e*x)^2*(a + b*ArcTa
nh[c*x^3]))/(2*e) + (b*d*Log[1 - c^(2/3)*x^2])/(2*c^(1/3)) + (b*e*Log[1 - c
^(1/3)*x + c^(2/3)*x^2])/(8*c^(2/3)) - (b*e*Log[1 + c^(1/3)*x + c^(2/3)*x^2
])/(8*c^(2/3)) - (b*d*Log[1 + c^(2/3)*x^2 + c^(4/3)*x^4])/(4*c^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rubi steps

$$\int (d + ex)\left(a + b\tanh^{-1}\left(cx^3\right)\right) dx = \frac{(d+ex)^2\left(a + b\tanh^{-1}\left(cx^3\right)\right)}{2e} - \frac{b\int \frac{3cx^2(d+ex)^2}{1-c^2x^6} dx}{2e}$$

$$= \frac{(d+ex)^2\left(a + b\tanh^{-1}\left(cx^3\right)\right)}{2e} - \frac{(3bc)\int \frac{x^2(d+ex)^2}{1-c^2x^6} dx}{2e}$$

$$= \frac{(d+ex)^2\left(a + b\tanh^{-1}\left(cx^3\right)\right)}{2e} - \frac{(3bc)\int \left(\frac{d^2x^2}{1-c^2x^6} + \frac{2dex^3}{1-c^2x^6} + \frac{e^2x^4}{1-c^2x^6}\right) dx}{2e}$$

$$= \frac{(d+ex)^2\left(a + b\tanh^{-1}\left(cx^3\right)\right)}{2e} - (3bcd)\int \frac{x^3}{1-c^2x^6} dx - \frac{\left(3bcd^2\right)\int \frac{x^2}{1-c^2x^6} dx}{2e}$$

$$= \frac{(d+ex)^2\left(a + b\tanh^{-1}\left(cx^3\right)\right)}{2e} - \frac{1}{2}(3bcd)\operatorname{Subst}\left(\int \frac{x}{1-c^2x^3} dx, x, x^2\right) - \frac{\left(b\text{…}\right.}{}$$

$$= -\frac{be\tanh^{-1}\left(\sqrt[3]{c}\,x\right)}{2c^{2/3}} - \frac{bd^2\tanh^{-1}\left(cx^3\right)}{2e} + \frac{(d+ex)^2\left(a + b\tanh^{-1}\left(cx^3\right)\right)}{2e} - \frac{1}{2}\text{…}$$

$$= -\frac{be\tanh^{-1}\left(\sqrt[3]{c}\,x\right)}{2c^{2/3}} - \frac{bd^2\tanh^{-1}\left(cx^3\right)}{2e} + \frac{(d+ex)^2\left(a + b\tanh^{-1}\left(cx^3\right)\right)}{2e} + \frac{b\text{…}}{}$$

$$= -\frac{\sqrt{3}\,be\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{4c^{2/3}} + \frac{\sqrt{3}\,be\tan^{-1}\left(\frac{1+2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{4c^{2/3}} - \frac{be\tanh^{-1}\left(\sqrt[3]{c}\,x\right)}{2c^{2/3}} - \frac{bd^2\,\text{t…}}{}$$

$$= -\frac{\sqrt{3}\,be\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{4c^{2/3}} + \frac{\sqrt{3}\,be\tan^{-1}\left(\frac{1+2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{4c^{2/3}} + \frac{\sqrt{3}\,bd\tan^{-1}\left(\frac{1+2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}} - \text{…}$$

Mathematica [A]   time = 0.10, size = 333, normalized size = 1.17

$$adx + \frac{1}{2}aex^2 - \frac{bd\left(\log\left(c^{2/3}x^2 - \sqrt[3]{c}\,x + 1\right) + \log\left(c^{2/3}x^2 + \sqrt[3]{c}\,x + 1\right) - 2\log\left(1 - \sqrt[3]{c}\,x\right) - 2\log\left(\sqrt[3]{c}\,x + 1\right) - 2\sqrt{3}\,\text{ta…}\right)}{4\sqrt[3]{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*ArcTanh[c*x^3]),x]

[Out] a*d*x + (a*e*x^2)/2 + (Sqrt[3]*b*e*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]])/(4*c
^(2/3)) + (Sqrt[3]*b*e*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/(4*c^(2/3)) + b*d
*x*ArcTanh[c*x^3] + (b*e*x^2*ArcTanh[c*x^3])/2 + (b*e*Log[1 - c^(1/3)*x])/(
4*c^(2/3)) - (b*e*Log[1 + c^(1/3)*x])/(4*c^(2/3)) + (b*e*Log[1 - c^(1/3)*x
```

```
+ c^(2/3)*x^2])/(8*c^(2/3)) - (b*e*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/(8*c^(
2/3)) - (b*d*(-2*Sqrt[3]*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]] + 2*Sqrt[3]*Arc
Tan[(1 + 2*c^(1/3)*x)/Sqrt[3]] - 2*Log[1 - c^(1/3)*x] - 2*Log[1 + c^(1/3)*x
] + Log[1 - c^(1/3)*x + c^(2/3)*x^2] + Log[1 + c^(1/3)*x + c^(2/3)*x^2]))/(
4*c^(1/3))
```

fricas [C]   time = 2.10, size = 3928, normalized size = 13.78

<div align="center">result too large to display</div>

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x^3)),x, algorithm="fricas")

[Out] 1/2*a*e*x^2 + a*d*x + 1/8*(4*(1/2)^(2/3)*b^2*d*e*(-I*sqrt(3) + 1)/(((8*c*d^
3 + e^3)*b^3/c^2 + (8*c*d^3 - e^3)*b^3/c^2)^(1/3)*c) - (1/2)^(1/3)*((8*c*d^
3 + e^3)*b^3/c^2 + (8*c*d^3 - e^3)*b^3/c^2)^(1/3)*(I*sqrt(3) + 1))*log(2*(4
*(1/2)^(2/3)*b^2*d*e*(-I*sqrt(3) + 1)/(((8*c*d^3 + e^3)*b^3/c^2 + (8*c*d^3
- e^3)*b^3/c^2)^(1/3)*c) - (1/2)^(1/3)*((8*c*d^3 + e^3)*b^3/c^2 + (8*c*d^3
- e^3)*b^3/c^2)^(1/3)*(I*sqrt(3) + 1))*b*c*d^2 + 4*b^2*d*e^2 + 1/4*(4*(1/2)
^(2/3)*b^2*d*e*(-I*sqrt(3) + 1)/(((8*c*d^3 + e^3)*b^3/c^2 + (8*c*d^3 - e^3)
*b^3/c^2)^(1/3)*c) - (1/2)^(1/3)*((8*c*d^3 + e^3)*b^3/c^2 + (8*c*d^3 - e^3)
*b^3/c^2)^(1/3)*(I*sqrt(3) + 1))^2*c*e + (8*b^2*c*d^3 + b^2*e^3)*x) - 1/16*
(4*(1/2)^(2/3)*b^2*d*e*(-I*sqrt(3) + 1)/(((8*c*d^3 + e^3)*b^3/c^2 + (8*c*d^
3 - e^3)*b^3/c^2)^(1/3)*c) - (1/2)^(1/3)*((8*c*d^3 + e^3)*b^3/c^2 + (8*c*d^
3 - e^3)*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - 2*sqrt(3/2)*sqrt(1/2)*sqrt(-(32*b
^2*d*e + (4*(1/2)^(2/3)*b^2*d*e*(-I*sqrt(3) + 1)/(((8*c*d^3 + e^3)*b^3/c^2
+ (8*c*d^3 - e^3)*b^3/c^2)^(1/3)*c) - (1/2)^(1/3)*((8*c*d^3 + e^3)*b^3/c^2
+ (8*c*d^3 - e^3)*b^3/c^2)^(1/3)*(I*sqrt(3) + 1))^2*c)/c))*log(-(4*(1/2)^(2
/3)*b^2*d*e*(-I*sqrt(3) + 1)/(((8*c*d^3 + e^3)*b^3/c^2 + (8*c*d^3 - e^3)*b^
3/c^2)^(1/3)*c) - (1/2)^(1/3)*((8*c*d^3 + e^3)*b^3/c^2 + (8*c*d^3 - e^3)*b^
3/c^2)^(1/3)*(I*sqrt(3) + 1))*b*c*d^2 - 2*b^2*d*e^2 - 1/8*(4*(1/2)^(2/3)*b^
2*d*e*(-I*sqrt(3) + 1)/(((8*c*d^3 + e^3)*b^3/c^2 + (8*c*d^3 - e^3)*b^3/c^2)
^(1/3)*c) - (1/2)^(1/3)*((8*c*d^3 + e^3)*b^3/c^2 + (8*c*d^3 - e^3)*b^3/c^2)
^(1/3)*(I*sqrt(3) + 1))^2*c*e + 1/4*sqrt(3/2)*sqrt(1/2)*(8*b*c*d^2 - (4*(1/
2)^(2/3)*b^2*d*e*(-I*sqrt(3) + 1)/(((8*c*d^3 + e^3)*b^3/c^2 + (8*c*d^3 - e^
3)*b^3/c^2)^(1/3)*c) - (1/2)^(1/3)*((8*c*d^3 + e^3)*b^3/c^2 + (8*c*d^3 - e^
3)*b^3/c^2)^(1/3)*(I*sqrt(3) + 1))*c*e)*sqrt(-(32*b^2*d*e + (4*(1/2)^(2/3)*
b^2*d*e*(-I*sqrt(3) + 1)/(((8*c*d^3 + e^3)*b^3/c^2 + (8*c*d^3 - e^3)*b^3/c^
2)^(1/3)*c) - (1/2)^(1/3)*((8*c*d^3 + e^3)*b^3/c^2 + (8*c*d^3 - e^3)*b^3/c^
2)^(1/3)*(I*sqrt(3) + 1))^2*c)/c) + (8*b^2*c*d^3 + b^2*e^3)*x) - 1/16*(4*(1
/2)^(2/3)*b^2*d*e*(-I*sqrt(3) + 1)/(((8*c*d^3 + e^3)*b^3/c^2 + (8*c*d^3 - e
^3)*b^3/c^2)^(1/3)*c) - (1/2)^(1/3)*((8*c*d^3 + e^3)*b^3/c^2 + (8*c*d^3 - e
^3)*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*sqrt(3/2)*sqrt(1/2)*sqrt(-(32*b^2*d*
e + (4*(1/2)^(2/3)*b^2*d*e*(-I*sqrt(3) + 1)/(((8*c*d^3 + e^3)*b^3/c^2 + (8*
c*d^3 - e^3)*b^3/c^2)^(1/3)*c) - (1/2)^(1/3)*((8*c*d^3 + e^3)*b^3/c^2 + (8*
```

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c*d^3 - e^3)*b^3/c^2)^(1/3)*(I*sqrt(3) + 1))^2*c)/c))*log(-(4*(1/2)^(2/3)*b
^2*d*e*(-I*sqrt(3) + 1)/(((8*c*d^3 + e^3)*b^3/c^2 + (8*c*d^3 - e^3)*b^3/c^2
)^(1/3)*c) - (1/2)^(1/3)*((8*c*d^3 + e^3)*b^3/c^2 + (8*c*d^3 - e^3)*b^3/c^2
)^(1/3)*(I*sqrt(3) + 1))*b*c*d^2 - 2*b^2*d*e^2 - 1/8*(4*(1/2)^(2/3)*b^2*d*e
*(-I*sqrt(3) + 1)/(((8*c*d^3 + e^3)*b^3/c^2 + (8*c*d^3 - e^3)*b^3/c^2)^(1/3
)*c) - (1/2)^(1/3)*((8*c*d^3 + e^3)*b^3/c^2 + (8*c*d^3 - e^3)*b^3/c^2)^(1/3
)*(I*sqrt(3) + 1))^2*c*e - 1/4*sqrt(3/2)*sqrt(1/2)*(8*b*c*d^2 - (4*(1/2)^(2
/3)*b^2*d*e*(-I*sqrt(3) + 1)/(((8*c*d^3 + e^3)*b^3/c^2 + (8*c*d^3 - e^3)*b^
3/c^2)^(1/3)*c) - (1/2)^(1/3)*((8*c*d^3 + e^3)*b^3/c^2 + (8*c*d^3 - e^3)*b^
3/c^2)^(1/3)*(I*sqrt(3) + 1))*c*e)*sqrt(-(32*b^2*d*e + (4*(1/2)^(2/3)*b^2*d
e(-I*sqrt(3) + 1)/(((8*c*d^3 + e^3)*b^3/c^2 + (8*c*d^3 - e^3)*b^3/c^2)^(1
/3)*c) - (1/2)^(1/3)*((8*c*d^3 + e^3)*b^3/c^2 + (8*c*d^3 - e^3)*b^3/c^2)^(1
/3)*(I*sqrt(3) + 1))^2*c)/c) + (8*b^2*c*d^3 + b^2*e^3)*x) + 1/16*(4*(1/2)^(
2/3)*b^2*d*e*(-I*sqrt(3) + 1)/(((8*c*d^3 + e^3)*b^3/c^2 - (8*c*d^3 - e^3)*b
^3/c^2)^(1/3)*c) + (1/2)^(1/3)*((8*c*d^3 + e^3)*b^3/c^2 - (8*c*d^3 - e^3)*b
^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*sqrt(3/2)*sqrt(1/2)*sqrt((32*b^2*d*e - (4
*(1/2)^(2/3)*b^2*d*e*(-I*sqrt(3) + 1)/(((8*c*d^3 + e^3)*b^3/c^2 - (8*c*d^3
- e^3)*b^3/c^2)^(1/3)*c) + (1/2)^(1/3)*((8*c*d^3 + e^3)*b^3/c^2 - (8*c*d^3
- e^3)*b^3/c^2)^(1/3)*(I*sqrt(3) + 1))^2*c)/c))*log((4*(1/2)^(2/3)*b^2*d*e*
(-I*sqrt(3) + 1)/(((8*c*d^3 + e^3)*b^3/c^2 - (8*c*d^3 - e^3)*b^3/c^2)^(1/3)
c) + (1/2)^(1/3)((8*c*d^3 + e^3)*b^3/c^2 - (8*c*d^3 - e^3)*b^3/c^2)^(1/3)
*(I*sqrt(3) + 1))*b*c*d^2 - 2*b^2*d*e^2 + 1/8*(4*(1/2)^(2/3)*b^2*d*e*(-I*sq
rt(3) + 1)/(((8*c*d^3 + e^3)*b^3/c^2 - (8*c*d^3 - e^3)*b^3/c^2)^(1/3)*c) +
(1/2)^(1/3)*((8*c*d^3 + e^3)*b^3/c^2 - (8*c*d^3 - e^3)*b^3/c^2)^(1/3)*(I*sq
rt(3) + 1))^2*c*e + 1/4*sqrt(3/2)*sqrt(1/2)*(8*b*c*d^2 - (4*(1/2)^(2/3)*b^2
*d*e*(-I*sqrt(3) + 1)/(((8*c*d^3 + e^3)*b^3/c^2 - (8*c*d^3 - e^3)*b^3/c^2)^
(1/3)*c) + (1/2)^(1/3)*((8*c*d^3 + e^3)*b^3/c^2 - (8*c*d^3 - e^3)*b^3/c^2)^
(1/3)*(I*sqrt(3) + 1))*c*e)*sqrt((32*b^2*d*e - (4*(1/2)^(2/3)*b^2*d*e*(-I*s
qrt(3) + 1)/(((8*c*d^3 + e^3)*b^3/c^2 - (8*c*d^3 - e^3)*b^3/c^2)^(1/3)*c) +
 (1/2)^(1/3)*((8*c*d^3 + e^3)*b^3/c^2 - (8*c*d^3 - e^3)*b^3/c^2)^(1/3)*(I*s
qrt(3) + 1))^2*c)/c) - (8*b^2*c*d^3 - b^2*e^3)*x) + 1/16*(4*(1/2)^(2/3)*b^2
*d*e*(-I*sqrt(3) + 1)/(((8*c*d^3 + e^3)*b^3/c^2 - (8*c*d^3 - e^3)*b^3/c^2)^
(1/3)*c) + (1/2)^(1/3)*((8*c*d^3 + e^3)*b^3/c^2 - (8*c*d^3 - e^3)*b^3/c^2)^
(1/3)*(I*sqrt(3) + 1) - 2*sqrt(3/2)*sqrt(1/2)*sqrt((32*b^2*d*e - (4*(1/2)^(
2/3)*b^2*d*e*(-I*sqrt(3) + 1)/(((8*c*d^3 + e^3)*b^3/c^2 - (8*c*d^3 - e^3)*b
^3/c^2)^(1/3)*c) + (1/2)^(1/3)*((8*c*d^3 + e^3)*b^3/c^2 - (8*c*d^3 - e^3)*b
^3/c^2)^(1/3)*(I*sqrt(3) + 1))^2*c)/c))*log((4*(1/2)^(2/3)*b^2*d*e*(-I*sqrt
(3) + 1)/(((8*c*d^3 + e^3)*b^3/c^2 - (8*c*d^3 - e^3)*b^3/c^2)^(1/3)*c) + (1
/2)^(1/3)*((8*c*d^3 + e^3)*b^3/c^2 - (8*c*d^3 - e^3)*b^3/c^2)^(1/3)*(I*sqrt
(3) + 1))*b*c*d^2 - 2*b^2*d*e^2 + 1/8*(4*(1/2)^(2/3)*b^2*d*e*(-I*sqrt(3) +
1)/(((8*c*d^3 + e^3)*b^3/c^2 - (8*c*d^3 - e^3)*b^3/c^2)^(1/3)*c) + (1/2)^(1
/3)*((8*c*d^3 + e^3)*b^3/c^2 - (8*c*d^3 - e^3)*b^3/c^2)^(1/3)*(I*sqrt(3) +
1))^2*c*e - 1/4*sqrt(3/2)*sqrt(1/2)*(8*b*c*d^2 - (4*(1/2)^(2/3)*b^2*d*e*(-I
*sqrt(3) + 1)/(((8*c*d^3 + e^3)*b^3/c^2 - (8*c*d^3 - e^3)*b^3/c^2)^(1/3)*c)
 + (1/2)^(1/3)*((8*c*d^3 + e^3)*b^3/c^2 - (8*c*d^3 - e^3)*b^3/c^2)^(1/3)*(I

```
*sqrt(3) + 1))*c*e)*sqrt((32*b^2*d*e - (4*(1/2)^(2/3)*b^2*d*e*(-I*sqrt(3) +
 1)/(((8*c*d^3 + e^3)*b^3/c^2 - (8*c*d^3 - e^3)*b^3/c^2)^(1/3)*c) + (1/2)^(
1/3)*((8*c*d^3 + e^3)*b^3/c^2 - (8*c*d^3 - e^3)*b^3/c^2)^(1/3)*(I*sqrt(3) +
 1))^2*c)/c) - (8*b^2*c*d^3 - b^2*e^3)*x) - 1/8*(4*(1/2)^(2/3)*b^2*d*e*(-I*
sqrt(3) + 1)/(((8*c*d^3 + e^3)*b^3/c^2 - (8*c*d^3 - e^3)*b^3/c^2)^(1/3)*c)
+ (1/2)^(1/3)*((8*c*d^3 + e^3)*b^3/c^2 - (8*c*d^3 - e^3)*b^3/c^2)^(1/3)*(I*
sqrt(3) + 1))*log(-2*(4*(1/2)^(2/3)*b^2*d*e*(-I*sqrt(3) + 1)/(((8*c*d^3 + e
^3)*b^3/c^2 - (8*c*d^3 - e^3)*b^3/c^2)^(1/3)*c) + (1/2)^(1/3)*((8*c*d^3 + e
^3)*b^3/c^2 - (8*c*d^3 - e^3)*b^3/c^2)^(1/3)*(I*sqrt(3) + 1))*b*c*d^2 + 4*b
^2*d*e^2 - 1/4*(4*(1/2)^(2/3)*b^2*d*e*(-I*sqrt(3) + 1)/(((8*c*d^3 + e^3)*b^
3/c^2 - (8*c*d^3 - e^3)*b^3/c^2)^(1/3)*c) + (1/2)^(1/3)*((8*c*d^3 + e^3)*b^
3/c^2 - (8*c*d^3 - e^3)*b^3/c^2)^(1/3)*(I*sqrt(3) + 1))^2*c*e - (8*b^2*c*d^
3 - b^2*e^3)*x) + 1/4*(b*e*x^2 + 2*b*d*x)*log(-(c*x^3 + 1)/(c*x^3 - 1))
```

giac [A]   time = 2.18, size = 304, normalized size = 1.07

$$\frac{1}{4} bx^2 e \log\left(-\frac{cx^3+1}{cx^3-1}\right) + \frac{1}{2} ax^2 e + \frac{1}{2} bdx \log\left(-\frac{cx^3+1}{cx^3-1}\right) + adx - \frac{\sqrt{3}\left(2\,bcd|c|^{\frac{2}{3}} - bc|c|^{\frac{1}{3}}e\right)\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+\frac{1}{|c|^{\frac{1}{3}}}\right)|c|\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/4*b*x^2*e*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/2*a*x^2*e + 1/2*b*d*x*log(-(c
*x^3 + 1)/(c*x^3 - 1)) + a*d*x - 1/4*sqrt(3)*(2*b*c*d*abs(c)^(2/3) - b*c*ab
s(c)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + 1/abs(c)^(1/3))*abs(c)^(1/3))/c^2 +
 1/4*sqrt(3)*(2*b*c*d*abs(c)^(2/3) + b*c*abs(c)^(1/3)*e)*arctan(1/3*sqrt(3)
*(2*x - 1/abs(c)^(1/3))*abs(c)^(1/3))/c^2 - 1/8*(2*b*c*d*abs(c)^(2/3) + b*c
*abs(c)^(1/3)*e)*log(x^2 + x/abs(c)^(1/3) + 1/abs(c)^(2/3))/c^2 - 1/8*(2*b*
c*d*abs(c)^(2/3) - b*c*abs(c)^(1/3)*e)*log(x^2 - x/abs(c)^(1/3) + 1/abs(c)^
(2/3))/c^2 + 1/4*(2*b*c*d*abs(c)^(2/3) - b*c*abs(c)^(1/3)*e)*log(abs(x + 1/
abs(c)^(1/3)))/c^2 + 1/4*(2*b*c*d*abs(c)^(2/3) + b*c*abs(c)^(1/3)*e)*log(ab
s(x - 1/abs(c)^(1/3)))/c^2
```

maple [A]   time = 0.03, size = 362, normalized size = 1.27

$$bd\sqrt{3}\ \text{ar}$$

$$\frac{a\,x^2 e}{2} + adx + \frac{b\arctanh\left(c\,x^3\right)x^2 e}{2} + b\arctanh\left(c\,x^3\right)dx + \frac{bd\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c}\right)^{\frac{2}{3}}} - \frac{bd\ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c}\right)^{\frac{2}{3}}} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arctanh(c*x^3)),x)
```

```
[Out] 1/2*a*x^2*e+a*d*x+1/2*b*arctanh(c*x^3)*x^2*e+b*arctanh(c*x^3)*d*x+1/2*b*d/c
/(1/c)^(2/3)*ln(x-(1/c)^(1/3))-1/4*b*d/c/(1/c)^(2/3)*ln(x^2+(1/c)^(1/3)*x+(
1/c)^(2/3))-1/2*b*d/c/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)
*x+1))+1/4*b*e/c/(1/c)^(1/3)*ln(x-(1/c)^(1/3))-1/8*b*e/c/(1/c)^(1/3)*ln(x^2
+(1/c)^(1/3)*x+(1/c)^(2/3))+1/4*b*e*3^(1/2)/c/(1/c)^(1/3)*arctan(1/3*3^(1/2
)*(2/(1/c)^(1/3)*x+1))+1/2*b*d/c/(1/c)^(2/3)*ln(x+(1/c)^(1/3))-1/4*b*d/c/(1
/c)^(2/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/3))+1/2*b*d/c/(1/c)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))-1/4*b*e/c/(1/c)^(1/3)*ln(x+(1/c)^(1/
3))+1/8*b*e/c/(1/c)^(1/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/3))+1/4*b*e*3^(1/2)
/c/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))
```

maxima [A]   time = 0.42, size = 248, normalized size = 0.87

$$\frac{1}{2}aex^2+\frac{1}{4}\left(\left(\left(c\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{4}{3}}x^2+c^{\frac{2}{3}}\right)}{3c^{\frac{2}{3}}}\right)}{c^{\frac{4}{3}}}-\frac{\log\left(c^{\frac{4}{3}}x^4+c^{\frac{2}{3}}x^2+1\right)}{c^{\frac{4}{3}}}+\frac{2\log\left(\frac{c^{\frac{2}{3}}x^2-1}{c^{\frac{2}{3}}}\right)}{c^{\frac{4}{3}}}\right)+4x\,\mathrm{artanh}\left(cx^3\right)\right)bd+\frac{1}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 + 1/4*(c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 + c^(2/3)
)/c^(2/3))/c^(4/3) - log(c^(4/3)*x^4 + c^(2/3)*x^2 + 1)/c^(4/3) + 2*log((c^
(2/3)*x^2 - 1)/c^(2/3))/c^(4/3)) + 4*x*arctanh(c*x^3))*b*d + 1/8*(4*x^2*arc
tanh(c*x^3) + c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*x + c^(1/3))/c^(1/
3))/c^(5/3) + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*x - c^(1/3))/c^(1/3))
/c^(5/3) - log(c^(2/3)*x^2 + c^(1/3)*x + 1)/c^(5/3) + log(c^(2/3)*x^2 - c^(
1/3)*x + 1)/c^(5/3) - 2*log((c^(1/3)*x + 1)/c^(1/3))/c^(5/3) + 2*log((c^(1/
3)*x - 1)/c^(1/3))/c^(5/3)))*b*e + a*d*x
```

mupad [B]   time = 1.03, size = 621, normalized size = 2.18

$$\left(\sum_{k=1}^{3}\ln\left(-\mathrm{root}\left(64\,c^2\,z^3+24\,b^2\,c\,d\,e\,z-8\,b^3\,c\,d^3+b^3\,e^3,z,k\right)\left(\mathrm{root}\left(64\,c^2\,z^3+24\,b^2\,c\,d\,e\,z-8\,b^3\,c\,d^3+b^3\,e^3,z,k\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))*(d + e*x),x)
```

```
[Out] symsum(log((243*b^5*c^9*d^4*e)/2 - root(64*c^2*z^3 + 24*b^2*c*d*e*z - 8*b^3
*c*d^3 + b^3*e^3, z, k)*(root(64*c^2*z^3 + 24*b^2*c*d*e*z - 8*b^3*c*d^3 + b
^3*e^3, z, k)*(root(64*c^2*z^3 + 24*b^2*c*d*e*z - 8*b^3*c*d^3 + b^3*e^3, z,
 k)*(486*b^2*c^10*e^2*x - 1944*b^2*c^10*d*e + 3888*root(64*c^2*z^3 + 24*b^2
*c*d*e*z - 8*b^3*c*d^3 + b^3*e^3, z, k)*b*c^11*d*x) - (243*b^3*c^9*e^3)/2)
 - 486*b^4*c^10*d^4*x) + (243*b^5*c^9*d^3*e^2*x)/4)*root(64*c^2*z^3 + 24*b^2
*c*d*e*z - 8*b^3*c*d^3 + b^3*e^3, z, k), k, 1, 3) + symsum(log((243*b^5*c^9
*d^4*e)/2 - root(64*c^2*z^3 - 24*b^2*c*d*e*z - 8*b^3*c*d^3 - b^3*e^3, z, k)
*(root(64*c^2*z^3 - 24*b^2*c*d*e*z - 8*b^3*c*d^3 - b^3*e^3, z, k)*(root(64*
c^2*z^3 - 24*b^2*c*d*e*z - 8*b^3*c*d^3 - b^3*e^3, z, k)*(486*b^2*c^10*e^2*x
 - 1944*b^2*c^10*d*e + 3888*root(64*c^2*z^3 - 24*b^2*c*d*e*z - 8*b^3*c*d^3
 - b^3*e^3, z, k)*b*c^11*d*x) - (243*b^3*c^9*e^3)/2) - 486*b^4*c^10*d^4*x) +
 (243*b^5*c^9*d^3*e^2*x)/4)*root(64*c^2*z^3 - 24*b^2*c*d*e*z - 8*b^3*c*d^3
 - b^3*e^3, z, k), k, 1, 3) + log(c*x^3 + 1)*((b*d*x)/2 + (b*e*x^2)/4) - log
(1 - c*x^3)*((b*d*x)/2 + (b*e*x^2)/4) + a*d*x + (a*e*x^2)/2
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*atanh(c*x**3)),x)
```

```
[Out] Timed out
```

3.34 $\displaystyle\int \frac{a+b\tanh^{-1}\left(cx^3\right)}{d+ex}\,dx$

Optimal. Leaf size=523

$$\frac{\log(d+ex)\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{e}-\frac{b\operatorname{Li}_2\left(\frac{\sqrt[3]{c}\,(d+ex)}{\sqrt[3]{c}\,d-e}\right)}{2e}+\frac{b\operatorname{Li}_2\left(\frac{\sqrt[3]{c}\,(d+ex)}{\sqrt[3]{c}\,d+e}\right)}{2e}+\frac{b\operatorname{Li}_2\left(\frac{\sqrt[3]{c}\,(d+ex)}{\sqrt[3]{c}\,d-\sqrt[3]{-1}\,e}\right)}{2e}-\frac{b\operatorname{Li}_2\left(\frac{\sqrt[3]{c}\,(d+ex)}{\sqrt[3]{c}\,d+\sqrt[3]{-1}\,e}\right)}{2e}-\frac{b\operatorname{Li}_2\Bigg($$

```
[Out] (a+b*arctanh(c*x^3))*ln(e*x+d)/e+1/2*b*ln(e*(1-c^(1/3)*x)/(c^(1/3)*d+e))*ln
(e*x+d)/e-1/2*b*ln(-e*(1+c^(1/3)*x)/(c^(1/3)*d-e))*ln(e*x+d)/e+1/2*b*ln(-e*
((-1)^(1/3)+c^(1/3)*x)/(c^(1/3)*d-(-1)^(1/3)*e))*ln(e*x+d)/e-1/2*b*ln(-e*((
-1)^(2/3)+c^(1/3)*x)/(c^(1/3)*d-(-1)^(2/3)*e))*ln(e*x+d)/e+1/2*b*ln((-1)^(2
/3)*e*(1+(-1)^(1/3)*c^(1/3)*x)/(c^(1/3)*d+(-1)^(2/3)*e))*ln(e*x+d)/e-1/2*b*
ln((-1)^(1/3)*e*(1+(-1)^(2/3)*c^(1/3)*x)/(c^(1/3)*d+(-1)^(1/3)*e))*ln(e*x+d
)/e-1/2*b*polylog(2,c^(1/3)*(e*x+d)/(c^(1/3)*d-e))/e+1/2*b*polylog(2,c^(1/3
)*(e*x+d)/(c^(1/3)*d+e))/e+1/2*b*polylog(2,c^(1/3)*(e*x+d)/(c^(1/3)*d-(-1)^
(1/3)*e))/e-1/2*b*polylog(2,c^(1/3)*(e*x+d)/(c^(1/3)*d+(-1)^(1/3)*e))/e-1/2
*b*polylog(2,c^(1/3)*(e*x+d)/(c^(1/3)*d-(-1)^(2/3)*e))/e+1/2*b*polylog(2,c^
(1/3)*(e*x+d)/(c^(1/3)*d+(-1)^(2/3)*e))/e
```

Rubi [F] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.000, Rules used = {}

$$\int \frac{a+b\tanh^{-1}\left(cx^3\right)}{d+ex}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^3])/(d + e*x),x]
```

```
[Out] (a*Log[d + e*x])/e + b*Defer[Int][ArcTanh[c*x^3]/(d + e*x), x]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^3\right)}{d+ex}\,dx = \int\left(\frac{a}{d+ex}+\frac{b\tanh^{-1}\left(cx^3\right)}{d+ex}\right)dx$$

$$= \frac{a\log(d+ex)}{e}+b\int\frac{\tanh^{-1}\left(cx^3\right)}{d+ex}\,dx$$

Mathematica [C]   time = 100.90, size = 515, normalized size = 0.98

$$\frac{a\log(d+ex)}{e}+\frac{b\left(-\mathrm{Li}_2\left(\frac{\sqrt[3]{c}\,(d+ex)}{\sqrt[3]{c}\,d-e}\right)+\mathrm{Li}_2\left(\frac{\sqrt[3]{c}\,(d+ex)}{\sqrt[3]{c}\,d+e}\right)+\mathrm{Li}_2\left(\frac{2\sqrt[3]{c}\,(d+ex)}{2\sqrt[3]{c}\,d-i\sqrt{3}\,e-e}\right)-\mathrm{Li}_2\left(\frac{2\sqrt[3]{c}\,(d+ex)}{2\sqrt[3]{c}\,d-i\sqrt{3}\,e+e}\right)+\mathrm{Li}_2\left(\frac{2\sqrt[3]{c}\,(d+ex)}{2\sqrt[3]{c}\,d+i\sqrt{3}\,e-e}\right)}{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])/(d + e*x),x]
```

```
[Out] (a*Log[d + e*x])/e + (b*(2*ArcTanh[c*x^3]*Log[d + e*x] - Log[(e*(1 - I*Sqrt
[3] - 2*c^(1/3)*x))/(2*c^(1/3)*d + e - I*Sqrt[3]*e)]*Log[d + e*x] + Log[(e*
(-I + Sqrt[3] - (2*I)*c^(1/3)*x))/((2*I)*c^(1/3)*d + (-I + Sqrt[3])*e)]*Log
[d + e*x] + Log[(e*(I + Sqrt[3] + (2*I)*c^(1/3)*x))/((-2*I)*c^(1/3)*d + (I
+ Sqrt[3])*e)]*Log[d + e*x] - Log[-((e*(1 + c^(1/3)*x))/(c^(1/3)*d - e))]*L
og[d + e*x] - Log[-((e*(-1 - I*Sqrt[3] + 2*c^(1/3)*x))/(2*c^(1/3)*d + e + I
*Sqrt[3]*e))]*Log[d + e*x] + Log[d + e*x]*Log[(e - c^(1/3)*e*x)/(c^(1/3)*d
+ e)] - PolyLog[2, (c^(1/3)*(d + e*x))/(c^(1/3)*d - e)] + PolyLog[2, (c^(1/
3)*(d + e*x))/(c^(1/3)*d + e)] + PolyLog[2, (2*c^(1/3)*(d + e*x))/(2*c^(1/3
)*d - e - I*Sqrt[3]*e)] - PolyLog[2, (2*c^(1/3)*(d + e*x))/(2*c^(1/3)*d + e
 - I*Sqrt[3]*e)] + PolyLog[2, (2*c^(1/3)*(d + e*x))/(2*c^(1/3)*d - e + I*Sq
rt[3]*e)] - PolyLog[2, (2*c^(1/3)*(d + e*x))/(2*c^(1/3)*d + e + I*Sqrt[3]*e
)]))/(2*e)
```

fricas [F]   time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b\,\mathrm{artanh}\left(cx^3\right)+a}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x^3) + a)/(e*x + d), x)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{artanh}\left(cx^3\right)+a}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)/(e*x + d), x)
```

maple [C]   time = 0.15, size = 182, normalized size = 0.35

$$\frac{a\ln(ex+d)}{e}+\frac{b\ln(ex+d)\operatorname{arctanh}\left(c\,x^3\right)}{e}-\frac{b\left(\displaystyle\sum_{\_R1=\operatorname{RootOf}\left(\_Z^3c-3cd\_Z^2+3c\,d^2\_Z-c\,d^3+e^3\right)}\left(\ln(ex+d)\ln\left(\frac{-ex+\_R1-d}{\_R1}\right)+d\right.\right.}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/(e*x+d),x)
```

```
[Out] a*ln(e*x+d)/e+b*ln(e*x+d)/e*arctanh(c*x^3)-1/2*b/e*sum(ln(e*x+d)*ln((-e*x+_
R1-d)/_R1)+dilog((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*c-3*_Z^2*c*d+3*_Z*c*d^2-
c*d^3+e^3))+1/2*b/e*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-e*x+_R1-d)/_
R1),_R1=RootOf(_Z^3*c-3*_Z^2*c*d+3*_Z*c*d^2-c*d^3-e^3))
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}b\int\frac{\log\left(cx^3+1\right)-\log\left(-cx^3+1\right)}{ex+d}\,dx+\frac{a\log(ex+d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*b*integrate((log(c*x^3 + 1) - log(-c*x^3 + 1))/(e*x + d), x) + a*log(e*
x + d)/e
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{a+b\operatorname{atanh}\left(c\,x^3\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))/(d + e*x),x)
```

```
[Out] int((a + b*atanh(c*x^3))/(d + e*x), x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

<div align="center">Timed out</div>

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))/(e*x+d),x)
```

```
[Out] Timed out
```

3.35 $\int \dfrac{a + b\tanh^{-1}(cx^3)}{(d+ex)^2}\, dx$

Optimal. Leaf size=414

$$-\frac{a + b\tanh^{-1}\left(cx^3\right)}{e(d + ex)} - \frac{b\sqrt[3]{c}\left(\sqrt[3]{c}\,d + e\right)\log\left(c^{2/3}x^2 - \sqrt[3]{c}\,x + 1\right)}{4\left(cd^3 - e^3\right)} - \frac{b\sqrt[3]{c}\left(\sqrt[3]{c}\,d - e\right)\log\left(c^{2/3}x^2 + \sqrt[3]{c}\,x + 1\right)}{4\left(cd^3 + e^3\right)} - \frac{\sqrt{3}\,b\sqrt[3]{c}}{2\left(c^{2/3}d^{\cdots}\right.}$$

```
[Out] (-a-b*arctanh(c*x^3))/e/(e*x+d)+1/2*b*c^(1/3)*(c^(1/3)*d-e)*ln(1-c^(1/3)*x)
/(c*d^3+e^3)+1/2*b*c^(1/3)*(c^(1/3)*d+e)*ln(1+c^(1/3)*x)/(c*d^3-e^3)-3*b*c*
d^2*e^2*ln(e*x+d)/(c^2*d^6-e^6)-1/4*b*c^(1/3)*(c^(1/3)*d+e)*ln(1-c^(1/3)*x+
c^(2/3)*x^2)/(c*d^3-e^3)-1/4*b*c^(1/3)*(c^(1/3)*d-e)*ln(1+c^(1/3)*x+c^(2/3)
*x^2)/(c*d^3+e^3)-1/2*b*c*d^2*ln(-c*x^3+1)/e/(c*d^3+e^3)+1/2*b*c*d^2*ln(c*x
^3+1)/e/(c*d^3-e^3)-1/2*b*c^(1/3)*arctan(1/3*(1-2*c^(1/3)*x)*3^(1/2))*3^(1/
2)/(c^(2/3)*d^2+c^(1/3)*d*e+e^2)-1/2*b*c^(1/3)*(c^(1/3)*d+e)*arctan(1/3*(1+
2*c^(1/3)*x)*3^(1/2))*3^(1/2)/(c*d^3+e^3)
```

Rubi [A]   time = 0.77, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6273, 12, 6725, 1871, 1861, 31, 634, 617, 204, 628, 260, 1860}

$$-\frac{a + b\tanh^{-1}\left(cx^3\right)}{e(d + ex)} - \frac{b\sqrt[3]{c}\left(\sqrt[3]{c}\,d + e\right)\log\left(c^{2/3}x^2 - \sqrt[3]{c}\,x + 1\right)}{4\left(cd^3 - e^3\right)} - \frac{b\sqrt[3]{c}\left(\sqrt[3]{c}\,d - e\right)\log\left(c^{2/3}x^2 + \sqrt[3]{c}\,x + 1\right)}{4\left(cd^3 + e^3\right)} - \frac{3bcd^2e^2\,\cdots}{c^2d^{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])/(d + e*x)^2,x]
```

```
[Out] -(Sqrt[3]*b*c^(1/3)*ArcTan[(1 - 2*c^(1/3)*x)/Sqrt[3]])/(2*(c^(2/3)*d^2 + c^
(1/3)*d*e + e^2)) - (Sqrt[3]*b*c^(1/3)*(c^(1/3)*d + e)*ArcTan[(1 + 2*c^(1/3
)*x)/Sqrt[3]])/(2*(c*d^3 + e^3)) - (a + b*ArcTanh[c*x^3])/(e*(d + e*x)) + (
b*c^(1/3)*(c^(1/3)*d - e)*Log[1 - c^(1/3)*x])/(2*(c*d^3 + e^3)) + (b*c^(1/3
)*(c^(1/3)*d + e)*Log[1 + c^(1/3)*x])/(2*(c*d^3 - e^3)) - (3*b*c*d^2*e^2*Lo
g[d + e*x])/(c^2*d^6 - e^6) - (b*c^(1/3)*(c^(1/3)*d + e)*Log[1 - c^(1/3)*x
+ c^(2/3)*x^2])/(4*(c*d^3 - e^3)) - (b*c^(1/3)*(c^(1/3)*d - e)*Log[1 + c^(1
/3)*x + c^(2/3)*x^2])/(4*(c*d^3 + e^3)) - (b*c*d^2*Log[1 - c*x^3])/(2*e*(c*
d^3 + e^3)) + (b*c*d^2*Log[1 + c*x^3])/(2*e*(c*d^3 - e^3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1861

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-(a/b), 3]], s = Denominator[Rt[-(a/b), 3]]}, Dist[(r*(B*r + A*s))/
(3*a*s), Int[1/(r - s*x), x], x] - Dist[r/(3*a*s), Int[(r*(B*r - 2*A*s) - s
*(B*r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x]
&& NeQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&& !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{a + b\tanh^{-1}\left(cx^3\right)}{(d+ex)^2}\,dx = -\frac{a + b\tanh^{-1}\left(cx^3\right)}{e(d+ex)} + \frac{b\int \frac{3cx^2}{(d+ex)\left(1-c^2x^6\right)}\,dx}{e}$$

$$= -\frac{a + b\tanh^{-1}\left(cx^3\right)}{e(d+ex)} + \frac{(3bc)\int \frac{x^2}{(d+ex)\left(1-c^2x^6\right)}\,dx}{e}$$

$$= -\frac{a + b\tanh^{-1}\left(cx^3\right)}{e(d+ex)} + \frac{(3bc)\int \left(\frac{d^2e^4}{\left(-cd^3+e^3\right)\left(cd^3+e^3\right)(d+ex)} + \frac{de-e^2x-cd^2x^2}{2\left(cd^3+e^3\right)\left(-1+cx^3\right)} + \frac{de-e^2x+cd^2x^2}{2\left(cd^3-e^3\right)\left(1+cx\right)}\right)}{e}$$

$$= -\frac{a + b\tanh^{-1}\left(cx^3\right)}{e(d+ex)} - \frac{3bcd^2e^2\log(d+ex)}{c^2d^6 - e^6} + \frac{(3bc)\int \frac{de-e^2x+cd^2x^2}{1+cx^3}\,dx}{2e\left(cd^3 - e^3\right)} + \frac{(3bc)\int \frac{de-e^2x-}{-1+c}}{2e\left(cd^3 + e\right)}$$

$$= -\frac{a + b\tanh^{-1}\left(cx^3\right)}{e(d+ex)} - \frac{3bcd^2e^2\log(d+ex)}{c^2d^6 - e^6} + \frac{(3bc)\int \frac{de-e^2x}{1+cx^3}\,dx}{2e\left(cd^3 - e^3\right)} + \frac{\left(3bc^2d^2\right)\int \frac{x^2}{1+cx^3}\,dx}{2e\left(cd^3 - e^3\right)}$$

$$= -\frac{a + b\tanh^{-1}\left(cx^3\right)}{e(d+ex)} - \frac{3bcd^2e^2\log(d+ex)}{c^2d^6 - e^6} - \frac{bcd^2\log\left(1-cx^3\right)}{2e\left(cd^3 + e^3\right)} + \frac{bcd^2\log\left(1+cx^3\right)}{2e\left(cd^3 - e^3\right)}$$

$$= -\frac{a + b\tanh^{-1}\left(cx^3\right)}{e(d+ex)} + \frac{b\sqrt[3]{c}\left(\sqrt[3]{c}\,d - e\right)\log\left(1 - \sqrt[3]{c}\,x\right)}{2\left(cd^3 + e^3\right)} + \frac{b\sqrt[3]{c}\left(\sqrt[3]{c}\,d + e\right)\log\left(1 + \sqrt[3]{c}\,x\right)}{2\left(cd^3 - e^3\right)}$$

$$= -\frac{a + b\tanh^{-1}\left(cx^3\right)}{e(d+ex)} + \frac{b\sqrt[3]{c}\left(\sqrt[3]{c}\,d - e\right)\log\left(1 - \sqrt[3]{c}\,x\right)}{2\left(cd^3 + e^3\right)} + \frac{b\sqrt[3]{c}\left(\sqrt[3]{c}\,d + e\right)\log\left(1 + \sqrt[3]{c}\,x\right)}{2\left(cd^3 - e^3\right)}$$

$$= -\frac{\sqrt{3}\,b\sqrt[3]{c}\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{2\left(c^{2/3}d^2 + \sqrt[3]{c}\,de + e^2\right)} - \frac{\sqrt{3}\,b\sqrt[3]{c}\left(\sqrt[3]{c}\,d + e\right)\tan^{-1}\left(\frac{1+2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{2\left(cd^3 + e^3\right)} - \frac{a + b\tanh^{-1}\left(cx^3\right)}{e(d+ex)}$$

Mathematica [A]   time = 0.52, size = 534, normalized size = 1.29

$$\frac{1}{4}\left(-\frac{4a}{e(d+ex)} + \frac{2\sqrt{3}\,b\sqrt[3]{c}\tan^{-1}\left(\frac{2\sqrt[3]{c}\,x-1}{\sqrt{3}}\right)}{c^{2/3}d^2 + \sqrt[3]{c}\,de + e^2} + \frac{2bcd^2e^2\log\left(1 - c^2x^6\right)}{c^2d^6 - e^6} - \frac{12bcd^2e^2\log(d+ex)}{c^2d^6 - e^6} + \frac{b\sqrt[3]{c}\left(2c^{5/3}d^5 - c^{4/3}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])/(d + e*x)^2,x]
```

```
[Out] ((-4*a)/(e*(d + e*x)) + (2*Sqrt[3]*b*c^(1/3)*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt
[3]])/(c^(2/3)*d^2 + c^(1/3)*d*e + e^2) - (2*Sqrt[3]*b*c^(1/3)*(c^(1/3)*d +
 e)*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/(c*d^3 + e^3) - (4*b*ArcTanh[c*x^3])
/(e*(d + e*x)) + (2*b*c^(1/3)*(c^(5/3)*d^5 - c^(4/3)*d^4*e + c*d^3*e^2 + c^
(1/3)*d*e^4 - e^5)*Log[1 - c^(1/3)*x])/(-(c^2*d^6*e) + e^7) - (2*b*c^(1/3)*
(c^(5/3)*d^5 + c^(4/3)*d^4*e + c*d^3*e^2 + c^(1/3)*d*e^4 + e^5)*Log[1 + c^(
1/3)*x])/(-(c^2*d^6*e) + e^7) - (12*b*c*d^2*e^2*Log[d + e*x])/(c^2*d^6 - e^
6) + (b*c^(1/3)*(2*c^(5/3)*d^5 - c^(4/3)*d^4*e - c*d^3*e^2 - c^(1/3)*d*e^4
- e^5)*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(c^2*d^6*e - e^7) + (b*c^(1/3)*(2*
c^(5/3)*d^5 + c^(4/3)*d^4*e - c*d^3*e^2 - c^(1/3)*d*e^4 + e^5)*Log[1 + c^(1
/3)*x + c^(2/3)*x^2])/(-(c^2*d^6*e) + e^7) + (2*b*c*d^2*e^2*Log[1 - c^2*x^6
])/(c^2*d^6 - e^6))/4
```

fricas $[\mathbf{F(-1)}]$ time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac $[\mathbf{A}]$ time = 28.28, size = 595, normalized size = 1.44

$$-\frac{bcd^2\log\left(\left|cx^3-1\right|\right)}{2\left(cd^3e+e^4\right)}+\frac{bcd^2\log\left(\left|-cx^3-1\right|\right)}{2\left(cd^3e-e^4\right)}-\frac{\sqrt{3}\,bc|c|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\,\sqrt{3}\,c^{\frac{1}{3}}\left(2x+\frac{1}{c^{\frac{1}{3}}}\right)\right)}{2\left(c^2d^2-cd|c|^{\frac{2}{3}}e+|c|^{\frac{4}{3}}e^2\right)}-\frac{\sqrt{3}\,bc\arctan\left(\dfrac{\sqrt{3}\left(2x+\left(-\frac{1}{c}\right)\right)}{3\left(-\frac{1}{c}\right)^{\frac{1}{3}}}\right)}{2\left(cde+\left(-c^2\right)^{\frac{2}{3}}d^2-\left(-c^2\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/2*b*c*d^2*log(abs(c*x^3 - 1))/(c*d^3*e + e^4) + 1/2*b*c*d^2*log(abs(-c*x
^3 - 1))/(c*d^3*e - e^4) - 1/2*sqrt(3)*b*c*abs(c)^(2/3)*arctan(1/3*sqrt(3)*
c^(1/3)*(2*x + 1/c^(1/3)))/(c^2*d^2 - c*d*abs(c)^(2/3)*e + abs(c)^(4/3)*e^2
) - 1/2*sqrt(3)*b*c*arctan(1/3*sqrt(3)*(2*x + (-1/c)^(1/3))/(-1/c)^(1/3))/(
c*d*e + (-c^2)^(2/3)*d^2 - (-c^2)^(1/3)*e^2) - 1/2*(b*c^3*d^4*e^2 - b*c^3*d
^3*(-1/c)^(1/3)*e^3 - b*c^2*d*e^5 + b*c^2*(-1/c)^(1/3)*e^6)*(-1/c)^(1/3)*lo
g(abs(x - (-1/c)^(1/3)))/(c^3*d^6*e^2 - 2*c^2*d^3*e^5 + c*e^8) + 1/4*((-c^2
)^(1/3)*b*c*d - (-c^2)^(2/3)*b*e)*log(x^2 + x*(-1/c)^(1/3) + (-1/c)^(2/3))/
(c^2*d^3 - c*e^3) - 1/4*(b*c*d*abs(c)^(2/3) - b*abs(c)^(4/3)*e)*log(x^2 + x
/c^(1/3) + 1/c^(2/3))/(c^2*d^3 + c*e^3) + 1/2*(b*c^3*d^4*e^2 - b*c^(8/3)*d^
```

```
3*e^3 + b*c^2*d*e^5 - b*c^(5/3)*e^6)*log(abs(x - 1/c^(1/3)))/((c^3*d^6*e^2
+ 2*c^2*d^3*e^5 + c*e^8)*c^(1/3)) - 1/2*(b*c^2*d^6*log(-(c*x^3 + 1)/(c*x^3
- 1)) + 2*a*c^2*d^6 + 6*b*c*d^2*x*e^4*log(x*e + d) + 6*b*c*d^3*e^3*log(x*e
+ d) - b*e^6*log(-(c*x^3 + 1)/(c*x^3 - 1)) - 2*a*e^6)/(c^2*d^6*x*e^2 + c^2*
d^7*e - x*e^8 - d*e^7)
```

maple [A]   time = 0.04, size = 591, normalized size = 1.43

$$-\frac{a}{(ex+d)e}-\frac{b\operatorname{arctanh}\left(c\,x^3\right)}{(ex+d)e}+\frac{bd\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{\left(2c\,d^3+2e^3\right)\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{bd\ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{2\left(2c\,d^3+2e^3\right)\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{bd\sqrt{3}\;\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{\left(2c\,d^3+2e^3\right)\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{be}{\left(2c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/(e*x+d)^2,x)

[Out] -a/(e*x+d)/e-b/(e*x+d)/e*arctanh(c*x^3)+b/(2*c*d^3+2*e^3)*d/(1/c)^(2/3)*ln(
x-(1/c)^(1/3))-1/2*b/(2*c*d^3+2*e^3)*d/(1/c)^(2/3)*ln(x^2+(1/c)^(1/3)*x+(1/
c)^(2/3))-b/(2*c*d^3+2*e^3)*d/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/
c)^(1/3)*x+1))-b*e/(2*c*d^3+2*e^3)/(1/c)^(1/3)*ln(x-(1/c)^(1/3))+1/2*b*e/(2
*c*d^3+2*e^3)/(1/c)^(1/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))-b*e/(2*c*d^3+2*
e^3)*3^(1/2)/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))-b/e*c/(2*c
*d^3+2*e^3)*d^2*ln(c*x^3-1)+b/(2*c*d^3-2*e^3)*d/(1/c)^(2/3)*ln(x+(1/c)^(1/3
))-1/2*b/(2*c*d^3-2*e^3)*d/(1/c)^(2/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/3))+b/
(2*c*d^3-2*e^3)*d/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1
))+b*e/(2*c*d^3-2*e^3)/(1/c)^(1/3)*ln(x+(1/c)^(1/3))-1/2*b*e/(2*c*d^3-2*e^3
)/(1/c)^(1/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/3))-b*e/(2*c*d^3-2*e^3)*3^(1/2)
/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))+b/e*c/(2*c*d^3-2*e^3)*
d^2*ln(c*x^3+1)-3*b*e^2*c*d^2/(c*d^3+e^3)/(c*d^3-e^3)*ln(e*x+d)
```

maxima [A]   time = 0.43, size = 394, normalized size = 0.95

$$-\frac{1}{4}\left(\left(\frac{12\,d^2e^2\log\left(ex+d\right)}{c^2d^6-e^6}+\frac{2\sqrt{3}\left(cde+c^{\frac{2}{3}}e^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{2}{3}}x+c^{\frac{1}{3}}\right)}{3\,c^{\frac{1}{3}}}\right)}{\left(c^2d^3e+ce^4\right)c^{\frac{1}{3}}}-\frac{2\sqrt{3}\left(cde-c^{\frac{2}{3}}e^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{2}{3}}x-c^{\frac{1}{3}}\right)}{3\,c^{\frac{1}{3}}}\right)}{\left(c^2d^3e-ce^4\right)c^{\frac{1}{3}}}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*((12*d^2*e^2*log(e*x + d)/(c^2*d^6 - e^6) + 2*sqrt(3)*(c*d*e + c^(2/3)
*e^2)*arctan(1/3*sqrt(3)*(2*c^(2/3)*x + c^(1/3))/c^(1/3))/((c^2*d^3*e + c*e
^4)*c^(1/3)) - 2*sqrt(3)*(c*d*e - c^(2/3)*e^2)*arctan(1/3*sqrt(3)*(2*c^(2/3
)*x - c^(1/3))/c^(1/3))/((c^2*d^3*e - c*e^4)*c^(1/3)) + (2*c*d^2 + c^(2/3)*
d*e - c^(1/3)*e^2)*log(c^(2/3)*x^2 + c^(1/3)*x + 1)/(c^2*d^3*e + c*e^4) - (
2*c*d^2 - c^(2/3)*d*e - c^(1/3)*e^2)*log(c^(2/3)*x^2 - c^(1/3)*x + 1)/(c^2*
d^3*e - c*e^4) - 2*(c*d^2 + c^(2/3)*d*e + c^(1/3)*e^2)*log((c^(1/3)*x + 1)/
c^(1/3))/(c^2*d^3*e - c*e^4) + 2*(c*d^2 - c^(2/3)*d*e + c^(1/3)*e^2)*log((c
^(1/3)*x - 1)/c^(1/3))/(c^2*d^3*e + c*e^4))*c + 4*arctanh(c*x^3)/(e^2*x + d
*e))*b - a/(e^2*x + d*e)
```

mupad **[B]**   time = 1.41, size = 2638, normalized size = 6.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))/(d + e*x)^2,x)
```

```
[Out] symsum(log(-(729*b^6*c^14*d*e^2 + 54432*root(8*c*d^3*e^3*z^3 - 8*e^6*z^3 -
12*b*c*d^2*e^2*z^2 + 6*b^2*c*d*e*z - b^3*c, z, k)^6*c^12*e^15*x + 729*b^6*c
^14*e^3*x + 31104*root(8*c*d^3*e^3*z^3 - 8*e^6*z^3 - 12*b*c*d^2*e^2*z^2 + 6
*b^2*c*d*e*z - b^3*c, z, k)^6*c^14*d^7*e^8 + 243*root(8*c*d^3*e^3*z^3 - 8*e
^6*z^3 - 12*b*c*d^2*e^2*z^2 + 6*b^2*c*d*e*z - b^3*c, z, k)*b^5*c^15*d^5 + 6
2208*root(8*c*d^3*e^3*z^3 - 8*e^6*z^3 - 12*b*c*d^2*e^2*z^2 + 6*b^2*c*d*e*z
- b^3*c, z, k)^6*c^12*d*e^14 - 5832*root(8*c*d^3*e^3*z^3 - 8*e^6*z^3 - 12*b
*c*d^2*e^2*z^2 + 6*b^2*c*d*e*z - b^3*c, z, k)^2*b^4*c^14*d^3*e^4 - 1944*roo
t(8*c*d^3*e^3*z^3 - 8*e^6*z^3 - 12*b*c*d^2*e^2*z^2 + 6*b^2*c*d*e*z - b^3*c,
 z, k)^3*b^3*c^15*d^7*e^2 + 15552*root(8*c*d^3*e^3*z^3 - 8*e^6*z^3 - 12*b*c
*d^2*e^2*z^2 + 6*b^2*c*d*e*z - b^3*c, z, k)^4*b^2*c^14*d^5*e^6 + 10692*root
(8*c*d^3*e^3*z^3 - 8*e^6*z^3 - 12*b*c*d^2*e^2*z^2 + 6*b^2*c*d*e*z - b^3*c,
z, k)^3*b^3*c^13*d*e^8 + 101088*root(8*c*d^3*e^3*z^3 - 8*e^6*z^3 - 12*b*c*d
^2*e^2*z^2 + 6*b^2*c*d*e*z - b^3*c, z, k)^5*b*c^13*d^3*e^10 + 3888*root(8*c
*d^3*e^3*z^3 - 8*e^6*z^3 - 12*b*c*d^2*e^2*z^2 + 6*b^2*c*d*e*z - b^3*c, z, k
)^5*b*c^15*d^9*e^4 + 12636*root(8*c*d^3*e^3*z^3 - 8*e^6*z^3 - 12*b*c*d^2*e^
2*z^2 + 6*b^2*c*d*e*z - b^3*c, z, k)^3*b^3*c^13*e^9*x + 38880*root(8*c*d^3*
e^3*z^3 - 8*e^6*z^3 - 12*b*c*d^2*e^2*z^2 + 6*b^2*c*d*e*z - b^3*c, z, k)^6*c
^14*d^6*e^9*x + 116640*root(8*c*d^3*e^3*z^3 - 8*e^6*z^3 - 12*b*c*d^2*e^2*z^
2 + 6*b^2*c*d*e*z - b^3*c, z, k)^5*b*c^13*d^2*e^11*x + 11664*root(8*c*d^3*e
^3*z^3 - 8*e^6*z^3 - 12*b*c*d^2*e^2*z^2 + 6*b^2*c*d*e*z - b^3*c, z, k)^5*b*
c^15*d^8*e^5*x - 11664*root(8*c*d^3*e^3*z^3 - 8*e^6*z^3 - 12*b*c*d^2*e^2*z^
2 + 6*b^2*c*d*e*z - b^3*c, z, k)^2*b^4*c^14*d^2*e^5*x - 3888*root(8*c*d^3*e
^3*z^3 - 8*e^6*z^3 - 12*b*c*d^2*e^2*z^2 + 6*b^2*c*d*e*z - b^3*c, z, k)^3*b^
3*c^15*d^6*e^3*x + 38880*root(8*c*d^3*e^3*z^3 - 8*e^6*z^3 - 12*b*c*d^2*e^2*
```

z^2 + 6*b^2*c*d*e*z - b^3*c, z, k)^4*b^2*c^14*d^4*e^7*x + 243*root(8*c*d^3*
e^3*z^3 - 8*e^6*z^3 - 12*b*c*d^2*e^2*z^2 + 6*b^2*c*d*e*z - b^3*c, z, k)*b^5
*c^15*d^4*e*x)/e^4)*root(8*c*d^3*e^3*z^3 - 8*e^6*z^3 - 12*b*c*d^2*e^2*z^2 +
 6*b^2*c*d*e*z - b^3*c, z, k), k, 1, 3) + symsum(log(-(729*b^6*c^14*d*e^2 +
 54432*root(8*c*d^3*e^3*z^3 + 8*e^6*z^3 + 12*b*c*d^2*e^2*z^2 + 6*b^2*c*d*e*
z + b^3*c, z, k)^6*c^12*e^15*x + 729*b^6*c^14*e^3*x + 31104*root(8*c*d^3*e^
3*z^3 + 8*e^6*z^3 + 12*b*c*d^2*e^2*z^2 + 6*b^2*c*d*e*z + b^3*c, z, k)^6*c^1
4*d^7*e^8 + 243*root(8*c*d^3*e^3*z^3 + 8*e^6*z^3 + 12*b*c*d^2*e^2*z^2 + 6*b
^2*c*d*e*z + b^3*c, z, k)*b^5*c^15*d^5 + 62208*root(8*c*d^3*e^3*z^3 + 8*e^6
*z^3 + 12*b*c*d^2*e^2*z^2 + 6*b^2*c*d*e*z + b^3*c, z, k)^6*c^12*d*e^14 - 58
32*root(8*c*d^3*e^3*z^3 + 8*e^6*z^3 + 12*b*c*d^2*e^2*z^2 + 6*b^2*c*d*e*z +
b^3*c, z, k)^2*b^4*c^14*d^3*e^4 - 1944*root(8*c*d^3*e^3*z^3 + 8*e^6*z^3 + 1
2*b*c*d^2*e^2*z^2 + 6*b^2*c*d*e*z + b^3*c, z, k)^3*b^3*c^15*d^7*e^2 + 15552
*root(8*c*d^3*e^3*z^3 + 8*e^6*z^3 + 12*b*c*d^2*e^2*z^2 + 6*b^2*c*d*e*z + b^
3*c, z, k)^4*b^2*c^14*d^5*e^6 + 10692*root(8*c*d^3*e^3*z^3 + 8*e^6*z^3 + 12
*b*c*d^2*e^2*z^2 + 6*b^2*c*d*e*z + b^3*c, z, k)^3*b^3*c^13*d*e^8 + 101088*r
oot(8*c*d^3*e^3*z^3 + 8*e^6*z^3 + 12*b*c*d^2*e^2*z^2 + 6*b^2*c*d*e*z + b^3*
c, z, k)^5*b*c^13*d^3*e^10 + 3888*root(8*c*d^3*e^3*z^3 + 8*e^6*z^3 + 12*b*c
*d^2*e^2*z^2 + 6*b^2*c*d*e*z + b^3*c, z, k)^5*b*c^15*d^9*e^4 + 12636*root(8
*c*d^3*e^3*z^3 + 8*e^6*z^3 + 12*b*c*d^2*e^2*z^2 + 6*b^2*c*d*e*z + b^3*c, z,
 k)^3*b^3*c^13*e^9*x + 38880*root(8*c*d^3*e^3*z^3 + 8*e^6*z^3 + 12*b*c*d^2*
e^2*z^2 + 6*b^2*c*d*e*z + b^3*c, z, k)^6*c^14*d^6*e^9*x + 116640*root(8*c*d
^3*e^3*z^3 + 8*e^6*z^3 + 12*b*c*d^2*e^2*z^2 + 6*b^2*c*d*e*z + b^3*c, z, k)^
5*b*c^13*d^2*e^11*x + 11664*root(8*c*d^3*e^3*z^3 + 8*e^6*z^3 + 12*b*c*d^2*e
^2*z^2 + 6*b^2*c*d*e*z + b^3*c, z, k)^5*b*c^15*d^8*e^5*x - 11664*root(8*c*d
^3*e^3*z^3 + 8*e^6*z^3 + 12*b*c*d^2*e^2*z^2 + 6*b^2*c*d*e*z + b^3*c, z, k)^
2*b^4*c^14*d^2*e^5*x - 3888*root(8*c*d^3*e^3*z^3 + 8*e^6*z^3 + 12*b*c*d^2*e
^2*z^2 + 6*b^2*c*d*e*z + b^3*c, z, k)^3*b^3*c^15*d^6*e^3*x + 38880*root(8*c
*d^3*e^3*z^3 + 8*e^6*z^3 + 12*b*c*d^2*e^2*z^2 + 6*b^2*c*d*e*z + b^3*c, z, k
)^4*b^2*c^14*d^4*e^7*x + 243*root(8*c*d^3*e^3*z^3 + 8*e^6*z^3 + 12*b*c*d^2*
e^2*z^2 + 6*b^2*c*d*e*z + b^3*c, z, k)*b^5*c^15*d^4*e*x)/e^4)*root(8*c*d^3*
e^3*z^3 + 8*e^6*z^3 + 12*b*c*d^2*e^2*z^2 + 6*b^2*c*d*e*z + b^3*c, z, k), k,
 1, 3) - a/(d*e + e^2*x) - (b*log(c*x^3 + 1))/(2*(d*e + e^2*x)) + (b*log(1
- c*x^3))/(2*d*e + 2*e^2*x) + (3*b*c*d^2*e^2*log(d + e*x))/(e^6 - c^2*d^6)

sympy $[\textbf{F(-1)}]$ time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))/(e*x+d)**2,x)

[Out] Timed out
```

3.36 $\int \dfrac{x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{1-c^2x}\,dx$

Optimal. Leaf size=195

$$-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{bc^8}+\frac{2\log\left(\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^8}-\frac{x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^6}-\frac{x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{2c^4}$$

```
[Out] -5/18*b*x^(3/2)/c^5-1/15*b*x^(5/2)/c^3+11/6*b*arctanh(c*x^(1/2))/c^8-x*(a+b
*arctanh(c*x^(1/2)))/c^6-1/2*x^2*(a+b*arctanh(c*x^(1/2)))/c^4-1/3*x^3*(a+b*
arctanh(c*x^(1/2)))/c^2-(a+b*arctanh(c*x^(1/2)))^2/b/c^8+2*(a+b*arctanh(c*x
^(1/2)))*ln(2/(1-c*x^(1/2)))/c^8+b*polylog(2,1-2/(1-c*x^(1/2)))/c^8-11/6*b*
x^(1/2)/c^7
```

Rubi [A]   time = 0.60, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 10, integrand size = 26, $\dfrac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {43, 5980, 5916, 302, 206, 321, 5984, 5918, 2402, 2315}

$$\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1-c\sqrt{x}}\right)}{c^8}-\frac{x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{3c^2}-\frac{x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{2c^4}-\frac{x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^6}-\frac{\left(a+b\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTanh[c*Sqrt[x]]))/(1 - c^2*x),x]
```

```
[Out] (-11*b*Sqrt[x])/(6*c^7) - (5*b*x^(3/2))/(18*c^5) - (b*x^(5/2))/(15*c^3) + (
11*b*ArcTanh[c*Sqrt[x]])/(6*c^8) - (x*(a + b*ArcTanh[c*Sqrt[x]]))/c^6 - (x^
2*(a + b*ArcTanh[c*Sqrt[x]]))/(2*c^4) - (x^3*(a + b*ArcTanh[c*Sqrt[x]]))/(3
*c^2) - (a + b*ArcTanh[c*Sqrt[x]])^2/(b*c^8) + (2*(a + b*ArcTanh[c*Sqrt[x]]
)*Log[2/(1 - c*Sqrt[x])])/c^8 + (b*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])])/c^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```


Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
```

```
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

<u>Rule 5984</u>

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

<u>Rubi steps</u>

$$\int \frac{x^3 \left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{1 - c^2 x}\, dx = 2\,\mathrm{Subst}\left(\int \frac{x^7 \left(a + b\tanh^{-1}(cx)\right)}{1 - c^2 x^2}\, dx, x, \sqrt{x}\right)$$

$$= -\frac{2\,\mathrm{Subst}\left(\int x^5 \left(a + b\tanh^{-1}(cx)\right)\, dx, x, \sqrt{x}\right)}{c^2} + \frac{2\,\mathrm{Subst}\left(\int \frac{x^5\left(a + b\tanh^{-1}(cx)\right)}{1 - c^2 x^2}\, dx\right)}{c^2}$$

$$= -\frac{x^3 \left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{3c^2} - \frac{2\,\mathrm{Subst}\left(\int x^3 \left(a + b\tanh^{-1}(cx)\right)\, dx, x, \sqrt{x}\right)}{c^4} + \frac{2}{}$$

$$= -\frac{x^2 \left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{2c^4} - \frac{x^3 \left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{3c^2} - \frac{2\,\mathrm{Subst}\left(\int x \left(a + b\,\mathrm{ta}\right.\right.}{c}$$

$$= -\frac{b\sqrt{x}}{3c^7} - \frac{bx^{3/2}}{9c^5} - \frac{bx^{5/2}}{15c^3} - \frac{x \left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^6} - \frac{x^2 \left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{2c^4}$$

$$= -\frac{11b\sqrt{x}}{6c^7} - \frac{5bx^{3/2}}{18c^5} - \frac{bx^{5/2}}{15c^3} + \frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{3c^8} - \frac{x \left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^6} - \frac{x^2\left(}{}$$

$$= -\frac{11b\sqrt{x}}{6c^7} - \frac{5bx^{3/2}}{18c^5} - \frac{bx^{5/2}}{15c^3} + \frac{11b\tanh^{-1}\left(c\sqrt{x}\right)}{6c^8} - \frac{x \left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^6} - \frac{x}{}$$

$$= -\frac{11b\sqrt{x}}{6c^7} - \frac{5bx^{3/2}}{18c^5} - \frac{bx^{5/2}}{15c^3} + \frac{11b\tanh^{-1}\left(c\sqrt{x}\right)}{6c^8} - \frac{x \left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^6} - \frac{x}{}$$

Mathematica [A]   time = 0.57, size = 160, normalized size = 0.82

$$-\frac{30ac^6x^3 + 45ac^4x^2 + 90ac^2x + 90a\log\left(1 - c^2x\right) + 6bc^5x^{5/2} + 25bc^3x^{3/2} + 15b\tanh^{-1}\left(c\sqrt{x}\right)\left(2c^6x^3 + 3c^4x^2 + 6\right.}{90c^8}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcTanh[c*Sqrt[x]]))/(1 - c^2*x),x]

[Out] -1/90*(165*b*c*Sqrt[x] + 90*a*c^2*x + 25*b*c^3*x^(3/2) + 45*a*c^4*x^2 + 6*b
*c^5*x^(5/2) + 30*a*c^6*x^3 - 90*b*ArcTanh[c*Sqrt[x]]^2 + 15*b*ArcTanh[c*Sq
rt[x]]*(-11 + 6*c^2*x + 3*c^4*x^2 + 2*c^6*x^3 - 12*Log[1 + E^(-2*ArcTanh[c*
Sqrt[x]])]) + 90*a*Log[1 - c^2*x] + 90*b*PolyLog[2, -E^(-2*ArcTanh[c*Sqrt[x
]])])/c^8
```

fricas [F]   time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{bx^3\operatorname{artanh}\left(c\sqrt{x}\right) + ax^3}{c^2x - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x, algorithm="fricas")

[Out] integral(-(b*x^3*arctanh(c*sqrt(x)) + a*x^3)/(c^2*x - 1), x)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(b\operatorname{artanh}\left(c\sqrt{x}\right) + a\right)x^3}{c^2x - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x, algorithm="giac")

[Out] integrate(-(b*arctanh(c*sqrt(x)) + a)*x^3/(c^2*x - 1), x)
```

maple [A]   time = 0.06, size = 309, normalized size = 1.58

$$-\frac{ax^3}{3c^2} - \frac{x^2a}{2c^4} - \frac{xa}{c^6} - \frac{a\ln\left(c\sqrt{x} - 1\right)}{c^8} - \frac{a\ln\left(1 + c\sqrt{x}\right)}{c^8} - \frac{b\operatorname{arctanh}\left(c\sqrt{x}\right)x^3}{3c^2} - \frac{b\operatorname{arctanh}\left(c\sqrt{x}\right)x^2}{2c^4} - \frac{b\operatorname{arctanh}\left(c\sqrt{x}\right)x}{c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x)
```

```
[Out] -1/3/c^2*a*x^3-1/2/c^4*x^2*a-1/c^6*x*a-1/c^8*a*ln(c*x^(1/2)-1)-1/c^8*a*ln(1
+c*x^(1/2))-1/3/c^2*b*arctanh(c*x^(1/2))*x^3-1/2/c^4*b*arctanh(c*x^(1/2))*x
^2-1/c^6*b*arctanh(c*x^(1/2))*x-1/c^8*b*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)-
1/c^8*b*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))-1/4/c^8*b*ln(c*x^(1/2)-1)^2+1/c^
8*b*dilog(1/2+1/2*c*x^(1/2))+1/2/c^8*b*ln(c*x^(1/2)-1)*ln(1/2+1/2*c*x^(1/2)
)+1/4/c^8*b*ln(1+c*x^(1/2))^2-1/2/c^8*b*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c*x^(1/
2))+1/2/c^8*b*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2+1/2*c*x^(1/2))-1/15*b*x^(5/2)/c
^3-5/18*b*x^(3/2)/c^5-11/6*b*x^(1/2)/c^7-11/12/c^8*b*ln(c*x^(1/2)-1)+11/12/
c^8*b*ln(1+c*x^(1/2))
```

<u>maxima</u> **[A]**   time = 0.56, size = 246, normalized size = 1.26

$$
-\frac{1}{6}a\left(\frac{2c^4x^3 + 3c^2x^2 + 6x}{c^6} + \frac{6\log\left(c^2x - 1\right)}{c^8}\right) - \frac{\left(\log\left(c\sqrt{x} + 1\right)\log\left(-\frac{1}{2}c\sqrt{x} + \frac{1}{2}\right) + \mathrm{Li}_2\left(\frac{1}{2}c\sqrt{x} + \frac{1}{2}\right)\right)b}{c^8} + \frac{11\,b\,\mathrm{lc}}{}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x, algorithm="maxima")
```

```
[Out] -1/6*a*((2*c^4*x^3 + 3*c^2*x^2 + 6*x)/c^6 + 6*log(c^2*x - 1)/c^8) - (log(c*
sqrt(x) + 1)*log(-1/2*c*sqrt(x) + 1/2) + dilog(1/2*c*sqrt(x) + 1/2))*b/c^8
+ 11/12*b*log(c*sqrt(x) + 1)/c^8 - 11/12*b*log(c*sqrt(x) - 1)/c^8 - 1/180*(
12*b*c^5*x^(5/2) + 50*b*c^3*x^(3/2) + 45*b*log(c*sqrt(x) + 1)^2 - 45*b*log(
-c*sqrt(x) + 1)^2 + 330*b*c*sqrt(x) + 15*(2*b*c^6*x^3 + 3*b*c^4*x^2 + 6*b*c
^2*x)*log(c*sqrt(x) + 1) - 15*(2*b*c^6*x^3 + 3*b*c^4*x^2 + 6*b*c^2*x + 6*b*
log(c*sqrt(x) + 1))*log(-c*sqrt(x) + 1))/c^8
```

<u>mupad</u> **[F]**   time = 0.00, size = -1, normalized size = -0.01

$$
\int -\frac{x^3\left(a + b\,\mathrm{atanh}\left(c\,\sqrt{x}\right)\right)}{c^2\,x - 1}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(a + b*atanh(c*x^(1/2))))/(c^2*x - 1),x)
```

```
[Out] int(-(x^3*(a + b*atanh(c*x^(1/2))))/(c^2*x - 1), x)
```

<u>sympy</u> **[F]**   time = 0.00, size = 0, normalized size = 0.00

$$
-\int \frac{ax^3}{c^2x - 1}\,dx - \int \frac{bx^3\,\mathrm{atanh}\left(c\sqrt{x}\right)}{c^2x - 1}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x**(1/2)))/(-c**2*x+1),x)
```

[Out] -Integral(a*x**3/(c**2*x - 1), x) - Integral(b*x**3*atanh(c*sqrt(x))/(c**2*
x - 1), x)

3.37 $\displaystyle\int \frac{x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{1-c^2x}\,dx$

Optimal. Leaf size=160

$$-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{bc^6}+\frac{2\log\left(\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^6}-\frac{x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^4}-\frac{x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{2c^2}$$

```
[Out] -1/6*b*x^(3/2)/c^3+3/2*b*arctanh(c*x^(1/2))/c^6-x*(a+b*arctanh(c*x^(1/2)))/
c^4-1/2*x^2*(a+b*arctanh(c*x^(1/2)))/c^2-(a+b*arctanh(c*x^(1/2)))^2/b/c^6+2
*(a+b*arctanh(c*x^(1/2)))*ln(2/(1-c*x^(1/2)))/c^6+b*polylog(2,1-2/(1-c*x^(1
/2)))/c^6-3/2*b*x^(1/2)/c^5
```

Rubi [A]   time = 0.43, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26, $\dfrac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {43, 5980, 5916, 302, 206, 321, 5984, 5918, 2402, 2315}

$$\frac{b\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-c\sqrt{x}}\right)}{c^6}-\frac{x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{2c^2}-\frac{x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^4}-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{bc^6}+\frac{2\log\left(\frac{}{1}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTanh[c*Sqrt[x]]))/(1 - c^2*x),x]
```

```
[Out] (-3*b*Sqrt[x])/(2*c^5) - (b*x^(3/2))/(6*c^3) + (3*b*ArcTanh[c*Sqrt[x]])/(2*
c^6) - (x*(a + b*ArcTanh[c*Sqrt[x]]))/c^4 - (x^2*(a + b*ArcTanh[c*Sqrt[x]])
)/(2*c^2) - (a + b*ArcTanh[c*Sqrt[x]])^2/(b*c^6) + (2*(a + b*ArcTanh[c*Sqrt
[x]])*Log[2/(1 - c*Sqrt[x])])/c^6 + (b*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])])/c
^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
```

]

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2 \left(a + b \tanh^{-1}\left(c\sqrt{x}\right)\right)}{1 - c^2 x}\, dx = 2\,\text{Subst}\left(\int \frac{x^5 \left(a + b \tanh^{-1}(cx)\right)}{1 - c^2 x^2}\, dx, x, \sqrt{x}\right)$$

$$= -\frac{2\,\text{Subst}\left(\int x^3 \left(a + b \tanh^{-1}(cx)\right)\, dx, x, \sqrt{x}\right)}{c^2} + \frac{2\,\text{Subst}\left(\int \frac{x^3\left(a + b\tanh^{-1}(cx)\right)}{1 - c^2 x^2}\, dx}{c^2}$$

$$= -\frac{x^2 \left(a + b \tanh^{-1}\left(c\sqrt{x}\right)\right)}{2c^2} - \frac{2\,\text{Subst}\left(\int x \left(a + b \tanh^{-1}(cx)\right)\, dx, x, \sqrt{x}\right)}{c^4} + \frac{2 S}{}$$

$$= -\frac{x \left(a + b \tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^4} - \frac{x^2 \left(a + b \tanh^{-1}\left(c\sqrt{x}\right)\right)}{2c^2} - \frac{\left(a + b \tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{bc^6}$$

$$= -\frac{3b\sqrt{x}}{2c^5} - \frac{bx^{3/2}}{6c^3} - \frac{x \left(a + b \tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^4} - \frac{x^2 \left(a + b \tanh^{-1}\left(c\sqrt{x}\right)\right)}{2c^2} - \frac{\left(a + \right.}{}$$

$$= -\frac{3b\sqrt{x}}{2c^5} - \frac{bx^{3/2}}{6c^3} + \frac{3b \tanh^{-1}\left(c\sqrt{x}\right)}{2c^6} - \frac{x \left(a + b \tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^4} - \frac{x^2 \left(a + b \tan\right.}{2c}$$

$$= -\frac{3b\sqrt{x}}{2c^5} - \frac{bx^{3/2}}{6c^3} + \frac{3b \tanh^{-1}\left(c\sqrt{x}\right)}{2c^6} - \frac{x \left(a + b \tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^4} - \frac{x^2 \left(a + b \tan\right.}{2c}$$

Mathematica [A]   time = 0.39, size = 130, normalized size = 0.81

$$-\frac{3ac^4 x^2 + 6ac^2 x + 6a \log\left(1 - c^2 x\right) + bc^3 x^{3/2} + 3b \tanh^{-1}\left(c\sqrt{x}\right)\left(c^4 x^2 + 2c^2 x - 4 \log\left(e^{-2 \tanh^{-1}\left(c\sqrt{x}\right)} + 1\right) - 3\right)}{6c^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*Sqrt[x]]))/(1 - c^2*x),x]
```

```
[Out] -1/6*(9*b*c*Sqrt[x] + 6*a*c^2*x + b*c^3*x^(3/2) + 3*a*c^4*x^2 - 6*b*ArcTanh
[c*Sqrt[x]]^2 + 3*b*ArcTanh[c*Sqrt[x]]*(-3 + 2*c^2*x + c^4*x^2 - 4*Log[1 +
E^(-2*ArcTanh[c*Sqrt[x]])]) + 6*a*Log[1 - c^2*x] + 6*b*PolyLog[2, -E^(-2*Ar
cTanh[c*Sqrt[x]])])/c^6
```

fricas [F]   time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{bx^2 \operatorname{artanh}\left(c\sqrt{x}\right) + ax^2}{c^2 x - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x, algorithm="fricas")
```

```
[Out] integral(-(b*x^2*arctanh(c*sqrt(x)) + a*x^2)/(c^2*x - 1), x)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(b \operatorname{artanh}\left(c\sqrt{x}\right) + a\right)x^2}{c^2 x - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x, algorithm="giac")
```

```
[Out] integrate(-(b*arctanh(c*sqrt(x)) + a)*x^2/(c^2*x - 1), x)
```

maple [B]   time = 0.06, size = 276, normalized size = 1.72

$$-\frac{x^2 a}{2c^2} - \frac{xa}{c^4} - \frac{a\ln\left(c\sqrt{x}-1\right)}{c^6} - \frac{a\ln\left(1+c\sqrt{x}\right)}{c^6} - \frac{b\arctanh\left(c\sqrt{x}\right)x^2}{2c^2} - \frac{b\arctanh\left(c\sqrt{x}\right)x}{c^4} - \frac{b\arctanh\left(c\sqrt{x}\right)\ln\left(c\sqrt{x}\right)}{c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x)
```

```
[Out] -1/2/c^2*x^2*a-1/c^4*x*a-1/c^6*a*ln(c*x^(1/2)-1)-1/c^6*a*ln(1+c*x^(1/2))-1/
2/c^2*b*arctanh(c*x^(1/2))*x^2-1/c^4*b*arctanh(c*x^(1/2))*x-1/c^6*b*arctanh
(c*x^(1/2))*ln(c*x^(1/2)-1)-1/c^6*b*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))-1/4/
c^6*b*ln(c*x^(1/2)-1)^2+1/c^6*b*dilog(1/2+1/2*c*x^(1/2))+1/2/c^6*b*ln(c*x^(
1/2)-1)*ln(1/2+1/2*c*x^(1/2))+1/4/c^6*b*ln(1+c*x^(1/2))^2+1/2/c^6*b*ln(-1/2
*c*x^(1/2)+1/2)*ln(1/2+1/2*c*x^(1/2))-1/2/c^6*b*ln(-1/2*c*x^(1/2)+1/2)*ln(1
+c*x^(1/2))-1/6*b*x^(3/2)/c^3-3/2*b*x^(1/2)/c^5-3/4/c^6*b*ln(c*x^(1/2)-1)+3
/4/c^6*b*ln(1+c*x^(1/2))
```

maxima [A]   time = 0.56, size = 208, normalized size = 1.30

$$-\frac{1}{2}\,a\left(\frac{c^2x^2+2\,x}{c^4}+\frac{2\,\log\left(c^2x-1\right)}{c^6}\right)-\frac{\left(\log\left(c\sqrt{x}+1\right)\log\left(-\frac{1}{2}\,c\sqrt{x}+\frac{1}{2}\right)+\mathrm{Li}_2\left(\frac{1}{2}\,c\sqrt{x}+\frac{1}{2}\right)\right)b}{c^6}+\frac{3\,b\log\left(c\sqrt{x}+1\right)}{4\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x, algorithm="maxima")
```

```
[Out] -1/2*a*((c^2*x^2 + 2*x)/c^4 + 2*log(c^2*x - 1)/c^6) - (log(c*sqrt(x) + 1)*l
og(-1/2*c*sqrt(x) + 1/2) + dilog(1/2*c*sqrt(x) + 1/2))*b/c^6 + 3/4*b*log(c*
sqrt(x) + 1)/c^6 - 3/4*b*log(c*sqrt(x) - 1)/c^6 - 1/12*(2*b*c^3*x^(3/2) + 3
*b*log(c*sqrt(x) + 1)^2 - 3*b*log(-c*sqrt(x) + 1)^2 + 18*b*c*sqrt(x) + 3*(b
*c^4*x^2 + 2*b*c^2*x)*log(c*sqrt(x) + 1) - 3*(b*c^4*x^2 + 2*b*c^2*x + 2*b*l
og(c*sqrt(x) + 1))*log(-c*sqrt(x) + 1))/c^6
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int-\frac{x^2\left(a+b\,\mathrm{atanh}\left(c\,\sqrt{x}\right)\right)}{c^2\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(a + b*atanh(c*x^(1/2))))/(c^2*x - 1),x)
```

```
[Out] int(-(x^2*(a + b*atanh(c*x^(1/2))))/(c^2*x - 1), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{ax^2}{c^2x-1}\,dx-\int\frac{bx^2\,\mathrm{atanh}\left(c\sqrt{x}\right)}{c^2x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**(1/2)))/(-c**2*x+1),x)
```

```
[Out] -Integral(a*x**2/(c**2*x - 1), x) - Integral(b*x**2*atanh(c*sqrt(x))/(c**2*
x - 1), x)
```

3.38 $\int \dfrac{x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{1-c^2x}\,dx$

Optimal. Leaf size=120

$$-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{bc^4}+\frac{2\log\left(\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^4}-\frac{x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^2}+\frac{b\mathrm{Li}_2\left(1-\frac{2}{1-c\sqrt{x}}\right)}{c^4}+\frac{b\tan}{}$$

[Out] b*arctanh(c*x^(1/2))/c^4-x*(a+b*arctanh(c*x^(1/2)))/c^2-(a+b*arctanh(c*x^(1/2)))^2/b/c^4+2*(a+b*arctanh(c*x^(1/2)))*ln(2/(1-c*x^(1/2)))/c^4+b*polylog(2,1-2/(1-c*x^(1/2)))/c^4-b*x^(1/2)/c^3

Rubi [A]   time = 0.26, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {43, 5980, 5916, 321, 206, 5984, 5918, 2402, 2315}

$$\frac{b\mathrm{PolyLog}\left(2,1-\frac{2}{1-c\sqrt{x}}\right)}{c^4}-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{bc^4}-\frac{x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^2}+\frac{2\log\left(\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^4}$$

Antiderivative was successfully verified.

[In] Int[(x*(a + b*ArcTanh[c*Sqrt[x]]))/(1 - c^2*x),x]

[Out] -((b*Sqrt[x])/c^3) + (b*ArcTanh[c*Sqrt[x]])/c^4 - (x*(a + b*ArcTanh[c*Sqrt[x]]))/c^2 - (a + b*ArcTanh[c*Sqrt[x]])^2/(b*c^4) + (2*(a + b*ArcTanh[c*Sqrt[x]])*Log[2/(1 - c*Sqrt[x])])/c^4 + (b*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])])/c^4

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 321

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[

(a*c^n*(m - n + 1))/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2315

Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]

Rule 2402

Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]

Rule 5916

Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
p)/(d(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]

Rule 5918

Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]

Rule 5980

Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]

Rule 5984

Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e

```
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{1 - c^2 x}\, dx = 2\,\text{Subst}\left(\int \frac{x^3\left(a + b\tanh^{-1}(cx)\right)}{1 - c^2 x^2}\, dx, x, \sqrt{x}\right)$$

$$= -\frac{2\,\text{Subst}\left(\int x\left(a + b\tanh^{-1}(cx)\right) dx, x, \sqrt{x}\right)}{c^2} + \frac{2\,\text{Subst}\left(\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{1-c^2 x^2}\, dx, x, \cdot\right)}{c^2}$$

$$= -\frac{x\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^2} - \frac{\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{bc^4} + \frac{2\,\text{Subst}\left(\int \frac{a+b\tanh^{-1}(cx)}{1-cx}\, dx\right)}{c^3}$$

$$= -\frac{b\sqrt{x}}{c^3} - \frac{x\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^2} - \frac{\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{bc^4} + \frac{2\left(a + b\tanh^{-1}\left(c\cdot\right)\right)}{c^{\cdot}}$$

$$= -\frac{b\sqrt{x}}{c^3} + \frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{c^4} - \frac{x\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^2} - \frac{\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{bc^4} +$$

$$= -\frac{b\sqrt{x}}{c^3} + \frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{c^4} - \frac{x\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^2} - \frac{\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{bc^4} +$$

Mathematica [A]   time = 0.22, size = 96, normalized size = 0.80

$$-\frac{ac^2 x + a\log\left(1 - c^2 x\right) + b\tanh^{-1}\left(c\sqrt{x}\right)\left(c^2 x - 2\log\left(e^{-2\tanh^{-1}\left(c\sqrt{x}\right)} + 1\right) - 1\right) + b\text{Li}_2\left(-e^{-2\tanh^{-1}\left(c\sqrt{x}\right)}\right) + bc\sqrt{x}}{c^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcTanh[c*Sqrt[x]]))/(1 - c^2*x),x]

[Out] -((b*c*Sqrt[x] + a*c^2*x - b*ArcTanh[c*Sqrt[x]]^2 + b*ArcTanh[c*Sqrt[x]]*(-
1 + c^2*x - 2*Log[1 + E^(-2*ArcTanh[c*Sqrt[x]])]) + a*Log[1 - c^2*x] + b*Po
lyLog[2, -E^(-2*ArcTanh[c*Sqrt[x]])])/c^4)
```

fricas [F]   time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{bx\,\text{artanh}\left(c\sqrt{x}\right) + ax}{c^2 x - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x, algorithm="fricas")
```

```
[Out] integral(-(b*x*arctanh(c*sqrt(x)) + a*x)/(c^2*x - 1), x)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(b\operatorname{artanh}\left(c\sqrt{x}\right)+a\right)x}{c^2x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x, algorithm="giac")
```

```
[Out] integrate(-(b*arctanh(c*sqrt(x)) + a)*x/(c^2*x - 1), x)
```

maple [B]   time = 0.06, size = 243, normalized size = 2.02

$$-\frac{xa}{c^2}-\frac{a\ln\left(c\sqrt{x}-1\right)}{c^4}-\frac{a\ln\left(1+c\sqrt{x}\right)}{c^4}-\frac{b\operatorname{arctanh}\left(c\sqrt{x}\right)x}{c^2}-\frac{b\operatorname{arctanh}\left(c\sqrt{x}\right)\ln\left(c\sqrt{x}-1\right)}{c^4}-\frac{b\operatorname{arctanh}\left(c\sqrt{x}\right)\ln}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x)
```

```
[Out] -1/c^2*x*a-1/c^4*a*ln(c*x^(1/2)-1)-1/c^4*a*ln(1+c*x^(1/2))-1/c^2*b*arctanh(
c*x^(1/2))*x-1/c^4*b*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)-1/c^4*b*arctanh(c*x
^(1/2))*ln(1+c*x^(1/2))-b*x^(1/2)/c^3-1/2/c^4*b*ln(c*x^(1/2)-1)+1/2/c^4*b*l
n(1+c*x^(1/2))-1/4/c^4*b*ln(c*x^(1/2)-1)^2+1/c^4*b*dilog(1/2+1/2*c*x^(1/2))
+1/2/c^4*b*ln(c*x^(1/2)-1)*ln(1/2+1/2*c*x^(1/2))+1/4/c^4*b*ln(1+c*x^(1/2))^
2-1/2/c^4*b*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c*x^(1/2))+1/2/c^4*b*ln(-1/2*c*x^(1
/2)+1/2)*ln(1/2+1/2*c*x^(1/2))
```

maxima [A]   time = 0.55, size = 166, normalized size = 1.38

$$-a\left(\frac{x}{c^2}+\frac{\log\left(c^2x-1\right)}{c^4}\right)-\frac{\left(\log\left(c\sqrt{x}+1\right)\log\left(-\frac{1}{2}c\sqrt{x}+\frac{1}{2}\right)+\operatorname{Li}_2\left(\frac{1}{2}c\sqrt{x}+\frac{1}{2}\right)\right)b}{c^4}+\frac{b\log\left(c\sqrt{x}+1\right)}{2c^4}-\frac{b\log\left(c\sqrt{x}\right.}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x, algorithm="maxima")
```

```
[Out] -a*(x/c^2 + log(c^2*x - 1)/c^4) - (log(c*sqrt(x) + 1)*log(-1/2*c*sqrt(x) +
1/2) + dilog(1/2*c*sqrt(x) + 1/2))*b/c^4 + 1/2*b*log(c*sqrt(x) + 1)/c^4 - 1
/2*b*log(c*sqrt(x) - 1)/c^4 - 1/4*(2*b*c^2*x*log(c*sqrt(x) + 1) + b*log(c*s
qrt(x) + 1)^2 - b*log(-c*sqrt(x) + 1)^2 + 4*b*c*sqrt(x) - 2*(b*c^2*x + b*lo
g(c*sqrt(x) + 1))*log(-c*sqrt(x) + 1))/c^4
```

mupad [**F**]   time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x\left(a + b\operatorname{atanh}\left(c\sqrt{x}\right)\right)}{c^2 x - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x*(a + b*atanh(c*x^(1/2))))/(c^2*x - 1),x)

[Out] int(-(x*(a + b*atanh(c*x^(1/2))))/(c^2*x - 1), x)

sympy [**F**]   time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{c^2 x - 1}\, dx - \int \frac{bx\operatorname{atanh}\left(c\sqrt{x}\right)}{c^2 x - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*atanh(c*x**(1/2)))/(-c**2*x+1),x)

[Out] -Integral(a*x/(c**2*x - 1), x) - Integral(b*x*atanh(c*sqrt(x))/(c**2*x - 1)
, x)

3.39 $\displaystyle\int \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{1-c^2x}\,dx$

Optimal. Leaf size=78

$$-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{bc^2}+\frac{2\log\left(\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^2}+\frac{b\mathrm{Li}_2\left(1-\frac{2}{1-c\sqrt{x}}\right)}{c^2}$$

[Out] -(a+b*arctanh(c*x^(1/2)))^2/b/c^2+2*(a+b*arctanh(c*x^(1/2)))*ln(2/(1-c*x^(1/2)))/c^2+b*polylog(2,1-2/(1-c*x^(1/2)))/c^2

Rubi [A]   time $= 0.13$, antiderivative size $= 78$, normalized size of antiderivative $= 1.00$, number of steps used $= 5$, number of rules used $= 4$, integrand size $= 23$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used $= \{5984, 5918, 2402, 2315\}$

$$\frac{b\mathrm{PolyLog}\left(2,1-\frac{2}{1-c\sqrt{x}}\right)}{c^2}-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{bc^2}+\frac{2\log\left(\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*Sqrt[x]])/(1 - c^2*x),x]

[Out] -((a + b*ArcTanh[c*Sqrt[x]])^2/(b*c^2)) + (2*(a + b*ArcTanh[c*Sqrt[x]])*Log[2/(1 - c*Sqrt[x])])/c^2 + (b*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])])/c^2

Rule 2315

Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]

Rule 2402

Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]

Rule 5918

Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
]

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{a + b \tanh^{-1}\left(c\sqrt{x}\right)}{1 - c^2 x}\, dx = 2\,\mathrm{Subst}\left(\int \frac{x\left(a + b\tanh^{-1}(cx)\right)}{1 - c^2 x^2}\, dx, x, \sqrt{x}\right)$$

$$= -\frac{\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{bc^2} + \frac{2\,\mathrm{Subst}\left(\int \frac{a + b\tanh^{-1}(cx)}{1 - cx}\, dx, x, \sqrt{x}\right)}{c}$$

$$= -\frac{\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{bc^2} + \frac{2\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(\frac{2}{1 - c\sqrt{x}}\right)}{c^2} - \frac{(2b)\,\mathrm{Subst}\left(\int \frac{\log\left(}{1-}\right.}{c}$$

$$= -\frac{\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{bc^2} + \frac{2\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(\frac{2}{1 - c\sqrt{x}}\right)}{c^2} + \frac{(2b)\,\mathrm{Subst}\left(\int \frac{\log(}{1-}\right.}{c^2}$$

$$= -\frac{\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{bc^2} + \frac{2\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(\frac{2}{1 - c\sqrt{x}}\right)}{c^2} + \frac{b\,\mathrm{Li}_2\left(1 - \frac{2}{1 - c\sqrt{x}}\right)}{c^2}$$

Mathematica [A] time = 0.10, size = 75, normalized size = 0.96

$$-\frac{a\log\left(1 - c^2 x\right)}{c^2} - \frac{b\left(\mathrm{Li}_2\left(-e^{-2\tanh^{-1}\left(c\sqrt{x}\right)}\right) - \tanh^{-1}\left(c\sqrt{x}\right)\left(\tanh^{-1}\left(c\sqrt{x}\right) + 2\log\left(e^{-2\tanh^{-1}\left(c\sqrt{x}\right)} + 1\right)\right)\right)}{c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/(1 - c^2*x),x]
```

```
[Out] -((a*Log[1 - c^2*x])/c^2) - (b*(-(ArcTanh[c*Sqrt[x]]*(ArcTanh[c*Sqrt[x]] +
2*Log[1 + E^(-2*ArcTanh[c*Sqrt[x]])])) + PolyLog[2, -E^(-2*ArcTanh[c*Sqrt[x
]])]))/c^2
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{b\,\mathrm{artanh}\left(c\sqrt{x}\right) + a}{c^2 x - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x, algorithm="fricas")
```

```
[Out] integral(-(b*arctanh(c*sqrt(x)) + a)/(c^2*x - 1), x)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{b\,\text{artanh}\left(c\sqrt{x}\right) + a}{c^2 x - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x, algorithm="giac")
```

```
[Out] integrate(-(b*arctanh(c*sqrt(x)) + a)/(c^2*x - 1), x)
```

maple [B]   time = 0.05, size = 186, normalized size = 2.38

$$-\frac{a\ln\left(c\sqrt{x}-1\right)}{c^2}-\frac{a\ln\left(1+c\sqrt{x}\right)}{c^2}-\frac{b\arctanh\left(c\sqrt{x}\right)\ln\left(c\sqrt{x}-1\right)}{c^2}-\frac{b\arctanh\left(c\sqrt{x}\right)\ln\left(1+c\sqrt{x}\right)}{c^2}-\frac{b\ln\left(c\sqrt{x}-\right.}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x)
```

```
[Out] -1/c^2*a*ln(c*x^(1/2)-1)-1/c^2*a*ln(1+c*x^(1/2))-1/c^2*b*arctanh(c*x^(1/2))
*ln(c*x^(1/2)-1)-1/c^2*b*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))-1/4/c^2*b*ln(c*
x^(1/2)-1)^2+1/c^2*b*dilog(1/2+1/2*c*x^(1/2))+1/2/c^2*b*ln(c*x^(1/2)-1)*ln(
1/2+1/2*c*x^(1/2))+1/4/c^2*b*ln(1+c*x^(1/2))^2-1/2/c^2*b*ln(-1/2*c*x^(1/2)+
1/2)*ln(1+c*x^(1/2))+1/2/c^2*b*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2+1/2*c*x^(1/2))
```

maxima [A]   time = 0.60, size = 101, normalized size = 1.29

$$-\frac{\left(\log\left(c\sqrt{x}+1\right)\log\left(-\frac{1}{2}c\sqrt{x}+\frac{1}{2}\right)+\text{Li}_2\left(\frac{1}{2}c\sqrt{x}+\frac{1}{2}\right)\right)b}{c^2}-\frac{a\log\left(c^2 x-1\right)}{c^2}-\frac{b\log\left(c\sqrt{x}+1\right)^2-2b\log\left(c\sqrt{x}+1\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/(-c^2*x+1),x, algorithm="maxima")
```

```
[Out] -(log(c*sqrt(x) + 1)*log(-1/2*c*sqrt(x) + 1/2) + dilog(1/2*c*sqrt(x) + 1/2)
)*b/c^2 - a*log(c^2*x - 1)/c^2 - 1/4*(b*log(c*sqrt(x) + 1)^2 - 2*b*log(c*sq
rt(x) + 1)*log(-c*sqrt(x) + 1) - b*log(-c*sqrt(x) + 1)^2)/c^2
```

mupad [**F**]   time = 0.00, size = -1, normalized size = -0.01

$$
\int -\frac{a + b \operatorname{atanh}\left(c\sqrt{x}\right)}{c^2\,x - 1}\, dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*atanh(c*x^(1/2)))/(c^2*x - 1),x)
```

```
[Out] int(-(a + b*atanh(c*x^(1/2)))/(c^2*x - 1), x)
```

sympy [**F**]   time = 0.00, size = 0, normalized size = 0.00

$$
-\int \frac{a}{c^2 x - 1}\, dx - \int \frac{b \operatorname{atanh}\left(c\sqrt{x}\right)}{c^2 x - 1}\, dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))/(-c**2*x+1),x)
```

```
[Out] -Integral(a/(c**2*x - 1), x) - Integral(b*atanh(c*sqrt(x))/(c**2*x - 1), x)
```

3.40 $\displaystyle\int \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x\left(1-c^2x\right)}\,dx$

Optimal. Leaf size=69

$$\frac{\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{b} + 2\log\left(2 - \frac{2}{c\sqrt{x}+1}\right)\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right) - b\text{Li}_2\left(\frac{2}{\sqrt{x}\,c+1} - 1\right)$$

```
[Out] (a+b*arctanh(c*x^(1/2)))^2/b+2*(a+b*arctanh(c*x^(1/2)))*ln(2-2/(1+c*x^(1/2)
))-b*polylog(2,-1+2/(1+c*x^(1/2)))
```

Rubi [A]   time = 0.24, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {36, 29, 31, 1593, 5988, 5932, 2447}

$$-b\text{PolyLog}\left(2, \frac{2}{c\sqrt{x}+1} - 1\right) + \frac{\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{b} + 2\log\left(2 - \frac{2}{c\sqrt{x}+1}\right)\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])/(x*(1 - c^2*x)),x]
```

```
[Out] (a + b*ArcTanh[c*Sqrt[x]])^2/b + 2*(a + b*ArcTanh[c*Sqrt[x]])*Log[2 - 2/(1
+ c*Sqrt[x])] - b*PolyLog[2, -1 + 2/(1 + c*Sqrt[x])]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```


Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```


Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```


Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{a + b \tanh^{-1}\left(c\sqrt{x}\right)}{x\left(1 - c^2 x\right)}\, dx = 2\operatorname{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x - c^2 x^3}\, dx, x, \sqrt{x}\right)$$

$$= 2\operatorname{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x\left(1 - c^2 x^2\right)}\, dx, x, \sqrt{x}\right)$$

$$= \frac{\left(a + b \tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{b} + 2\operatorname{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x(1 + cx)}\, dx, x, \sqrt{x}\right)$$

$$= \frac{\left(a + b \tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{b} + 2\left(a + b \tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(2 - \frac{2}{1 + c\sqrt{x}}\right) - (2bc)\operatorname{Subst}\left($$

$$= \frac{\left(a + b \tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{b} + 2\left(a + b \tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(2 - \frac{2}{1 + c\sqrt{x}}\right) - b\operatorname{Li}_2\left(-1 + \frac{}{1}\right.$$

Mathematica [A]   time = 0.13, size = 72, normalized size = 1.04

$$-a \log\left(1 - c^2 x\right) + a \log(x) - b \mathrm{Li}_2\left(e^{-2\tanh^{-1}(c\sqrt{x})}\right) + b \tanh^{-1}\left(c\sqrt{x}\right)\left(\tanh^{-1}\left(c\sqrt{x}\right) + 2\log\left(1 - e^{-2\tanh^{-1}(c\sqrt{x})}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/(x*(1 - c^2*x)),x]
```

```
[Out] b*ArcTanh[c*Sqrt[x]]*(ArcTanh[c*Sqrt[x]] + 2*Log[1 - E^(-2*ArcTanh[c*Sqrt[x
]])]) + a*Log[x] - a*Log[1 - c^2*x] - b*PolyLog[2, E^(-2*ArcTanh[c*Sqrt[x]]]
)]
```

fricas [F]   time = 0.72, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{b\,\mathrm{artanh}\left(c\sqrt{x}\right) + a}{c^2 x^2 - x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x/(-c^2*x+1),x, algorithm="fricas")
```

```
[Out] integral(-(b*arctanh(c*sqrt(x)) + a)/(c^2*x^2 - x), x)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{b\,\mathrm{artanh}\left(c\sqrt{x}\right) + a}{\left(c^2 x - 1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x/(-c^2*x+1),x, algorithm="giac")
```

```
[Out] integrate(-(b*arctanh(c*sqrt(x)) + a)/((c^2*x - 1)*x), x)
```

maple [B]   time = 0.06, size = 217, normalized size = 3.14

$$2a\ln\left(c\sqrt{x}\right) - a\ln\left(c\sqrt{x} - 1\right) - a\ln\left(1 + c\sqrt{x}\right) + 2b\,\mathrm{arctanh}\left(c\sqrt{x}\right)\ln\left(c\sqrt{x}\right) - b\,\mathrm{arctanh}\left(c\sqrt{x}\right)\ln\left(c\sqrt{x} - 1\right) - b\,\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))/x/(-c^2*x+1),x)
```

```
[Out] 2*a*ln(c*x^(1/2))-a*ln(c*x^(1/2)-1)-a*ln(1+c*x^(1/2))+2*b*arctanh(c*x^(1/2)
)*ln(c*x^(1/2))-b*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)-b*arctanh(c*x^(1/2))*l
```

```
n(1+c*x^(1/2))-b*dilog(c*x^(1/2))-b*dilog(1+c*x^(1/2))-b*ln(c*x^(1/2))*ln(1
+c*x^(1/2))-1/4*b*ln(c*x^(1/2)-1)^2+b*dilog(1/2+1/2*c*x^(1/2))+1/2*b*ln(c*x
^(1/2)-1)*ln(1/2+1/2*c*x^(1/2))+1/4*b*ln(1+c*x^(1/2))^2-1/2*b*ln(-1/2*c*x^(
1/2)+1/2)*ln(1+c*x^(1/2))+1/2*b*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2+1/2*c*x^(1/2)
)
```

maxima [B]   time = 0.52, size = 159, normalized size = 2.30

$$-\frac{1}{4}b\log\left(c\sqrt{x}+1\right)^2+\frac{1}{2}b\log\left(c\sqrt{x}+1\right)\log\left(-c\sqrt{x}+1\right)+\frac{1}{4}b\log\left(-c\sqrt{x}+1\right)^2-\left(\log\left(c\sqrt{x}+1\right)\log\left(-\frac{1}{2}c\sqrt{x}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^(1/2)))/x/(-c^2*x+1),x, algorithm="maxima")

[Out] -1/4*b*log(c*sqrt(x) + 1)^2 + 1/2*b*log(c*sqrt(x) + 1)*log(-c*sqrt(x) + 1)
+ 1/4*b*log(-c*sqrt(x) + 1)^2 - (log(c*sqrt(x) + 1)*log(-1/2*c*sqrt(x) + 1/
2) + dilog(1/2*c*sqrt(x) + 1/2))*b - (log(c*sqrt(x))*log(-c*sqrt(x) + 1) +
dilog(-c*sqrt(x) + 1))*b + (log(c*sqrt(x) + 1)*log(-c*sqrt(x)) + dilog(c*sq
rt(x) + 1))*b - a*(log(c*sqrt(x) + 1) + log(c*sqrt(x) - 1) - log(x))

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{a+b\,\text{atanh}\left(c\,\sqrt{x}\right)}{x\,\left(c^2\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(a + b*atanh(c*x^(1/2)))/(x*(c^2*x - 1)),x)

[Out] -int((a + b*atanh(c*x^(1/2)))/(x*(c^2*x - 1)), x)

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{c^2x^2-x}\,dx-\int \frac{b\,\text{atanh}\left(c\sqrt{x}\right)}{c^2x^2-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x**(1/2)))/x/(-c**2*x+1),x)

[Out] -Integral(a/(c**2*x**2 - x), x) - Integral(b*atanh(c*sqrt(x))/(c**2*x**2 -
x), x)

3.41 $\int \dfrac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x^2\left(1-c^2x\right)}\,dx$

Optimal. Leaf size=117

$$\frac{c^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{b}+2c^2\log\left(2-\frac{2}{c\sqrt{x}+1}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x}-bc^2\text{Li}_2\left(\frac{2}{\sqrt{x}\,c+1}\right)$$

[Out] b*c^2*arctanh(c*x^(1/2))+(-a-b*arctanh(c*x^(1/2)))/x+c^2*(a+b*arctanh(c*x^(1/2)))^2/b+2*c^2*(a+b*arctanh(c*x^(1/2)))*ln(2-2/(1+c*x^(1/2)))-b*c^2*polylog(2,-1+2/(1+c*x^(1/2)))-b*c/x^(1/2)

Rubi [A]   time = 0.36, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 26, $\dfrac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {44, 1593, 5982, 5916, 325, 206, 5988, 5932, 2447}

$$-bc^2\text{PolyLog}\left(2,\frac{2}{c\sqrt{x}+1}-1\right)+\frac{c^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{b}+2c^2\log\left(2-\frac{2}{c\sqrt{x}+1}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{a+b}{}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*Sqrt[x]])/(x^2*(1 - c^2*x)),x]

[Out] -((b*c)/Sqrt[x]) + b*c^2*ArcTanh[c*Sqrt[x]] - (a + b*ArcTanh[c*Sqrt[x]])/x
+ (c^2*(a + b*ArcTanh[c*Sqrt[x]])^2)/b + 2*c^2*(a + b*ArcTanh[c*Sqrt[x]])*L
og[2 - 2/(1 + c*Sqrt[x])] - b*c^2*PolyLog[2, -1 + 2/(1 + c*Sqrt[x])]

Rule 44

Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

Rule 325

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
```

```
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{a + b\tanh^{-1}\left(c\sqrt{x}\right)}{x^2\left(1 - c^2 x\right)}\,dx = 2\,\mathrm{Subst}\left(\int \frac{a + b\tanh^{-1}(cx)}{x^3 - c^2 x^5}\,dx, x, \sqrt{x}\right)$$

$$= 2\,\mathrm{Subst}\left(\int \frac{a + b\tanh^{-1}(cx)}{x^3\left(1 - c^2 x^2\right)}\,dx, x, \sqrt{x}\right)$$

$$= 2\,\mathrm{Subst}\left(\int \frac{a + b\tanh^{-1}(cx)}{x^3}\,dx, x, \sqrt{x}\right) + \left(2c^2\right)\mathrm{Subst}\left(\int \frac{a + b\tanh^{-1}(cx)}{x\left(1 - c^2 x^2\right)}\,dx, x,\right.$$

$$= -\frac{a + b\tanh^{-1}\left(c\sqrt{x}\right)}{x} + \frac{c^2\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{b} + (bc)\,\mathrm{Subst}\left(\int \frac{1}{x^2\left(1 - c^2 x^2\right)}\,d\right.$$

$$= -\frac{bc}{\sqrt{x}} - \frac{a + b\tanh^{-1}\left(c\sqrt{x}\right)}{x} + \frac{c^2\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{b} + 2c^2\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right.$$

$$= -\frac{bc}{\sqrt{x}} + bc^2\tanh^{-1}\left(c\sqrt{x}\right) - \frac{a + b\tanh^{-1}\left(c\sqrt{x}\right)}{x} + \frac{c^2\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{b} + 2c^2$$

Mathematica [A]   time = 0.34, size = 118, normalized size = 1.01

$$2ac^2\log\left(\sqrt{x}\right) - ac^2\log\left(1 - c^2 x\right) - \frac{a}{x} - bc^2\left(-\tanh^{-1}\left(c\sqrt{x}\right)\left(-\frac{1 - c^2 x}{c^2 x} + \tanh^{-1}\left(c\sqrt{x}\right) + 2\log\left(1 - e^{-2\tanh^{-1}\left(c\sqrt{x}\right)}\right)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/(x^2*(1 - c^2*x)),x]

[Out] -(a/x) + 2*a*c^2*Log[Sqrt[x]] - a*c^2*Log[1 - c^2*x] - b*c^2*(1/(c*Sqrt[x])
 - ArcTanh[c*Sqrt[x]]*(-((1 - c^2*x)/(c^2*x)) + ArcTanh[c*Sqrt[x]] + 2*Log[
1 - E^(-2*ArcTanh[c*Sqrt[x]])]) + PolyLog[2, E^(-2*ArcTanh[c*Sqrt[x]])])
```

fricas [F]   time = 1.05, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{b\,\mathrm{artanh}\left(c\sqrt{x}\right) + a}{c^2 x^3 - x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^2/(-c^2*x+1),x, algorithm="fricas")
```

```
[Out] integral(-(b*arctanh(c*sqrt(x)) + a)/(c^2*x^3 - x^2), x)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{b\operatorname{artanh}\left(c\sqrt{x}\right)+a}{\left(c^2x-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^2/(-c^2*x+1),x, algorithm="giac")
```

```
[Out] integrate(-(b*arctanh(c*sqrt(x)) + a)/((c^2*x - 1)*x^2), x)
```

maple [B]   time = 0.07, size = 315, normalized size = 2.69

$$-\frac{a}{x}+2c^2a\ln\left(c\sqrt{x}\right)-c^2a\ln\left(c\sqrt{x}-1\right)-c^2a\ln\left(1+c\sqrt{x}\right)-\frac{b\operatorname{arctanh}\left(c\sqrt{x}\right)}{x}+2c^2b\operatorname{arctanh}\left(c\sqrt{x}\right)\ln\left(c\sqrt{x}\right)-c^2b\operatorname{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))/x^2/(-c^2*x+1),x)
```

```
[Out] -a/x+2*c^2*a*ln(c*x^(1/2))-c^2*a*ln(c*x^(1/2)-1)-c^2*a*ln(1+c*x^(1/2))-b*ar
ctanh(c*x^(1/2))/x+2*c^2*b*arctanh(c*x^(1/2))*ln(c*x^(1/2))-c^2*b*arctanh(c
*x^(1/2))*ln(c*x^(1/2)-1)-c^2*b*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))-b*c/x^(1
/2)-1/2*c^2*b*ln(c*x^(1/2)-1)+1/2*c^2*b*ln(1+c*x^(1/2))-c^2*b*dilog(c*x^(1/
2))-c^2*b*dilog(1+c*x^(1/2))-c^2*b*ln(c*x^(1/2))*ln(1+c*x^(1/2))-1/4*c^2*b*
ln(c*x^(1/2)-1)^2+c^2*b*dilog(1/2+1/2*c*x^(1/2))+1/2*c^2*b*ln(c*x^(1/2)-1)*
ln(1/2+1/2*c*x^(1/2))+1/4*c^2*b*ln(1+c*x^(1/2))^2-1/2*c^2*b*ln(-1/2*c*x^(1/
2)+1/2)*ln(1+c*x^(1/2))+1/2*c^2*b*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2+1/2*c*x^(1/
2))
```

maxima [B]   time = 0.57, size = 248, normalized size = 2.12

$$-\left(\log\left(c\sqrt{x}+1\right)\log\left(-\frac{1}{2}c\sqrt{x}+\frac{1}{2}\right)+\operatorname{Li}_2\left(\frac{1}{2}c\sqrt{x}+\frac{1}{2}\right)\right)bc^2-\left(\log\left(c\sqrt{x}\right)\log\left(-c\sqrt{x}+1\right)+\operatorname{Li}_2\left(-c\sqrt{x}+1\right)\right)bc^2-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^2/(-c^2*x+1),x, algorithm="maxima")
```

```
[Out] -(log(c*sqrt(x) + 1)*log(-1/2*c*sqrt(x) + 1/2) + dilog(1/2*c*sqrt(x) + 1/2)
)*b*c^2 - (log(c*sqrt(x))*log(-c*sqrt(x) + 1) + dilog(-c*sqrt(x) + 1))*b*c^
```

```
2 + (log(c*sqrt(x) + 1)*log(-c*sqrt(x)) + dilog(c*sqrt(x) + 1))*b*c^2 + 1/2
*b*c^2*log(c*sqrt(x) + 1) - 1/2*b*c^2*log(c*sqrt(x) - 1) - (c^2*log(c*sqrt(
x) + 1) + c^2*log(c*sqrt(x) - 1) - c^2*log(x) + 1/x)*a - 1/4*(b*c^2*x*log(c
*sqrt(x) + 1)^2 - b*c^2*x*log(-c*sqrt(x) + 1)^2 + 4*b*c*sqrt(x) + 2*b*log(c
*sqrt(x) + 1) - 2*(b*c^2*x*log(c*sqrt(x) + 1) + b)*log(-c*sqrt(x) + 1))/x
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$ -\int \frac{a + b \operatorname{atanh}\left(c \sqrt{x}\right)}{x^2 \left(c^2 x - 1\right)} \, dx $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*atanh(c*x^(1/2)))/(x^2*(c^2*x - 1)),x)
```

```
[Out] -int((a + b*atanh(c*x^(1/2)))/(x^2*(c^2*x - 1)), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$ -\int \frac{a}{c^2 x^3 - x^2} \, dx - \int \frac{b \operatorname{atanh}\left(c\sqrt{x}\right)}{c^2 x^3 - x^2} \, dx $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))/x**2/(-c**2*x+1),x)
```

```
[Out] -Integral(a/(c**2*x**3 - x**2), x) - Integral(b*atanh(c*sqrt(x))/(c**2*x**3
 - x**2), x)
```

3.42 $\displaystyle\int \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x^3\left(1-c^2x\right)}\,dx$

Optimal. Leaf size=157

$$\frac{c^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{b}+2c^4\log\left(2-\frac{2}{c\sqrt{x}+1}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{c^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{x}-\frac{a+b\tanh^{-1}}{2x^2}$$

```
[Out] -1/6*b*c/x^(3/2)+3/2*b*c^4*arctanh(c*x^(1/2))+1/2*(-a-b*arctanh(c*x^(1/2)))
/x^2-c^2*(a+b*arctanh(c*x^(1/2)))/x+c^4*(a+b*arctanh(c*x^(1/2)))^2/b+2*c^4*
(a+b*arctanh(c*x^(1/2)))*ln(2-2/(1+c*x^(1/2)))-b*c^4*polylog(2,-1+2/(1+c*x^
(1/2)))-3/2*b*c^3/x^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {44, 1593, 5982, 5916, 325, 206, 5988, 5932, 2447}

$$-bc^4\text{PolyLog}\left(2,\frac{2}{c\sqrt{x}+1}-1\right)+\frac{c^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{b}-\frac{c^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{x}+2c^4\log\left(2-\frac{2}{c\sqrt{x}+1}\right)\left(a+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])/(x^3*(1 - c^2*x)),x]
```

```
[Out] -(b*c)/(6*x^(3/2)) - (3*b*c^3)/(2*Sqrt[x]) + (3*b*c^4*ArcTanh[c*Sqrt[x]])/2
 - (a + b*ArcTanh[c*Sqrt[x]])/(2*x^2) - (c^2*(a + b*ArcTanh[c*Sqrt[x]]))/x
+ (c^4*(a + b*ArcTanh[c*Sqrt[x]])^2)/b + 2*c^4*(a + b*ArcTanh[c*Sqrt[x]])*L
og[2 - 2/(1 + c*Sqrt[x])] - b*c^4*PolyLog[2, -1 + 2/(1 + c*Sqrt[x])]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{a + b \tanh^{-1}\left(c\sqrt{x}\right)}{x^3 \left(1 - c^2 x\right)}\, dx = 2\,\mathrm{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x^5 - c^2 x^7}\, dx, x, \sqrt{x}\right)$$

$$= 2\,\mathrm{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x^5 \left(1 - c^2 x^2\right)}\, dx, x, \sqrt{x}\right)$$

$$= 2\,\mathrm{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x^5}\, dx, x, \sqrt{x}\right) + \left(2c^2\right)\mathrm{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x^3 \left(1 - c^2 x^2\right)}\, dx, x, \sqrt{\ }\right.$$

$$= -\frac{a + b \tanh^{-1}\left(c\sqrt{x}\right)}{2x^2} + \frac{1}{2}(bc)\,\mathrm{Subst}\left(\int \frac{1}{x^4 \left(1 - c^2 x^2\right)}\, dx, x, \sqrt{x}\right) + \left(2c^2\right)\mathrm{Subst}\left(\int\right.$$

$$= -\frac{bc}{6x^{3/2}} - \frac{a + b \tanh^{-1}\left(c\sqrt{x}\right)}{2x^2} - \frac{c^2\left(a + b \tanh^{-1}\left(c\sqrt{x}\right)\right)}{x} + \frac{c^4\left(a + b \tanh^{-1}\left(c\sqrt{x}\right)\right)}{b}$$

$$= -\frac{bc}{6x^{3/2}} - \frac{3bc^3}{2\sqrt{x}} - \frac{a + b \tanh^{-1}\left(c\sqrt{x}\right)}{2x^2} - \frac{c^2\left(a + b \tanh^{-1}\left(c\sqrt{x}\right)\right)}{x} + \frac{c^4\left(a + b \tanh^{-1}\right)}{b}$$

$$= -\frac{bc}{6x^{3/2}} - \frac{3bc^3}{2\sqrt{x}} + \frac{3}{2}bc^4 \tanh^{-1}\left(c\sqrt{x}\right) - \frac{a + b \tanh^{-1}\left(c\sqrt{x}\right)}{2x^2} - \frac{c^2\left(a + b \tanh^{-1}\left(c\sqrt{\ }\right.\right.}{x}$$

Mathematica [A]   time = 0.57, size = 158, normalized size = 1.01

$$-\frac{-6ac^4 x^2 \log(x) + 6ac^2 x + 6ac^4 x^2 \log\left(1 - c^2 x\right) + 3a - 6bc^4 x^2 \tanh^{-1}\left(c\sqrt{x}\right)^2 + 9bc^3 x^{3/2} - 3b \tanh^{-1}\left(c\sqrt{x}\right)\left(3c^4\right.}{6x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/(x^3*(1 - c^2*x)),x]
```

```
[Out] -1/6*(3*a + b*c*Sqrt[x] + 6*a*c^2*x + 9*b*c^3*x^(3/2) - 6*b*c^4*x^2*ArcTanh
[c*Sqrt[x]]^2 - 3*b*ArcTanh[c*Sqrt[x]]*(-1 - 2*c^2*x + 3*c^4*x^2 + 4*c^4*x^
2*Log[1 - E^(-2*ArcTanh[c*Sqrt[x]])]) - 6*a*c^4*x^2*Log[x] + 6*a*c^4*x^2*Lo
g[1 - c^2*x])/x^2 - b*c^4*PolyLog[2, E^(-2*ArcTanh[c*Sqrt[x]])]
```

fricas [F]   time = 1.22, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{b\,\mathrm{artanh}\left(c\sqrt{x}\right)+a}{c^2x^4-x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^3/(-c^2*x+1),x, algorithm="fricas")
```

```
[Out] integral(-(b*arctanh(c*sqrt(x)) + a)/(c^2*x^4 - x^3), x)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int-\frac{b\,\mathrm{artanh}\left(c\sqrt{x}\right)+a}{\left(c^2x-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^3/(-c^2*x+1),x, algorithm="giac")
```

```
[Out] integrate(-(b*arctanh(c*sqrt(x)) + a)/((c^2*x - 1)*x^3), x)
```

maple [B]   time = 0.07, size = 348, normalized size = 2.22

$$-\frac{a}{2x^2}-\frac{c^2a}{x}+2c^4a\ln\left(c\sqrt{x}\right)-c^4a\ln\left(c\sqrt{x}-1\right)-c^4a\ln\left(1+c\sqrt{x}\right)-\frac{b\,\mathrm{arctanh}\left(c\sqrt{x}\right)}{2x^2}-\frac{c^2b\,\mathrm{arctanh}\left(c\sqrt{x}\right)}{x}+2c^4b\,\mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))/x^3/(-c^2*x+1),x)
```

```
[Out] -1/2*a/x^2-c^2*a/x+2*c^4*a*ln(c*x^(1/2))-c^4*a*ln(c*x^(1/2)-1)-c^4*a*ln(1+c
*x^(1/2))-1/2*b*arctanh(c*x^(1/2))/x^2-c^2*b*arctanh(c*x^(1/2))/x+2*c^4*b*a
rctanh(c*x^(1/2))*ln(c*x^(1/2))-c^4*b*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)-c^
4*b*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))-c^4*b*dilog(c*x^(1/2))-c^4*b*dilog(1
+c*x^(1/2))-c^4*b*ln(c*x^(1/2))*ln(1+c*x^(1/2))-1/4*c^4*b*ln(c*x^(1/2)-1)^2
+c^4*b*dilog(1/2+1/2*c*x^(1/2))+1/2*c^4*b*ln(c*x^(1/2)-1)*ln(1/2+1/2*c*x^(1
/2))+1/4*c^4*b*ln(1+c*x^(1/2))^2-1/2*c^4*b*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c*x^
(1/2))+1/2*c^4*b*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2+1/2*c*x^(1/2))-1/6*b*c/x^(3/
2)-3/2*b*c^3/x^(1/2)-3/4*c^4*b*ln(c*x^(1/2)-1)+3/4*c^4*b*ln(1+c*x^(1/2))
```

maxima [B]   time = 0.57, size = 291, normalized size = 1.85

$$-\left(\log\left(c\sqrt{x}+1\right)\log\left(-\frac{1}{2}c\sqrt{x}+\frac{1}{2}\right)+\mathrm{Li}_2\left(\frac{1}{2}c\sqrt{x}+\frac{1}{2}\right)\right)bc^4-\left(\log\left(c\sqrt{x}\right)\log\left(-c\sqrt{x}+1\right)+\mathrm{Li}_2\left(-c\sqrt{x}+1\right)\right)bc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^3/(-c^2*x+1),x, algorithm="maxima")
```

```
[Out] -(log(c*sqrt(x) + 1)*log(-1/2*c*sqrt(x) + 1/2) + dilog(1/2*c*sqrt(x) + 1/2)
)*b*c^4 - (log(c*sqrt(x))*log(-c*sqrt(x) + 1) + dilog(-c*sqrt(x) + 1))*b*c^
4 + (log(c*sqrt(x) + 1)*log(-c*sqrt(x)) + dilog(c*sqrt(x) + 1))*b*c^4 + 3/4
*b*c^4*log(c*sqrt(x) + 1) - 3/4*b*c^4*log(c*sqrt(x) - 1) - 1/2*(2*c^4*log(c
*sqrt(x) + 1) + 2*c^4*log(c*sqrt(x) - 1) - 2*c^4*log(x) + (2*c^2*x + 1)/x^2
)*a - 1/12*(3*b*c^4*x^2*log(c*sqrt(x) + 1)^2 - 3*b*c^4*x^2*log(-c*sqrt(x) +
 1)^2 + 18*b*c^3*x^(3/2) + 2*b*c*sqrt(x) + 3*(2*b*c^2*x + b)*log(c*sqrt(x)
+ 1) - 3*(2*b*c^4*x^2*log(c*sqrt(x) + 1) + 2*b*c^2*x + b)*log(-c*sqrt(x) +
1))/x^2
```

mupad **[F]**   time = 0.00, size = -1, normalized size = -0.01

$$- \int \frac{a + b \operatorname{atanh}\left(c \sqrt{x}\right)}{x^3 \left(c^2 x - 1\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*atanh(c*x^(1/2)))/(x^3*(c^2*x - 1)),x)
```

```
[Out] -int((a + b*atanh(c*x^(1/2)))/(x^3*(c^2*x - 1)), x)
```

sympy **[F]**   time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{a}{c^2 x^4 - x^3} \, dx - \int \frac{b \operatorname{atanh}\left(c \sqrt{x}\right)}{c^2 x^4 - x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))/x**3/(-c**2*x+1),x)
```

```
[Out] -Integral(a/(c**2*x**4 - x**3), x) - Integral(b*atanh(c*sqrt(x))/(c**2*x**4
 - x**3), x)
```

3.43 $\quad \int \dfrac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x^4\left(1-c^2x\right)}\,dx$

Optimal. Leaf size=192

$$\frac{c^6\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{b}+2c^6\log\left(2-\frac{2}{c\sqrt{x}+1}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{c^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{x}-\frac{c^2\left(a+b\tan\right)}{2}$$

```
[Out] -1/15*b*c/x^(5/2)-5/18*b*c^3/x^(3/2)+11/6*b*c^6*arctanh(c*x^(1/2))+1/3*(-a-
b*arctanh(c*x^(1/2)))/x^3-1/2*c^2*(a+b*arctanh(c*x^(1/2)))/x^2-c^4*(a+b*arc
tanh(c*x^(1/2)))/x+c^6*(a+b*arctanh(c*x^(1/2)))^2/b+2*c^6*(a+b*arctanh(c*x^
(1/2)))*ln(2-2/(1+c*x^(1/2)))-b*c^6*polylog(2,-1+2/(1+c*x^(1/2)))-11/6*b*c^
5/x^(1/2)
```

Rubi [A]   time = 0.57, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 9, integrand size = 26, $\dfrac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {44, 1593, 5982, 5916, 325, 206, 5988, 5932, 2447}

$$-bc^6\text{PolyLog}\left(2,\frac{2}{c\sqrt{x}+1}-1\right)-\frac{c^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{2x^2}+\frac{c^6\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{b}-\frac{c^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])/(x^4*(1 - c^2*x)),x]
```

```
[Out] -(b*c)/(15*x^(5/2)) - (5*b*c^3)/(18*x^(3/2)) - (11*b*c^5)/(6*Sqrt[x]) + (11
*b*c^6*ArcTanh[c*Sqrt[x]])/6 - (a + b*ArcTanh[c*Sqrt[x]])/(3*x^3) - (c^2*(a
 + b*ArcTanh[c*Sqrt[x]]))/(2*x^2) - (c^4*(a + b*ArcTanh[c*Sqrt[x]]))/x + (c
^6*(a + b*ArcTanh[c*Sqrt[x]])^2)/b + 2*c^6*(a + b*ArcTanh[c*Sqrt[x]])*Log[2
 - 2/(1 + c*Sqrt[x])] - b*c^6*PolyLog[2, -1 + 2/(1 + c*Sqrt[x])]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5982

```
Int[(((a_) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{a + b \tanh^{-1}\left(c\sqrt{x}\right)}{x^4\left(1 - c^2 x\right)}\, dx = 2\,\mathrm{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x^7 - c^2 x^9}\, dx, x, \sqrt{x}\right)$$

$$= 2\,\mathrm{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x^7\left(1 - c^2 x^2\right)}\, dx, x, \sqrt{x}\right)$$

$$= 2\,\mathrm{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x^7}\, dx, x, \sqrt{x}\right) + \left(2c^2\right)\mathrm{Subst}\left(\int \frac{a + b \tanh^{-1}(cx)}{x^5\left(1 - c^2 x^2\right)}\, dx, x, \right.$$

$$= -\frac{a + b \tanh^{-1}\left(c\sqrt{x}\right)}{3x^3} + \frac{1}{3}(bc)\,\mathrm{Subst}\left(\int \frac{1}{x^6\left(1 - c^2 x^2\right)}\, dx, x, \sqrt{x}\right) + \left(2c^2\right)\mathrm{Subst}\left(\int \right.$$

$$= -\frac{bc}{15x^{5/2}} - \frac{a + b \tanh^{-1}\left(c\sqrt{x}\right)}{3x^3} - \frac{c^2\left(a + b \tanh^{-1}\left(c\sqrt{x}\right)\right)}{2x^2} + \frac{1}{3}\left(bc^3\right)\mathrm{Subst}\left(\int \frac{}{x^4\left(\right.}\right.$$

$$= -\frac{bc}{15x^{5/2}} - \frac{5bc^3}{18x^{3/2}} - \frac{a + b \tanh^{-1}\left(c\sqrt{x}\right)}{3x^3} - \frac{c^2\left(a + b \tanh^{-1}\left(c\sqrt{x}\right)\right)}{2x^2} - \frac{c^4\left(a + b\tan\right.}{x}$$

$$= -\frac{bc}{15x^{5/2}} - \frac{5bc^3}{18x^{3/2}} - \frac{11bc^5}{6\sqrt{x}} - \frac{a + b \tanh^{-1}\left(c\sqrt{x}\right)}{3x^3} - \frac{c^2\left(a + b \tanh^{-1}\left(c\sqrt{x}\right)\right)}{2x^2} - \frac{c^4\left(\right.}{}$$

$$= -\frac{bc}{15x^{5/2}} - \frac{5bc^3}{18x^{3/2}} - \frac{11bc^5}{6\sqrt{x}} + \frac{11}{6}bc^6 \tanh^{-1}\left(c\sqrt{x}\right) - \frac{a + b \tanh^{-1}\left(c\sqrt{x}\right)}{3x^3} - \frac{c^2\left(a + b\right.}{}$$

Mathematica [A]   time = 0.81, size = 187, normalized size = 0.97

$$-\frac{-90ac^6x^3\log(x) + 90ac^4x^2 + 45ac^2x + 90ac^6x^3\log\left(1 - c^2 x\right) + 30a - 90bc^6x^3\tanh^{-1}\left(c\sqrt{x}\right)^2 + 165bc^5x^{5/2} + }{90x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/(x^4*(1 - c^2*x)),x]
```

```
[Out] -1/90*(30*a + 6*b*c*Sqrt[x] + 45*a*c^2*x + 25*b*c^3*x^(3/2) + 90*a*c^4*x^2
+ 165*b*c^5*x^(5/2) - 90*b*c^6*x^3*ArcTanh[c*Sqrt[x]]^2 - 15*b*ArcTanh[c*Sq
rt[x]]*(-2 - 3*c^2*x - 6*c^4*x^2 + 11*c^6*x^3 + 12*c^6*x^3*Log[1 - E^(-2*Ar
cTanh[c*Sqrt[x]])]) - 90*a*c^6*x^3*Log[x] + 90*a*c^6*x^3*Log[1 - c^2*x])/x^
3 - b*c^6*PolyLog[2, E^(-2*ArcTanh[c*Sqrt[x]])])
```

fricas [F]   time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{b\,\text{artanh}\left(c\sqrt{x}\right)+a}{c^2x^5-x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^4/(-c^2*x+1),x, algorithm="fricas")

[Out] integral(-(b*arctanh(c*sqrt(x)) + a)/(c^2*x^5 - x^4), x)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int-\frac{b\,\text{artanh}\left(c\sqrt{x}\right)+a}{\left(c^2x-1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^4/(-c^2*x+1),x, algorithm="giac")

[Out] integrate(-(b*arctanh(c*sqrt(x)) + a)/((c^2*x - 1)*x^4), x)
```

maple [B]   time = 0.07, size = 381, normalized size = 1.98

$$-\frac{c^2b\,\text{arctanh}\left(c\sqrt{x}\right)}{2x^2}-c^6b\ln\left(c\sqrt{x}\right)\ln\left(1+c\sqrt{x}\right)+\frac{c^6b\ln\left(c\sqrt{x}-1\right)\ln\left(\frac{1}{2}+\frac{c\sqrt{x}}{2}\right)}{2}-\frac{c^6b\ln\left(-\frac{c\sqrt{x}}{2}+\frac{1}{2}\right)\ln\left(1+c\sqrt{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))/x^4/(-c^2*x+1),x)

[Out] -c^4*b*arctanh(c*x^(1/2))/x-c^6*b*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))-c^6*b*
ln(c*x^(1/2))*ln(1+c*x^(1/2))+2*c^6*b*arctanh(c*x^(1/2))*ln(c*x^(1/2))+1/2*
c^6*b*ln(c*x^(1/2)-1)*ln(1/2+1/2*c*x^(1/2))-1/2*c^6*b*ln(-1/2*c*x^(1/2)+1/2
)*ln(1+c*x^(1/2))+1/2*c^6*b*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2+1/2*c*x^(1/2))-c^
6*b*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)-1/2*c^2*b*arctanh(c*x^(1/2))/x^2-11/
6*b*c^5/x^(1/2)-5/18*b*c^3/x^(3/2)-1/15*b*c/x^(5/2)-c^6*b*dilog(c*x^(1/2))-
c^4*a/x-1/2*c^2*a/x^2-c^6*a*ln(c*x^(1/2)-1)-c^6*a*ln(1+c*x^(1/2))-1/4*c^6*b
*ln(c*x^(1/2)-1)^2+c^6*b*dilog(1/2+1/2*c*x^(1/2))+1/4*c^6*b*ln(1+c*x^(1/2))
```

```
^2-1/3*b*arctanh(c*x^(1/2))/x^3-11/12*c^6*b*ln(c*x^(1/2)-1)-c^6*b*dilog(1+c
*x^(1/2))+2*c^6*a*ln(c*x^(1/2))+11/12*c^6*b*ln(1+c*x^(1/2))-1/3*a/x^3
```

maxima **[B]**   time $= 0.56$, size $= 330$, normalized size $= 1.72$

$$-\left(\log\left(c\sqrt{x}+1\right)\log\left(-\frac{1}{2}c\sqrt{x}+\frac{1}{2}\right)+\operatorname{Li}_2\left(\frac{1}{2}c\sqrt{x}+\frac{1}{2}\right)\right)bc^6-\left(\log\left(c\sqrt{x}\right)\log\left(-c\sqrt{x}+1\right)+\operatorname{Li}_2\left(-c\sqrt{x}+1\right)\right)bc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^4/(-c^2*x+1),x, algorithm="maxima")
```

```
[Out] -(log(c*sqrt(x) + 1)*log(-1/2*c*sqrt(x) + 1/2) + dilog(1/2*c*sqrt(x) + 1/2)
)*b*c^6 - (log(c*sqrt(x))*log(-c*sqrt(x) + 1) + dilog(-c*sqrt(x) + 1))*b*c^
6 + (log(c*sqrt(x) + 1)*log(-c*sqrt(x)) + dilog(c*sqrt(x) + 1))*b*c^6 + 11/
12*b*c^6*log(c*sqrt(x) + 1) - 11/12*b*c^6*log(c*sqrt(x) - 1) - 1/6*(6*c^6*l
og(c*sqrt(x) + 1) + 6*c^6*log(c*sqrt(x) - 1) - 6*c^6*log(x) + (6*c^4*x^2 +
3*c^2*x + 2)/x^3)*a - 1/180*(45*b*c^6*x^3*log(c*sqrt(x) + 1)^2 - 45*b*c^6*x
^3*log(-c*sqrt(x) + 1)^2 + 330*b*c^5*x^(5/2) + 50*b*c^3*x^(3/2) + 12*b*c*sq
rt(x) + 15*(6*b*c^4*x^2 + 3*b*c^2*x + 2*b)*log(c*sqrt(x) + 1) - 15*(6*b*c^6
*x^3*log(c*sqrt(x) + 1) + 6*b*c^4*x^2 + 3*b*c^2*x + 2*b)*log(-c*sqrt(x) + 1
))/x^3
```

mupad **[F]**   time $= 0.00$, size $= -1$, normalized size $= -0.01$

$$-\int\frac{a+b\operatorname{atanh}\left(c\sqrt{x}\right)}{x^4\left(c^2x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*atanh(c*x^(1/2)))/(x^4*(c^2*x - 1)),x)
```

```
[Out] -int((a + b*atanh(c*x^(1/2)))/(x^4*(c^2*x - 1)), x)
```

sympy **[F]**   time $= 0.00$, size $= 0$, normalized size $= 0.00$

$$-\int\frac{a}{c^2x^5-x^4}\,dx-\int\frac{b\operatorname{atanh}\left(c\sqrt{x}\right)}{c^2x^5-x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))/x**4/(-c**2*x+1),x)
```

```
[Out] -Integral(a/(c**2*x**5 - x**4), x) - Integral(b*atanh(c*sqrt(x))/(c**2*x**5
 - x**4), x)
```

3.44 $\displaystyle\int \frac{x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{d+ex}\,dx$

Optimal. Leaf size=460

$$-\frac{2d^2\log\left(\frac{2}{c\sqrt{x}+1}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{e^3}+\frac{d^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,\sqrt{x}\right)}{\left(c\sqrt{x}+1\right)\left(c\sqrt{-d}-\sqrt{e}\right)}\right)}{e^3}+\frac{d^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{e^3}$$

[Out] 1/6*b*x^(3/2)/c/e+b*d*arctanh(c*x^(1/2))/c^2/e^2-1/2*b*arctanh(c*x^(1/2))/c
^4/e-d*x*(a+b*arctanh(c*x^(1/2)))/e^2+1/2*x^2*(a+b*arctanh(c*x^(1/2)))/e-2*
d^2*(a+b*arctanh(c*x^(1/2)))*ln(2/(1+c*x^(1/2)))/e^3+d^2*(a+b*arctanh(c*x^(
1/2)))*ln(2*c*((-d)^(1/2)-e^(1/2)*x^(1/2))/(c*(-d)^(1/2)-e^(1/2))/(1+c*x^(1
/2)))/e^3+d^2*(a+b*arctanh(c*x^(1/2)))*ln(2*c*((-d)^(1/2)+e^(1/2)*x^(1/2)))/
(c*(-d)^(1/2)+e^(1/2))/(1+c*x^(1/2)))/e^3+b*d^2*polylog(2,1-2/(1+c*x^(1/2))
)/e^3-1/2*b*d^2*polylog(2,1-2*c*((-d)^(1/2)-e^(1/2)*x^(1/2))/(c*(-d)^(1/2)-
e^(1/2))/(1+c*x^(1/2)))/e^3-1/2*b*d^2*polylog(2,1-2*c*((-d)^(1/2)+e^(1/2)*x
^(1/2))/(c*(-d)^(1/2)+e^(1/2))/(1+c*x^(1/2)))/e^3-b*d*x^(1/2)/c/e^2+1/2*b*x
^(1/2)/c^3/e

Rubi [A]   time = 0.78, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {43, 5980, 5916, 302, 206, 321, 6044, 5920, 2402, 2315, 2447}

$$\frac{bd^2\mathrm{PolyLog}\left(2,1-\frac{2}{c\sqrt{x}+1}\right)}{e^3}-\frac{bd^2\mathrm{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}\,\sqrt{x}\right)}{\left(c\sqrt{x}+1\right)\left(c\sqrt{-d}-\sqrt{e}\right)}\right)}{2e^3}-\frac{bd^2\mathrm{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}\,\sqrt{x}\right)}{\left(c\sqrt{x}+1\right)\left(c\sqrt{-d}+\sqrt{e}\right)}\right)}{2e^3}-2d^2$$

Antiderivative was successfully verified.

[In] Int[(x^2*(a + b*ArcTanh[c*Sqrt[x]]))/(d + e*x),x]

[Out] -((b*d*Sqrt[x])/(c*e^2)) + (b*Sqrt[x])/(2*c^3*e) + (b*x^(3/2))/(6*c*e) + (b
*d*ArcTanh[c*Sqrt[x]])/(c^2*e^2) - (b*ArcTanh[c*Sqrt[x]])/(2*c^4*e) - (d*x*
(a + b*ArcTanh[c*Sqrt[x]]))/e^2 + (x^2*(a + b*ArcTanh[c*Sqrt[x]]))/(2*e) -
(2*d^2*(a + b*ArcTanh[c*Sqrt[x]])*Log[2/(1 + c*Sqrt[x])])/e^3 + (d^2*(a + b
*ArcTanh[c*Sqrt[x]])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*Sqrt[x]))/((c*Sqrt[-d] -
Sqrt[e])*(1 + c*Sqrt[x]))])/e^3 + (d^2*(a + b*ArcTanh[c*Sqrt[x]])*Log[(2*c*
(Sqrt[-d] + Sqrt[e]*Sqrt[x]))/((c*Sqrt[-d] + Sqrt[e])*(1 + c*Sqrt[x]))])/e^
3 + (b*d^2*PolyLog[2, 1 - 2/(1 + c*Sqrt[x])])/e^3 - (b*d^2*PolyLog[2, 1 - (
2*c*(Sqrt[-d] - Sqrt[e]*Sqrt[x]))/((c*Sqrt[-d] - Sqrt[e])*(1 + c*Sqrt[x]))]
)/(2*e^3) - (b*d^2*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*Sqrt[x]))/((c*Sq
rt[-d] + Sqrt[e])*(1 + c*Sqrt[x]))])/(2*e^3)

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6044

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTanh[c*
x])^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && (GtQ[q, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^2\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{d + ex}\, dx = 2\operatorname{Subst}\left(\int \frac{x^5\left(a + b\tanh^{-1}(cx)\right)}{d + ex^2}\, dx, x, \sqrt{x}\right)$$

$$= \frac{2\operatorname{Subst}\left(\int x^3\left(a + b\tanh^{-1}(cx)\right)\, dx, x, \sqrt{x}\right)}{e} - \frac{(2d)\operatorname{Subst}\left(\int \frac{x^3\left(a + b\tanh^{-1}(cx)\right)}{d + ex^2}\, d\right)}{e}$$

$$= \frac{x^2\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{2e} - \frac{(2d)\operatorname{Subst}\left(\int x\left(a + b\tanh^{-1}(cx)\right)\, dx, x, \sqrt{x}\right)}{e^2} + $$

$$= -\frac{dx\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{e^2} + \frac{x^2\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{2e} + \frac{(bcd)\operatorname{Subst}\left(\int \frac{x^2}{1 - c^2 x^2}\right)}{e^2}$$

$$= -\frac{bd\sqrt{x}}{ce^2} + \frac{b\sqrt{x}}{2c^3 e} + \frac{bx^{3/2}}{6ce} - \frac{dx\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{e^2} + \frac{x^2\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{2e}$$

$$= -\frac{bd\sqrt{x}}{ce^2} + \frac{b\sqrt{x}}{2c^3 e} + \frac{bx^{3/2}}{6ce} + \frac{bd\tanh^{-1}\left(c\sqrt{x}\right)}{c^2 e^2} - \frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{2c^4 e} - \frac{dx\left(a + b\tan\right)}{e}$$

$$= -\frac{bd\sqrt{x}}{ce^2} + \frac{b\sqrt{x}}{2c^3 e} + \frac{bx^{3/2}}{6ce} + \frac{bd\tanh^{-1}\left(c\sqrt{x}\right)}{c^2 e^2} - \frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{2c^4 e} - \frac{dx\left(a + b\tan\right)}{e}$$

$$= -\frac{bd\sqrt{x}}{ce^2} + \frac{b\sqrt{x}}{2c^3 e} + \frac{bx^{3/2}}{6ce} + \frac{bd\tanh^{-1}\left(c\sqrt{x}\right)}{c^2 e^2} - \frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{2c^4 e} - \frac{dx\left(a + b\tan\right)}{e}$$

Mathematica [C]   time = 2.93, size = 558, normalized size = 1.21

$$6ad^2\log(d + ex) - 6adex + 3ae^2 x^2 + \frac{b\left(-6c^4 d^2\left(\tanh^{-1}\left(c\sqrt{x}\right)\left(\tanh^{-1}\left(c\sqrt{x}\right) + 2\log\left(e^{-2\tanh^{-1}\left(c\sqrt{x}\right)} + 1\right)\right) - \operatorname{Li}_2\left(-e^{-2\tanh^{-1}\left(c\sqrt{x}\right)}\right)\right) + 2ce\sqrt{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*Sqrt[x]]))/(d + e*x),x]

[Out] (-6*a*d*e*x + 3*a*e^2*x^2 + 6*a*d^2*Log[d + e*x] + (b*(2*c*e*(-3*c^2*d + 2*
e)*Sqrt[x] + c*e^2*Sqrt[x]*(-1 + c^2*x) - 6*(c^2*d - e)*e*(-1 + c^2*x)*ArcT
```

```
anh[c*Sqrt[x]] + 3*e^2*(-1 + c^2*x)^2*ArcTanh[c*Sqrt[x]] - 6*c^4*d^2*(ArcTa
nh[c*Sqrt[x]]*(ArcTanh[c*Sqrt[x]] + 2*Log[1 + E^(-2*ArcTanh[c*Sqrt[x]])]) -
 PolyLog[2, -E^(-2*ArcTanh[c*Sqrt[x]])]) + 3*c^4*d^2*(2*ArcTanh[c*Sqrt[x]]^
2 - (4*I)*ArcSin[Sqrt[(c^2*d)/(c^2*d + e)]]*ArcTanh[(c*e*Sqrt[x])/Sqrt[-(c^
2*d*e)]] + 2*((-I)*ArcSin[Sqrt[(c^2*d)/(c^2*d + e)]] + ArcTanh[c*Sqrt[x]])*
Log[(-2*Sqrt[-(c^2*d*e)] + e*(-1 + E^(2*ArcTanh[c*Sqrt[x]])) + c^2*d*(1 + E
^(2*ArcTanh[c*Sqrt[x]])))/((c^2*d + e)*E^(2*ArcTanh[c*Sqrt[x]]))] + 2*(I*Ar
cSin[Sqrt[(c^2*d)/(c^2*d + e)]] + ArcTanh[c*Sqrt[x]])*Log[(2*Sqrt[-(c^2*d*e
)] + e*(-1 + E^(2*ArcTanh[c*Sqrt[x]])) + c^2*d*(1 + E^(2*ArcTanh[c*Sqrt[x]]
)))/((c^2*d + e)*E^(2*ArcTanh[c*Sqrt[x]]))] - PolyLog[2, (-(c^2*d) + e - 2*
Sqrt[-(c^2*d*e)])/((c^2*d + e)*E^(2*ArcTanh[c*Sqrt[x]]))] - PolyLog[2, (-(c
^2*d) + e + 2*Sqrt[-(c^2*d*e)])/((c^2*d + e)*E^(2*ArcTanh[c*Sqrt[x]]))])))/
c^4)/(6*e^3)
```

__fricas__ [F]   time = 0.83, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{bx^2\,\mathrm{artanh}\left(c\sqrt{x}\right)+ax^2}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))/(e*x+d),x, algorithm="fricas")

[Out] integral((b*x^2*arctanh(c*sqrt(x)) + a*x^2)/(e*x + d), x)

__giac__ [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,\mathrm{artanh}\left(c\sqrt{x}\right)+a\right)x^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))/(e*x+d),x, algorithm="giac")

[Out] integrate((b*arctanh(c*sqrt(x)) + a)*x^2/(e*x + d), x)

__maple__ [A]   time = 0.10, size = 651, normalized size = 1.42

$$-\frac{adx}{e^2}+\frac{ax^2}{2e}+\frac{ad^2\ln\left(c^2ex+c^2d\right)}{e^3}-\frac{b\,\mathrm{arctanh}\left(c\sqrt{x}\right)xd}{e^2}+\frac{b\,\mathrm{arctanh}\left(c\sqrt{x}\right)x^2}{2e}+\frac{b\,\mathrm{arctanh}\left(c\sqrt{x}\right)d^2\ln\left(c^2ex+c^2d\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a+b*arctanh(c*x^(1/2)))/(e*x+d),x)

```
[Out] -a*d*x/e^2+1/2*a*x^2/e+a*d^2/e^3*ln(c^2*e*x+c^2*d)-b*arctanh(c*x^(1/2))*x*d
/e^2+1/2*b*arctanh(c*x^(1/2))*x^2/e+b*arctanh(c*x^(1/2))*d^2/e^3*ln(c^2*e*x
+c^2*d)+1/6*b*x^(3/2)/c/e-b*d*x^(1/2)/c/e^2+1/2*b*x^(1/2)/c^3/e-1/2/c^2*b/e
^2*ln(c*x^(1/2)-1)*d+1/4/c^4*b*ln(c*x^(1/2)-1)/e+1/2/c^2*b/e^2*ln(1+c*x^(1/
2))*d-1/4/c^4*b*ln(1+c*x^(1/2))/e+1/2*b*d^2/e^3*ln(c*x^(1/2)-1)*ln(c^2*e*x+
c^2*d)-1/2*b*d^2/e^3*ln(c*x^(1/2)-1)*ln((c*(-d*e)^(1/2)-e*(c*x^(1/2)-1)-e)/
(c*(-d*e)^(1/2)-e))-1/2*b*d^2/e^3*ln(c*x^(1/2)-1)*ln((c*(-d*e)^(1/2)+e*(c*x
^(1/2)-1)+e)/(c*(-d*e)^(1/2)+e))-1/2*b*d^2/e^3*dilog((c*(-d*e)^(1/2)-e*(c*x
^(1/2)-1)-e)/(c*(-d*e)^(1/2)-e))-1/2*b*d^2/e^3*dilog((c*(-d*e)^(1/2)+e*(c*x
^(1/2)-1)+e)/(c*(-d*e)^(1/2)+e))-1/2*b*d^2/e^3*ln(1+c*x^(1/2))*ln(c^2*e*x+c
^2*d)+1/2*b*d^2/e^3*ln(1+c*x^(1/2))*ln((c*(-d*e)^(1/2)-e*(1+c*x^(1/2))+e)/(
c*(-d*e)^(1/2)+e))+1/2*b*d^2/e^3*ln(1+c*x^(1/2))*ln((c*(-d*e)^(1/2)+e*(1+c*
x^(1/2))-e)/(c*(-d*e)^(1/2)-e))+1/2*b*d^2/e^3*dilog((c*(-d*e)^(1/2)-e*(1+c*
x^(1/2))+e)/(c*(-d*e)^(1/2)+e))+1/2*b*d^2/e^3*dilog((c*(-d*e)^(1/2)+e*(1+c*
x^(1/2))-e)/(c*(-d*e)^(1/2)-e))
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$
\frac{1}{2}\,a\!\left(\frac{2\,d^2\log{(ex+d)}}{e^3}+\frac{ex^2-2\,dx}{e^2}\right)+b\int\frac{x^2\log\left(c\sqrt{x}+1\right)}{2\,(ex+d)}\,dx-b\int\frac{x^2\log\left(-c\sqrt{x}+1\right)}{2\,(ex+d)}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) + b*integrate(1/2*x^2*
log(c*sqrt(x) + 1)/(e*x + d), x) - b*integrate(1/2*x^2*log(-c*sqrt(x) + 1)/
(e*x + d), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$
\int\frac{x^2\left(a+b\,\mathrm{atanh}\left(c\,\sqrt{x}\right)\right)}{d+e\,x}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atanh(c*x^(1/2))))/(d + e*x),x)
```

```
[Out] int((x^2*(a + b*atanh(c*x^(1/2))))/(d + e*x), x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**(1/2)))/(e*x+d),x)
```

```
[Out] Timed out
```

3.45 $\int \dfrac{x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{d+ex}\,dx$

Optimal. Leaf size=374

$$\frac{2d\log\left(\frac{2}{c\sqrt{x}+1}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{e^2}-\frac{d\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,\sqrt{x}\right)}{\left(c\sqrt{x}+1\right)\left(c\sqrt{-d}-\sqrt{e}\right)}\right)}{e^2}-\frac{d\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\mathrm{l}}{e^2}$$

[Out] -b*arctanh(c*x^(1/2))/c^2/e+x*(a+b*arctanh(c*x^(1/2)))/e+2*d*(a+b*arctanh(c
*x^(1/2)))*ln(2/(1+c*x^(1/2)))/e^2-d*(a+b*arctanh(c*x^(1/2)))*ln(2*c*((-d)^
(1/2)-e^(1/2)*x^(1/2))/(c*(-d)^(1/2)-e^(1/2))/(1+c*x^(1/2)))/e^2-d*(a+b*arc
tanh(c*x^(1/2)))*ln(2*c*((-d)^(1/2)+e^(1/2)*x^(1/2))/(c*(-d)^(1/2)+e^(1/2))
/(1+c*x^(1/2)))/e^2-b*d*polylog(2,1-2/(1+c*x^(1/2)))/e^2+1/2*b*d*polylog(2,
1-2*c*((-d)^(1/2)-e^(1/2)*x^(1/2))/(c*(-d)^(1/2)-e^(1/2))/(1+c*x^(1/2)))/e^
2+1/2*b*d*polylog(2,1-2*c*((-d)^(1/2)+e^(1/2)*x^(1/2))/(c*(-d)^(1/2)+e^(1/2
))/(1+c*x^(1/2)))/e^2+b*x^(1/2)/c/e

Rubi [A] time = 0.48, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {43, 5980, 5916, 321, 206, 6044, 5920, 2402, 2315, 2447}

$$-\frac{bd\mathrm{PolyLog}\left(2,1-\frac{2}{c\sqrt{x}+1}\right)}{e^2}+\frac{bd\mathrm{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}\,\sqrt{x}\right)}{\left(c\sqrt{x}+1\right)\left(c\sqrt{-d}-\sqrt{e}\right)}\right)}{2e^2}+\frac{bd\mathrm{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}\,\sqrt{x}\right)}{\left(c\sqrt{x}+1\right)\left(c\sqrt{-d}+\sqrt{e}\right)}\right)}{2e^2}+\frac{2d\,\mathrm{l}}{}$$

Antiderivative was successfully verified.

[In] Int[(x*(a + b*ArcTanh[c*Sqrt[x]]))/(d + e*x),x]

[Out] (b*Sqrt[x])/(c*e) - (b*ArcTanh[c*Sqrt[x]])/(c^2*e) + (x*(a + b*ArcTanh[c*Sq
rt[x]]))/e + (2*d*(a + b*ArcTanh[c*Sqrt[x]])*Log[2/(1 + c*Sqrt[x])])/e^2 -
(d*(a + b*ArcTanh[c*Sqrt[x]])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*Sqrt[x]))/((c*Sq
rt[-d] - Sqrt[e])*(1 + c*Sqrt[x]))])/e^2 - (d*(a + b*ArcTanh[c*Sqrt[x]])*Lo
g[(2*c*(Sqrt[-d] + Sqrt[e]*Sqrt[x]))/((c*Sqrt[-d] + Sqrt[e])*(1 + c*Sqrt[x]
))])/e^2 - (b*d*PolyLog[2, 1 - 2/(1 + c*Sqrt[x])])/e^2 + (b*d*PolyLog[2, 1
- (2*c*(Sqrt[-d] - Sqrt[e]*Sqrt[x]))/((c*Sqrt[-d] - Sqrt[e])*(1 + c*Sqrt[x]
))])/(2*e^2) + (b*d*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*Sqrt[x]))/((c*S
qrt[-d] + Sqrt[e])*(1 + c*Sqrt[x]))])/(2*e^2)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le

Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

Rule 321

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2315

Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]

Rule 2402

Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]

Rule 2447

Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]

Rule 5916

Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
p)/(d(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6044

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTanh[c*
x])^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && (GtQ[q, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{d + ex}\, dx = 2\,\mathrm{Subst}\left(\int \frac{x^3\left(a + b\tanh^{-1}(cx)\right)}{d + ex^2}\, dx, x, \sqrt{x}\right)$$

$$= \frac{2\,\mathrm{Subst}\left(\int x\left(a + b\tanh^{-1}(cx)\right)\, dx, x, \sqrt{x}\right)}{e} - \frac{(2d)\,\mathrm{Subst}\left(\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{d+ex^2}\, dx, x, \right.}{e}$$

$$= \frac{x\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{e} - \frac{(bc)\,\mathrm{Subst}\left(\int \frac{x^2}{1-c^2x^2}\, dx, x, \sqrt{x}\right)}{e} - \frac{(2d)\,\mathrm{Subst}\left(\int \left(-\frac{}{2}\right)\right.}{}$$

$$= \frac{b\sqrt{x}}{ce} + \frac{x\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{e} + \frac{d\,\mathrm{Subst}\left(\int \frac{a+b\tanh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}\,x}\, dx, x, \sqrt{x}\right)}{e^{3/2}} - d\,\mathrm{Subs}$$

$$= \frac{b\sqrt{x}}{ce} - \frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{c^2 e} + \frac{x\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{e} + \frac{2d\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{e^2}$$

$$= \frac{b\sqrt{x}}{ce} - \frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{c^2 e} + \frac{x\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{e} + \frac{2d\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{e^2}$$

$$= \frac{b\sqrt{x}}{ce} - \frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{c^2 e} + \frac{x\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{e} + \frac{2d\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{e^2}$$

Mathematica [A]   time = 1.46, size = 337, normalized size = 0.90

$$-2ad\log(d + ex) + 2aex + \frac{2b\left(\tanh^{-1}\left(c\sqrt{x}\right)\left(2c^2 d\log\left(e^{-2\tanh^{-1}\left(c\sqrt{x}\right)}+1\right)+c^2 ex-e\right)-c^2 d\mathrm{Li}_2\left(-e^{-2\tanh^{-1}\left(c\sqrt{x}\right)}\right)+c^2 d\tanh^{-1}\left(c\sqrt{x}\right)^2+ce\sqrt{x}\right)}{c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcTanh[c*Sqrt[x]]))/(d + e*x),x]

[Out] (2*a*e*x - 2*a*d*Log[d + e*x] + (2*b*(c*e*Sqrt[x] + c^2*d*ArcTanh[c*Sqrt[x]
]^2 + ArcTanh[c*Sqrt[x]]*(-e + c^2*e*x + 2*c^2*d*Log[1 + E^(-2*ArcTanh[c*Sq
rt[x]])]) - c^2*d*PolyLog[2, -E^(-2*ArcTanh[c*Sqrt[x]])]))/c^2 - b*d*(2*Arc
Tanh[c*Sqrt[x]]*(-ArcTanh[c*Sqrt[x]] + Log[1 + ((c^2*d + e)*E^(2*ArcTanh[c*
Sqrt[x]]))/(c^2*d - 2*c*Sqrt[-d]*Sqrt[e] - e)] + Log[1 + ((c^2*d + e)*E^(2*
ArcTanh[c*Sqrt[x]]))/(c^2*d + 2*c*Sqrt[-d]*Sqrt[e] - e)]) + PolyLog[2, -(((
c^2*d + e)*E^(2*ArcTanh[c*Sqrt[x]]))/(c^2*d - 2*c*Sqrt[-d]*Sqrt[e] - e))] +
```

```
 PolyLog[2, -(((c^2*d + e)*E^(2*ArcTanh[c*Sqrt[x]]))/(c^2*d + 2*c*Sqrt[-d]*
Sqrt[e] - e))])])/(2*e^2)
```

__fricas__ [F]   time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx\operatorname{artanh}\left(c\sqrt{x}\right)+ax}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arctanh(c*x^(1/2)))/(e*x+d),x, algorithm="fricas")

[Out] integral((b*x*arctanh(c*sqrt(x)) + a*x)/(e*x + d), x)

__giac__ [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\operatorname{artanh}\left(c\sqrt{x}\right)+a\right)x}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arctanh(c*x^(1/2)))/(e*x+d),x, algorithm="giac")

[Out] integrate((b*arctanh(c*sqrt(x)) + a)*x/(e*x + d), x)

__maple__ [A]   time = 0.06, size = 539, normalized size = 1.44

$$\frac{ax}{e}-\frac{ad\ln\left(c^2ex+c^2d\right)}{e^2}+\frac{b\operatorname{arctanh}\left(c\sqrt{x}\right)x}{e}-\frac{b\operatorname{arctanh}\left(c\sqrt{x}\right)d\ln\left(c^2ex+c^2d\right)}{e^2}-\frac{bd\ln\left(c\sqrt{x}-1\right)\ln\left(c^2ex+c^2d\right)}{2e^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a+b*arctanh(c*x^(1/2)))/(e*x+d),x)

[Out] a*x/e-a/e^2*d*ln(c^2*e*x+c^2*d)+b*arctanh(c*x^(1/2))*x/e-b*arctanh(c*x^(1/2
))/e^2*d*ln(c^2*e*x+c^2*d)-1/2*b/e^2*d*ln(c*x^(1/2)-1)*ln(c^2*e*x+c^2*d)+1/
2*b/e^2*d*ln(c*x^(1/2)-1)*ln((c*(-d*e)^(1/2)-e*(c*x^(1/2)-1)-e)/(c*(-d*e)^(
1/2)-e))+1/2*b/e^2*d*ln(c*x^(1/2)-1)*ln((c*(-d*e)^(1/2)+e*(c*x^(1/2)-1)+e)/
(c*(-d*e)^(1/2)+e))+1/2*b/e^2*d*dilog((c*(-d*e)^(1/2)-e*(c*x^(1/2)-1)-e)/(c
*(-d*e)^(1/2)-e))+1/2*b/e^2*d*dilog((c*(-d*e)^(1/2)+e*(c*x^(1/2)-1)+e)/(c*(
-d*e)^(1/2)+e))+1/2*b/e^2*d*ln(1+c*x^(1/2))*ln(c^2*e*x+c^2*d)-1/2*b/e^2*d*l
n(1+c*x^(1/2))*ln((c*(-d*e)^(1/2)-e*(1+c*x^(1/2))+e)/(c*(-d*e)^(1/2)+e))-1/
2*b/e^2*d*ln(1+c*x^(1/2))*ln((c*(-d*e)^(1/2)+e*(1+c*x^(1/2))-e)/(c*(-d*e)^(
1/2)-e))-1/2*b/e^2*d*dilog((c*(-d*e)^(1/2)-e*(1+c*x^(1/2))+e)/(c*(-d*e)^(1/
2)+e))-1/2*b/e^2*d*dilog((c*(-d*e)^(1/2)+e*(1+c*x^(1/2))-e)/(c*(-d*e)^(1/2)
-e))+b*x^(1/2)/c/e+1/2/c^2*b*ln(c*x^(1/2)-1)/e-1/2/c^2*b*ln(1+c*x^(1/2))/e
```

maxima [**F**]   time = 0.00, size = 0, normalized size = 0.00

$$a\left(\frac{x}{e} - \frac{d\log{(ex+d)}}{e^2}\right) + b\int \frac{x\log{\left(c\sqrt{x}+1\right)}}{2\,(ex+d)}\,dx - b\int \frac{x\log{\left(-c\sqrt{x}+1\right)}}{2\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arctanh(c*x^(1/2)))/(e*x+d),x, algorithm="maxima")

[Out] a*(x/e - d*log(e*x + d)/e^2) + b*integrate(1/2*x*log(c*sqrt(x) + 1)/(e*x + d), x) - b*integrate(1/2*x*log(-c*sqrt(x) + 1)/(e*x + d), x)

mupad [**F**]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(a + b\operatorname{atanh}\left(c\sqrt{x}\right)\right)}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a + b*atanh(c*x^(1/2))))/(d + e*x),x)

[Out] int((x*(a + b*atanh(c*x^(1/2))))/(d + e*x), x)

sympy [**F**]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a + b\operatorname{atanh}\left(c\sqrt{x}\right)\right)}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*atanh(c*x**(1/2)))/(e*x+d),x)

[Out] Integral(x*(a + b*atanh(c*sqrt(x)))/(d + e*x), x)

3.46 $\int \dfrac{a + b\tanh^{-1}\left(c\sqrt{x}\right)}{d + ex}\, dx$

Optimal. Leaf size=318

$$\dfrac{\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(\dfrac{2c\left(\sqrt{-d}-\sqrt{e}\,\sqrt{x}\right)}{\left(c\sqrt{x}+1\right)\left(c\sqrt{-d}-\sqrt{e}\right)}\right)}{e} + \dfrac{\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(\dfrac{2c\left(\sqrt{-d}+\sqrt{e}\,\sqrt{x}\right)}{\left(c\sqrt{x}+1\right)\left(c\sqrt{-d}+\sqrt{e}\right)}\right)}{e} - \dfrac{2\log\left(\dfrac{2}{c\sqrt{x}+1}\right)\left(a + \right.}$$

```
[Out] -2*(a+b*arctanh(c*x^(1/2)))*ln(2/(1+c*x^(1/2)))/e+(a+b*arctanh(c*x^(1/2)))*
ln(2*c*((-d)^(1/2)-e^(1/2)*x^(1/2))/(c*(-d)^(1/2)-e^(1/2)))/(1+c*x^(1/2)))/e
+(a+b*arctanh(c*x^(1/2)))*ln(2*c*((-d)^(1/2)+e^(1/2)*x^(1/2))/(c*(-d)^(1/2)
+e^(1/2))/(1+c*x^(1/2)))/e+b*polylog(2,1-2/(1+c*x^(1/2)))/e-1/2*b*polylog(2
,1-2*c*((-d)^(1/2)-e^(1/2)*x^(1/2))/(c*(-d)^(1/2)-e^(1/2)))/(1+c*x^(1/2)))/e
-1/2*b*polylog(2,1-2*c*((-d)^(1/2)+e^(1/2)*x^(1/2))/(c*(-d)^(1/2)+e^(1/2))/
(1+c*x^(1/2)))/e
```

Rubi [A]   time = 0.32, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 20, $\dfrac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6044, 5920, 2402, 2315, 2447}

$$-\dfrac{b\,\text{PolyLog}\left(2, 1 - \dfrac{2c\left(\sqrt{-d}-\sqrt{e}\,\sqrt{x}\right)}{\left(c\sqrt{x}+1\right)\left(c\sqrt{-d}-\sqrt{e}\right)}\right)}{2e} - \dfrac{b\,\text{PolyLog}\left(2, 1 - \dfrac{2c\left(\sqrt{-d}+\sqrt{e}\,\sqrt{x}\right)}{\left(c\sqrt{x}+1\right)\left(c\sqrt{-d}+\sqrt{e}\right)}\right)}{2e} + \dfrac{b\,\text{PolyLog}\left(2, 1 - \dfrac{2}{c\sqrt{x}+1}\right)}{e} + \dfrac{\left(a + b\,t\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])/(d + e*x),x]
```

```
[Out] (-2*(a + b*ArcTanh[c*Sqrt[x]])*Log[2/(1 + c*Sqrt[x])])/e + ((a + b*ArcTanh[
c*Sqrt[x]])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*Sqrt[x]))/((c*Sqrt[-d] - Sqrt[e])*
(1 + c*Sqrt[x]))])/e + ((a + b*ArcTanh[c*Sqrt[x]])*Log[(2*c*(Sqrt[-d] + Sqr
t[e]*Sqrt[x]))/((c*Sqrt[-d] + Sqrt[e])*(1 + c*Sqrt[x]))])/e + (b*PolyLog[2,
 1 - 2/(1 + c*Sqrt[x])])/e - (b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*Sqr
t[x]))/((c*Sqrt[-d] - Sqrt[e])*(1 + c*Sqrt[x]))])/(2*e) - (b*PolyLog[2, 1 -
 (2*c*(Sqrt[-d] + Sqrt[e]*Sqrt[x]))/((c*Sqrt[-d] + Sqrt[e])*(1 + c*Sqrt[x])
)])/(2*e)
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]

Rule 2447

Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]

Rule 5920

Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]

Rule 6044

Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTanh[c*
x])^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && (GtQ[q, 0] || IntegerQ[m])

Rubi steps

$$\int \frac{a + b\tanh^{-1}\left(c\sqrt{x}\right)}{d + ex}\,dx = 2\,\text{Subst}\left(\int \frac{x\left(a + b\tanh^{-1}(cx)\right)}{d + ex^2}\,dx, x, \sqrt{x}\right)$$

$$= 2\,\text{Subst}\left(\int \left(-\frac{a + b\tanh^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d} - \sqrt{e}\,x\right)} + \frac{a + b\tanh^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d} + \sqrt{e}\,x\right)}\right)dx, x, \sqrt{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a + b\tanh^{-1}(cx)}{\sqrt{-d} - \sqrt{e}\,x}\,dx, x, \sqrt{x}\right)}{\sqrt{e}} + \frac{\text{Subst}\left(\int \frac{a + b\tanh^{-1}(cx)}{\sqrt{-d} + \sqrt{e}\,x}\,dx, x, \sqrt{x}\right)}{\sqrt{e}}$$

$$= -\frac{2\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(\frac{2}{1 + c\sqrt{x}}\right)}{e} + \frac{\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(\frac{2c\left(\sqrt{-d} - \sqrt{e}\,\sqrt{x}\right)}{\left(c\sqrt{-d} - \sqrt{e}\right)\left(1 + c\sqrt{x}\right)}\right)}{e}$$

$$= -\frac{2\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(\frac{2}{1 + c\sqrt{x}}\right)}{e} + \frac{\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(\frac{2c\left(\sqrt{-d} - \sqrt{e}\,\sqrt{x}\right)}{\left(c\sqrt{-d} - \sqrt{e}\right)\left(1 + c\sqrt{x}\right)}\right)}{e}$$

$$= -\frac{2\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(\frac{2}{1 + c\sqrt{x}}\right)}{e} + \frac{\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(\frac{2c\left(\sqrt{-d} - \sqrt{e}\,\sqrt{x}\right)}{\left(c\sqrt{-d} - \sqrt{e}\right)\left(1 + c\sqrt{x}\right)}\right)}{e}$$

Mathematica [C]   time = 1.62, size = 432, normalized size = 1.36

$$\frac{a\log(d + ex)}{e} - \frac{b\left(\text{Li}_2\left(\frac{\left(-dc^2 + e - 2\sqrt{-c^2 de}\right)e^{-2\tanh^{-1}\left(c\sqrt{x}\right)}}{dc^2 + e}\right) + \text{Li}_2\left(\frac{\left(-dc^2 + e + 2\sqrt{-c^2 de}\right)e^{-2\tanh^{-1}\left(c\sqrt{x}\right)}}{dc^2 + e}\right) + 4i\sin^{-1}\left(\sqrt{\frac{c^2 d}{c^2 d + e}}\right)\tanh^{}}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/(d + e*x),x]

[Out] (a*Log[d + e*x])/e - (b*(-2*ArcTanh[c*Sqrt[x]]^2 + (4*I)*ArcSin[Sqrt[(c^2*d
)/(c^2*d + e)]]*ArcTanh[(c*e*Sqrt[x])/Sqrt[-(c^2*d*e)]] + 2*ArcTanh[c*Sqrt[
x]]*(ArcTanh[c*Sqrt[x]] + 2*Log[1 + E^(-2*ArcTanh[c*Sqrt[x]])]) - 2*((-I)*A
rcSin[Sqrt[(c^2*d)/(c^2*d + e)]] + ArcTanh[c*Sqrt[x]])*Log[(-2*Sqrt[-(c^2*d
*e)] + e*(-1 + E^(2*ArcTanh[c*Sqrt[x]])) + c^2*d*(1 + E^(2*ArcTanh[c*Sqrt[x
]])))/((c^2*d + e)*E^(2*ArcTanh[c*Sqrt[x]]))] - 2*(I*ArcSin[Sqrt[(c^2*d)/(c
^2*d + e)]] + ArcTanh[c*Sqrt[x]])*Log[(2*Sqrt[-(c^2*d*e)] + e*(-1 + E^(2*Ar
cTanh[c*Sqrt[x]])) + c^2*d*(1 + E^(2*ArcTanh[c*Sqrt[x]])))/((c^2*d + e)*E^(
2*ArcTanh[c*Sqrt[x]]))] - 2*PolyLog[2, -E^(-2*ArcTanh[c*Sqrt[x]])] + PolyLo
g[2, (-(c^2*d) + e - 2*Sqrt[-(c^2*d*e)])/((c^2*d + e)*E^(2*ArcTanh[c*Sqrt[x
```

```
]])))] + PolyLog[2, (-(c^2*d) + e + 2*Sqrt[-(c^2*d*e)])/((c^2*d + e)*E^(2*Ar
cTanh[c*Sqrt[x]]))])])))/(2*e)
```

fricas [**F**]   time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{artanh}\left(c\sqrt{x}\right)+a}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*sqrt(x)) + a)/(e*x + d), x)
```

giac [**F**]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{artanh}\left(c\sqrt{x}\right)+a}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)/(e*x + d), x)
```

maple [**A**]   time = 0.06, size = 462, normalized size = 1.45

$$\frac{a\ln\left(c^2ex+c^2d\right)}{e}+\frac{b\ln\left(c^2ex+c^2d\right)\arctanh\left(c\sqrt{x}\right)}{e}+\frac{b\ln\left(c\sqrt{x}-1\right)\ln\left(c^2ex+c^2d\right)}{2e}-\frac{b\ln\left(c\sqrt{x}-1\right)\ln\left(\frac{c\sqrt{-de}-e\left(}{c\sqrt{-}}\right)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))/(e*x+d),x)
```

```
[Out] a/e*ln(c^2*e*x+c^2*d)+b/e*ln(c^2*e*x+c^2*d)*arctanh(c*x^(1/2))+1/2*b/e*ln(c
*x^(1/2)-1)*ln(c^2*e*x+c^2*d)-1/2*b*ln(c*x^(1/2)-1)/e*ln((c*(-d*e)^(1/2)-e*
(c*x^(1/2)-1)-e)/(c*(-d*e)^(1/2)-e))-1/2*b*ln(c*x^(1/2)-1)/e*ln((c*(-d*e)^(
1/2)+e*(c*x^(1/2)-1)+e)/(c*(-d*e)^(1/2)+e))-1/2*b/e*dilog((c*(-d*e)^(1/2)-e
*(c*x^(1/2)-1)-e)/(c*(-d*e)^(1/2)-e))-1/2*b/e*dilog((c*(-d*e)^(1/2)+e*(c*x^
(1/2)-1)+e)/(c*(-d*e)^(1/2)+e))-1/2*b/e*ln(1+c*x^(1/2))*ln(c^2*e*x+c^2*d)+1
/2*b*ln(1+c*x^(1/2))/e*ln((c*(-d*e)^(1/2)-e*(1+c*x^(1/2))+e)/(c*(-d*e)^(1/2
)+e))+1/2*b*ln(1+c*x^(1/2))/e*ln((c*(-d*e)^(1/2)+e*(1+c*x^(1/2))-e)/(c*(-d*
e)^(1/2)-e))+1/2*b/e*dilog((c*(-d*e)^(1/2)-e*(1+c*x^(1/2))+e)/(c*(-d*e)^(1/
2)+e))+1/2*b/e*dilog((c*(-d*e)^(1/2)+e*(1+c*x^(1/2))-e)/(c*(-d*e)^(1/2)-e))
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$b \int \frac{\log\left(c\sqrt{x}+1\right)}{2\left(ex+d\right)}\,dx - b\int \frac{\log\left(-c\sqrt{x}+1\right)}{2\left(ex+d\right)}\,dx + \frac{a\log\left(ex+d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*integrate(1/2*log(c*sqrt(x) + 1)/(e*x + d), x) - b*integrate(1/2*log(-c*s
qrt(x) + 1)/(e*x + d), x) + a*log(e*x + d)/e
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\operatorname{atanh}\left(c\,\sqrt{x}\right)}{d + e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))/(d + e*x),x)
```

```
[Out] int((a + b*atanh(c*x^(1/2)))/(d + e*x), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{atanh}\left(c\sqrt{x}\right)}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))/(e*x+d),x)
```

```
[Out] Integral((a + b*atanh(c*sqrt(x)))/(d + e*x), x)
```

3.47 $\displaystyle\int \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x(d+ex)}\,dx$

Optimal. Leaf size=358

$$-\frac{\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\sqrt{x}\right)}{\left(c\sqrt{x}+1\right)\left(c\sqrt{-d}-\sqrt{e}\right)}\right)}{d} - \frac{\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}\sqrt{x}\right)}{\left(c\sqrt{x}+1\right)\left(c\sqrt{-d}+\sqrt{e}\right)}\right)}{d} + \frac{2\log\left(\frac{2}{c\sqrt{x}+1}\right)\left(}{}$$

```
[Out] a*ln(x)/d+2*(a+b*arctanh(c*x^(1/2)))*ln(2/(1+c*x^(1/2)))/d-(a+b*arctanh(c*x
^(1/2)))*ln(2*c*((-d)^(1/2)-e^(1/2)*x^(1/2))/(c*(-d)^(1/2)-e^(1/2))/(1+c*x^
(1/2)))/d-(a+b*arctanh(c*x^(1/2)))*ln(2*c*((-d)^(1/2)+e^(1/2)*x^(1/2))/(c*(
-d)^(1/2)+e^(1/2))/(1+c*x^(1/2)))/d-b*polylog(2,-c*x^(1/2))/d+b*polylog(2,c
*x^(1/2))/d-b*polylog(2,1-2/(1+c*x^(1/2)))/d+1/2*b*polylog(2,1-2*c*((-d)^(1
/2)-e^(1/2)*x^(1/2))/(c*(-d)^(1/2)-e^(1/2))/(1+c*x^(1/2)))/d+1/2*b*polylog(
2,1-2*c*((-d)^(1/2)+e^(1/2)*x^(1/2))/(c*(-d)^(1/2)+e^(1/2))/(1+c*x^(1/2)))/
d
```

Rubi **[A]**   time = 0.59, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {36, 29, 31, 1593, 5992, 5912, 6044, 5920, 2402, 2315, 2447}

$$\frac{b\text{PolyLog}\left(2, 1 - \frac{2c\left(\sqrt{-d}-\sqrt{e}\sqrt{x}\right)}{\left(c\sqrt{x}+1\right)\left(c\sqrt{-d}-\sqrt{e}\right)}\right)}{2d} + \frac{b\text{PolyLog}\left(2, 1 - \frac{2c\left(\sqrt{-d}+\sqrt{e}\sqrt{x}\right)}{\left(c\sqrt{x}+1\right)\left(c\sqrt{-d}+\sqrt{e}\right)}\right)}{2d} - \frac{b\text{PolyLog}\left(2, 1 - \frac{2}{c\sqrt{x}+1}\right)}{d} - b\text{PolyL}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])/(x*(d + e*x)),x]
```

```
[Out] (2*(a + b*ArcTanh[c*Sqrt[x]])*Log[2/(1 + c*Sqrt[x])])/d - ((a + b*ArcTanh[c
*Sqrt[x]])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*Sqrt[x]))/((c*Sqrt[-d] - Sqrt[e])*(
1 + c*Sqrt[x]))])/d - ((a + b*ArcTanh[c*Sqrt[x]])*Log[(2*c*(Sqrt[-d] + Sqrt
[e]*Sqrt[x]))/((c*Sqrt[-d] + Sqrt[e])*(1 + c*Sqrt[x]))])/d + (a*Log[x])/d -
 (b*PolyLog[2, 1 - 2/(1 + c*Sqrt[x])])/d + (b*PolyLog[2, 1 - (2*c*(Sqrt[-d]
 - Sqrt[e]*Sqrt[x]))/((c*Sqrt[-d] - Sqrt[e])*(1 + c*Sqrt[x]))])/(2*d) + (b*
PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*Sqrt[x]))/((c*Sqrt[-d] + Sqrt[e])*(
1 + c*Sqrt[x]))])/(2*d) - (b*PolyLog[2, -(c*Sqrt[x])])/d + (b*PolyLog[2, c*
Sqrt[x]])/d
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
```

```
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6044

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTanh[c*
x])^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && (GtQ[q, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a + b\tanh^{-1}\left(c\sqrt{x}\right)}{x(d + ex)}\,dx = 2\,\mathrm{Subst}\left(\int \frac{a + b\tanh^{-1}(cx)}{dx + ex^3}\,dx, x, \sqrt{x}\right)$$

$$= 2\,\mathrm{Subst}\left(\int \frac{a + b\tanh^{-1}(cx)}{x\left(d + ex^2\right)}\,dx, x, \sqrt{x}\right)$$

$$= 2\,\mathrm{Subst}\left(\int \left(\frac{a + b\tanh^{-1}(cx)}{dx} - \frac{ex\left(a + b\tanh^{-1}(cx)\right)}{d\left(d + ex^2\right)}\right)\,dx, x, \sqrt{x}\right)$$

$$= \frac{2\,\mathrm{Subst}\left(\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx, x, \sqrt{x}\right)}{d} - \frac{(2e)\,\mathrm{Subst}\left(\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{d+ex^2}\,dx, x, \sqrt{x}\right)}{d}$$

$$= \frac{a\log(x)}{d} - \frac{b\mathrm{Li}_2\left(-c\sqrt{x}\right)}{d} + \frac{b\mathrm{Li}_2\left(c\sqrt{x}\right)}{d} - \frac{(2e)\,\mathrm{Subst}\left(\int \left(-\frac{a+b\tanh^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{a+b\tanh^{-1}}{2\sqrt{e}\left(\sqrt{-d}-\right.}\right.\right.}{d}$$

$$= \frac{a\log(x)}{d} - \frac{b\mathrm{Li}_2\left(-c\sqrt{x}\right)}{d} + \frac{b\mathrm{Li}_2\left(c\sqrt{x}\right)}{d} + \frac{\sqrt{e}\,\mathrm{Subst}\left(\int \frac{a+b\tanh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}x}\,dx, x, \sqrt{x}\right)}{d} \quad \sqrt{}$$

$$= \frac{2\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(\frac{2}{1+c\sqrt{x}}\right)}{d} - \frac{\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\sqrt{x}\right)}{\left(c\sqrt{-d}-\sqrt{e}\right)\left(1+c\sqrt{x}\right)}\right)}{d}$$

$$= \frac{2\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(\frac{2}{1+c\sqrt{x}}\right)}{d} - \frac{\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\sqrt{x}\right)}{\left(c\sqrt{-d}-\sqrt{e}\right)\left(1+c\sqrt{x}\right)}\right)}{d}$$

$$= \frac{2\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(\frac{2}{1+c\sqrt{x}}\right)}{d} - \frac{\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\sqrt{x}\right)}{\left(c\sqrt{-d}-\sqrt{e}\right)\left(1+c\sqrt{x}\right)}\right)}{d}$$

Mathematica [A]   time = 1.15, size = 302, normalized size = 0.84

$$-\frac{2a\log(d + ex) - 2a\log(x) + b\mathrm{Li}_2\left(-\frac{(dc^2+e)e^{2\tanh^{-1}\left(c\sqrt{x}\right)}}{dc^2-2\sqrt{-d}\sqrt{e}\,c-e}\right) + b\mathrm{Li}_2\left(-\frac{(dc^2+e)e^{2\tanh^{-1}\left(c\sqrt{x}\right)}}{dc^2+2\sqrt{-d}\sqrt{e}\,c-e}\right) + 2b\tanh^{-1}\left(c\sqrt{x}\right)\log\left(\frac{(c^2d}{c^2}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/(x*(d + e*x)),x]

[Out] -1/2*(-4*b*ArcTanh[c*Sqrt[x]]^2 - 4*b*ArcTanh[c*Sqrt[x]]*Log[1 - E^(-2*ArcT
anh[c*Sqrt[x]])] + 2*b*ArcTanh[c*Sqrt[x]]*Log[1 + ((c^2*d + e)*E^(2*ArcTanh
```

```
[c*Sqrt[x]]))/(c^2*d - 2*c*Sqrt[-d]*Sqrt[e] - e)] + 2*b*ArcTanh[c*Sqrt[x]]*
Log[1 + ((c^2*d + e)*E^(2*ArcTanh[c*Sqrt[x]]))/(c^2*d + 2*c*Sqrt[-d]*Sqrt[e
] - e)] - 2*a*Log[x] + 2*a*Log[d + e*x] + 2*b*PolyLog[2, E^(-2*ArcTanh[c*Sq
rt[x]])] + b*PolyLog[2, -(((c^2*d + e)*E^(2*ArcTanh[c*Sqrt[x]]))/(c^2*d - 2
*c*Sqrt[-d]*Sqrt[e] - e))] + b*PolyLog[2, -(((c^2*d + e)*E^(2*ArcTanh[c*Sqr
t[x]]))/(c^2*d + 2*c*Sqrt[-d]*Sqrt[e] - e))])/d
```

fricas $[\mathbf{F}]$ time $= 0.71$, size $= 0$, normalized size $= 0.00$

$$\text{integral}\left(\frac{b\operatorname{artanh}\left(c\sqrt{x}\right)+a}{ex^2+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*sqrt(x)) + a)/(e*x^2 + d*x), x)
```

giac $[\mathbf{F}]$ time $= 0.00$, size $= 0$, normalized size $= 0.00$

$$\int \frac{b\operatorname{artanh}\left(c\sqrt{x}\right)+a}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)/((e*x + d)*x), x)
```

maple $[\mathbf{A}]$ time $= 0.07$, size $= 540$, normalized size $= 1.51$

$$\frac{2a\ln\left(c\sqrt{x}\right)}{d}-\frac{a\ln\left(c^2ex+c^2d\right)}{d}+\frac{2b\operatorname{arctanh}\left(c\sqrt{x}\right)\ln\left(c\sqrt{x}\right)}{d}-\frac{b\operatorname{arctanh}\left(c\sqrt{x}\right)\ln\left(c^2ex+c^2d\right)}{d}-\frac{b\ln\left(c\sqrt{x}-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))/x/(e*x+d),x)
```

```
[Out] 2*a/d*ln(c*x^(1/2))-a/d*ln(c^2*e*x+c^2*d)+2*b*arctanh(c*x^(1/2))/d*ln(c*x^(
1/2))-b*arctanh(c*x^(1/2))/d*ln(c^2*e*x+c^2*d)-1/2*b/d*ln(c*x^(1/2)-1)*ln(c
^2*e*x+c^2*d)+1/2*b/d*ln(c*x^(1/2)-1)*ln((c*(-d*e)^(1/2)-e*(c*x^(1/2)-1)-e)
/(c*(-d*e)^(1/2)-e))+1/2*b/d*ln(c*x^(1/2)-1)*ln((c*(-d*e)^(1/2)+e*(c*x^(1/2
)-1)+e)/(c*(-d*e)^(1/2)+e))+1/2*b/d*dilog((c*(-d*e)^(1/2)-e*(c*x^(1/2)-1)-e
)/(c*(-d*e)^(1/2)-e))+1/2*b/d*dilog((c*(-d*e)^(1/2)+e*(c*x^(1/2)-1)+e)/(c*(
-d*e)^(1/2)+e))+1/2*b/d*ln(1+c*x^(1/2))*ln(c^2*e*x+c^2*d)-1/2*b/d*ln(1+c*x^
```

```
(1/2))*ln((c*(-d*e)^(1/2)-e*(1+c*x^(1/2))+e)/(c*(-d*e)^(1/2)+e))-1/2*b/d*ln
(1+c*x^(1/2))*ln((c*(-d*e)^(1/2)+e*(1+c*x^(1/2))-e)/(c*(-d*e)^(1/2)-e))-1/2
*b/d*dilog((c*(-d*e)^(1/2)-e*(1+c*x^(1/2))+e)/(c*(-d*e)^(1/2)+e))-1/2*b/d*d
ilog((c*(-d*e)^(1/2)+e*(1+c*x^(1/2))-e)/(c*(-d*e)^(1/2)-e))-b/d*dilog(c*x^(
1/2))-b/d*dilog(1+c*x^(1/2))-b/d*ln(c*x^(1/2))*ln(1+c*x^(1/2))
```

maxima [**F**]   time = 0.00, size = 0, normalized size = 0.00

$$-a\left(\frac{\log{(ex+d)}}{d}-\frac{\log(x)}{d}\right)+b\int\frac{\log\left(c\sqrt{x}+1\right)}{2\left(ex^{\frac{3}{2}}+d\sqrt{x}\right)\sqrt{x}}\,dx-b\int\frac{\log\left(-c\sqrt{x}+1\right)}{2\left(ex^{\frac{3}{2}}+d\sqrt{x}\right)\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^(1/2)))/x/(e*x+d),x, algorithm="maxima")

[Out] -a*(log(e*x + d)/d - log(x)/d) + b*integrate(1/2*log(c*sqrt(x) + 1)/((e*x^(
3/2) + d*sqrt(x))*sqrt(x)), x) - b*integrate(1/2*log(-c*sqrt(x) + 1)/((e*x^
(3/2) + d*sqrt(x))*sqrt(x)), x)

mupad [**F**]   time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{a+b\operatorname{atanh}\left(c\sqrt{x}\right)}{x\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c*x^(1/2)))/(x*(d + e*x)),x)

[Out] int((a + b*atanh(c*x^(1/2)))/(x*(d + e*x)), x)

sympy [**F(-1)**]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x**(1/2)))/x/(e*x+d),x)

[Out] Timed out

3.48 $\displaystyle\int \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x^2(d+ex)}\,dx$

Optimal. Leaf size=413

$$-\frac{2e\log\left(\frac{2}{c\sqrt{x}+1}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{d^2}+\frac{e\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,\sqrt{x}\right)}{\left(c\sqrt{x}+1\right)\left(c\sqrt{-d}-\sqrt{e}\right)}\right)}{d^2}+\frac{e\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{}$$

```
[Out] b*c^2*arctanh(c*x^(1/2))/d+(-a-b*arctanh(c*x^(1/2)))/d/x-a*e*ln(x)/d^2-2*e*
(a+b*arctanh(c*x^(1/2)))*ln(2/(1+c*x^(1/2)))/d^2+e*(a+b*arctanh(c*x^(1/2)))
*ln(2*c*((-d)^(1/2)-e^(1/2)*x^(1/2))/(c*(-d)^(1/2)-e^(1/2))/(1+c*x^(1/2)))/
d^2+e*(a+b*arctanh(c*x^(1/2)))*ln(2*c*((-d)^(1/2)+e^(1/2)*x^(1/2))/(c*(-d)^
(1/2)+e^(1/2)))/(1+c*x^(1/2)))/d^2+b*e*polylog(2,-c*x^(1/2))/d^2-b*e*polylog
(2,c*x^(1/2))/d^2+b*e*polylog(2,1-2/(1+c*x^(1/2)))/d^2-1/2*b*e*polylog(2,1-
2*c*((-d)^(1/2)-e^(1/2)*x^(1/2))/(c*(-d)^(1/2)-e^(1/2))/(1+c*x^(1/2)))/d^2-
1/2*b*e*polylog(2,1-2*c*((-d)^(1/2)+e^(1/2)*x^(1/2))/(c*(-d)^(1/2)+e^(1/2))
/(1+c*x^(1/2)))/d^2-b*c/d/x^(1/2)
```

Rubi [A] time = 0.72, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {44, 1593, 5982, 5916, 325, 206, 5992, 5912, 6044, 5920, 2402, 2315, 2447}

$$\frac{be\text{PolyLog}\left(2,1-\frac{2}{c\sqrt{x}+1}\right)}{d^2}-\frac{be\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}\,\sqrt{x}\right)}{\left(c\sqrt{x}+1\right)\left(c\sqrt{-d}-\sqrt{e}\right)}\right)}{2d^2}-\frac{be\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}\,\sqrt{x}\right)}{\left(c\sqrt{x}+1\right)\left(c\sqrt{-d}+\sqrt{e}\right)}\right)}{2d^2}+\frac{be\text{Po}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])/(x^2*(d + e*x)),x]
```

```
[Out] -((b*c)/(d*Sqrt[x])) + (b*c^2*ArcTanh[c*Sqrt[x]])/d - (a + b*ArcTanh[c*Sqrt
[x]])/(d*x) - (2*e*(a + b*ArcTanh[c*Sqrt[x]])*Log[2/(1 + c*Sqrt[x])])/d^2 +
 (e*(a + b*ArcTanh[c*Sqrt[x]])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*Sqrt[x]))/((c*S
qrt[-d] - Sqrt[e])*(1 + c*Sqrt[x]))])/d^2 + (e*(a + b*ArcTanh[c*Sqrt[x]])*L
og[(2*c*(Sqrt[-d] + Sqrt[e]*Sqrt[x]))/((c*Sqrt[-d] + Sqrt[e])*(1 + c*Sqrt[x
]))])/d^2 - (a*e*Log[x])/d^2 + (b*e*PolyLog[2, 1 - 2/(1 + c*Sqrt[x])])/d^2
- (b*e*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*Sqrt[x]))/((c*Sqrt[-d] - Sqr
t[e])*(1 + c*Sqrt[x]))])/(2*d^2) - (b*e*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqr
t[e]*Sqrt[x]))/((c*Sqrt[-d] + Sqrt[e])*(1 + c*Sqrt[x]))])/(2*d^2) + (b*e*Po
lyLog[2, -(c*Sqrt[x])])/d^2 - (b*e*PolyLog[2, c*Sqrt[x]])/d^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6044

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTanh[c*
x])^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && (GtQ[q, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a + b\tanh^{-1}\left(c\sqrt{x}\right)}{x^2(d + ex)}\,dx = 2\,\text{Subst}\left(\int \frac{a + b\tanh^{-1}(cx)}{dx^3 + ex^5}\,dx, x, \sqrt{x}\right)$$

$$= 2\,\text{Subst}\left(\int \frac{a + b\tanh^{-1}(cx)}{x^3\left(d + ex^2\right)}\,dx, x, \sqrt{x}\right)$$

$$= \frac{2\,\text{Subst}\left(\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx, x, \sqrt{x}\right)}{d} - \frac{(2e)\,\text{Subst}\left(\int \frac{a+b\tanh^{-1}(cx)}{x\left(d+ex^2\right)}\,dx, x, \sqrt{x}\right)}{d}$$

$$= -\frac{a + b\tanh^{-1}\left(c\sqrt{x}\right)}{dx} + \frac{(bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(1-c^2x^2\right)}\,dx, x, \sqrt{x}\right)}{d} - \frac{(2e)\,\text{Subst}\left(\int \left(\frac{a+b\tanh}{dx}\right.\right.}{}$$

$$= -\frac{bc}{d\sqrt{x}} - \frac{a + b\tanh^{-1}\left(c\sqrt{x}\right)}{dx} + \frac{\left(bc^3\right)\,\text{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx, x, \sqrt{x}\right)}{d} - \frac{(2e)\,\text{Subst}\left(\int \frac{a+}{}\right.}{}$$

$$= -\frac{bc}{d\sqrt{x}} + \frac{bc^2\tanh^{-1}\left(c\sqrt{x}\right)}{d} - \frac{a + b\tanh^{-1}\left(c\sqrt{x}\right)}{dx} - \frac{ae\log(x)}{d^2} + \frac{be\text{Li}_2\left(-c\sqrt{x}\right)}{d^2} - \frac{b}{}$$

$$= -\frac{bc}{d\sqrt{x}} + \frac{bc^2\tanh^{-1}\left(c\sqrt{x}\right)}{d} - \frac{a + b\tanh^{-1}\left(c\sqrt{x}\right)}{dx} - \frac{ae\log(x)}{d^2} + \frac{be\text{Li}_2\left(-c\sqrt{x}\right)}{d^2} - \frac{b}{}$$

$$= -\frac{bc}{d\sqrt{x}} + \frac{bc^2\tanh^{-1}\left(c\sqrt{x}\right)}{d} - \frac{a + b\tanh^{-1}\left(c\sqrt{x}\right)}{dx} - \frac{2e\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(\frac{}{1}\right)}{d^2}$$

$$= -\frac{bc}{d\sqrt{x}} + \frac{bc^2\tanh^{-1}\left(c\sqrt{x}\right)}{d} - \frac{a + b\tanh^{-1}\left(c\sqrt{x}\right)}{dx} - \frac{2e\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(\frac{}{1}\right)}{d^2}$$

$$= -\frac{bc}{d\sqrt{x}} + \frac{bc^2\tanh^{-1}\left(c\sqrt{x}\right)}{d} - \frac{a + b\tanh^{-1}\left(c\sqrt{x}\right)}{dx} - \frac{2e\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(\frac{}{1}\right)}{d^2}$$

Mathematica [A]   time = 1.64, size = 360, normalized size = 0.87

$$-\frac{2ae\log\left(\sqrt{x}\right)}{d^2} + \frac{ae\log(d + ex)}{d^2} - \frac{a}{dx} + 2bc^4\left(\frac{e\left(\text{Li}_2\left(-\frac{(dc^2+e)e^{2\tanh^{-1}(c\sqrt{x})}}{dc^2-2\sqrt{-d}\sqrt{e}c-e}\right) + \text{Li}_2\left(-\frac{(dc^2+e)e^{2\tanh^{-1}(c\sqrt{x})}}{dc^2+2\sqrt{-d}\sqrt{e}c-e}\right)\right) + 2\tanh^{-1}\left(c\sqrt{}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/(x^2*(d + e*x)),x]

[Out] -(a/(d*x)) - (2*a*e*Log[Sqrt[x]])/d^2 + (a*e*Log[d + e*x])/d^2 + 2*b*c^4*(-
1/2*((c*d)/Sqrt[x] + ArcTanh[c*Sqrt[x]]*((d*(1 - c^2*x))/x + e*ArcTanh[c*Sq
rt[x]] + 2*e*Log[1 - E^(-2*ArcTanh[c*Sqrt[x]])]) - e*PolyLog[2, E^(-2*ArcTa
nh[c*Sqrt[x]])])/(c^4*d^2) + (e*(2*ArcTanh[c*Sqrt[x]]*(-ArcTanh[c*Sqrt[x]]
+ Log[1 + ((c^2*d + e)*E^(2*ArcTanh[c*Sqrt[x]]))/(c^2*d - 2*c*Sqrt[-d]*Sqrt
[e] - e)] + Log[1 + ((c^2*d + e)*E^(2*ArcTanh[c*Sqrt[x]]))/(c^2*d + 2*c*Sqr
t[-d]*Sqrt[e] - e)]) + PolyLog[2, -(((c^2*d + e)*E^(2*ArcTanh[c*Sqrt[x]]))/
(c^2*d - 2*c*Sqrt[-d]*Sqrt[e] - e))] + PolyLog[2, -(((c^2*d + e)*E^(2*ArcTa
nh[c*Sqrt[x]]))/(c^2*d + 2*c*Sqrt[-d]*Sqrt[e] - e))]))/(4*c^4*d^2))
```

fricas [**F**]   time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\operatorname{artanh}\left(c\sqrt{x}\right)+a}{ex^3+dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^2/(e*x+d),x, algorithm="fricas")

[Out] integral((b*arctanh(c*sqrt(x)) + a)/(e*x^3 + d*x^2), x)
```

giac [**F**]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{artanh}\left(c\sqrt{x}\right)+a}{(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^2/(e*x+d),x, algorithm="giac")

[Out] integrate((b*arctanh(c*sqrt(x)) + a)/((e*x + d)*x^2), x)
```

maple [**A**]   time = 0.07, size = 620, normalized size = 1.50

$$-\frac{a}{dx}-\frac{2ae\ln\left(c\sqrt{x}\right)}{d^2}+\frac{ae\ln\left(c^2ex+c^2d\right)}{d^2}-\frac{b\operatorname{arctanh}\left(c\sqrt{x}\right)}{dx}-\frac{2b\operatorname{arctanh}\left(c\sqrt{x}\right)e\ln\left(c\sqrt{x}\right)}{d^2}+\frac{b\operatorname{arctanh}\left(c\sqrt{x}\right)e\ln}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))/x^2/(e*x+d),x)
```

```
[Out] -a/d/x-2*a/d^2*e*ln(c*x^(1/2))+a*e/d^2*ln(c^2*e*x+c^2*d)-b*arctanh(c*x^(1/2
))/d/x-2*b*arctanh(c*x^(1/2))/d^2*e*ln(c*x^(1/2))+b*arctanh(c*x^(1/2))*e/d^
2*ln(c^2*e*x+c^2*d)-b*c/d/x^(1/2)-1/2*c^2*b/d*ln(c*x^(1/2)-1)+1/2*c^2*b/d*l
n(1+c*x^(1/2))+b/d^2*e*dilog(c*x^(1/2))+b/d^2*e*dilog(1+c*x^(1/2))+b/d^2*e*
ln(c*x^(1/2))*ln(1+c*x^(1/2))+1/2*b/d^2*e*ln(c*x^(1/2)-1)*ln(c^2*e*x+c^2*d)
-1/2*b/d^2*e*ln(c*x^(1/2)-1)*ln((c*(-d*e)^(1/2)-e*(c*x^(1/2)-1)-e)/(c*(-d*e
)^(1/2)-e))-1/2*b/d^2*e*ln(c*x^(1/2)-1)*ln((c*(-d*e)^(1/2)+e*(c*x^(1/2)-1)+
e)/(c*(-d*e)^(1/2)+e))-1/2*b/d^2*e*dilog((c*(-d*e)^(1/2)-e*(c*x^(1/2)-1)-e)
/(c*(-d*e)^(1/2)-e))-1/2*b/d^2*e*dilog((c*(-d*e)^(1/2)+e*(c*x^(1/2)-1)+e)/(
c*(-d*e)^(1/2)+e))-1/2*b/d^2*e*ln(1+c*x^(1/2))*ln(c^2*e*x+c^2*d)+1/2*b/d^2*
e*ln(1+c*x^(1/2))*ln((c*(-d*e)^(1/2)-e*(1+c*x^(1/2))+e)/(c*(-d*e)^(1/2)+e))
+1/2*b/d^2*e*ln(1+c*x^(1/2))*ln((c*(-d*e)^(1/2)+e*(1+c*x^(1/2))-e)/(c*(-d*e
)^(1/2)-e))+1/2*b/d^2*e*dilog((c*(-d*e)^(1/2)-e*(1+c*x^(1/2))+e)/(c*(-d*e)^
(1/2)+e))+1/2*b/d^2*e*dilog((c*(-d*e)^(1/2)+e*(1+c*x^(1/2))-e)/(c*(-d*e)^(1
/2)-e))
```

maxima $[\mathbf{F}]$ time = 0.00, size = 0, normalized size = 0.00

$$a\left(\frac{e\log(ex+d)}{d^2} - \frac{e\log(x)}{d^2} - \frac{1}{dx}\right) + b\int \frac{\log\left(c\sqrt{x}+1\right)}{2\left(ex^{\frac{5}{2}}+dx^{\frac{3}{2}}\right)\sqrt{x}}\,dx - b\int \frac{\log\left(-c\sqrt{x}+1\right)}{2\left(ex^{\frac{5}{2}}+dx^{\frac{3}{2}}\right)\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] a*(e*log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(d*x)) + b*integrate(1/2*log(c*sqr
t(x) + 1)/((e*x^(5/2) + d*x^(3/2))*sqrt(x)), x) - b*integrate(1/2*log(-c*sq
rt(x) + 1)/((e*x^(5/2) + d*x^(3/2))*sqrt(x)), x)
```

mupad $[\mathbf{F}]$ time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{atanh}\left(c\,\sqrt{x}\right)}{x^2\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))/(x^2*(d + e*x)),x)
```

```
[Out] int((a + b*atanh(c*x^(1/2)))/(x^2*(d + e*x)), x)
```

sympy $[\mathbf{F(-1)}]$ time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

3.49 $\int \dfrac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x^3(d+ex)}\,dx$

Optimal. Leaf size=506

$$\frac{2e^2\log\left(\frac{2}{c\sqrt{x}+1}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{d^3}-\frac{e^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,\sqrt{x}\right)}{\left(c\sqrt{x}+1\right)\left(c\sqrt{-d}-\sqrt{e}\right)}\right)}{d^3}-\frac{e^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{d}$$

[Out] -1/6*b*c/d/x^(3/2)+1/2*b*c^4*arctanh(c*x^(1/2))/d-b*c^2*e*arctanh(c*x^(1/2)
)/d^2+1/2*(-a-b*arctanh(c*x^(1/2)))/d/x^2+e*(a+b*arctanh(c*x^(1/2)))/d^2/x+
a*e^2*ln(x)/d^3+2*e^2*(a+b*arctanh(c*x^(1/2)))*ln(2/(1+c*x^(1/2)))/d^3-e^2*
(a+b*arctanh(c*x^(1/2)))*ln(2*c*((-d)^(1/2)-e^(1/2)*x^(1/2))/(c*(-d)^(1/2)-
e^(1/2))/(1+c*x^(1/2)))/d^3-e^2*(a+b*arctanh(c*x^(1/2)))*ln(2*c*((-d)^(1/2)
+e^(1/2)*x^(1/2))/(c*(-d)^(1/2)+e^(1/2))/(1+c*x^(1/2)))/d^3-b*e^2*polylog(2
,-c*x^(1/2))/d^3+b*e^2*polylog(2,c*x^(1/2))/d^3-b*e^2*polylog(2,1-2/(1+c*x^
(1/2)))/d^3+1/2*b*e^2*polylog(2,1-2*c*((-d)^(1/2)-e^(1/2)*x^(1/2))/(c*(-d)^
(1/2)-e^(1/2))/(1+c*x^(1/2)))/d^3+1/2*b*e^2*polylog(2,1-2*c*((-d)^(1/2)+e^(
1/2)*x^(1/2))/(c*(-d)^(1/2)+e^(1/2))/(1+c*x^(1/2)))/d^3-1/2*b*c^3/d/x^(1/2)
+b*c*e/d^2/x^(1/2)

Rubi [A]   time = 0.87, antiderivative size = 506, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {44, 1593, 5982, 5916, 325, 206, 5992, 5912, 6044, 5920, 2402, 2315, 2447}

$$-\frac{be^2\mathrm{PolyLog}\left(2,1-\frac{2}{c\sqrt{x}+1}\right)}{d^3}+\frac{be^2\mathrm{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}\,\sqrt{x}\right)}{\left(c\sqrt{x}+1\right)\left(c\sqrt{-d}-\sqrt{e}\right)}\right)}{2d^3}+\frac{be^2\mathrm{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}\,\sqrt{x}\right)}{\left(c\sqrt{x}+1\right)\left(c\sqrt{-d}+\sqrt{e}\right)}\right)}{2d^3}-be$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*Sqrt[x]])/(x^3*(d + e*x)),x]

[Out] -(b*c)/(6*d*x^(3/2)) - (b*c^3)/(2*d*Sqrt[x]) + (b*c*e)/(d^2*Sqrt[x]) + (b*c
^4*ArcTanh[c*Sqrt[x]])/(2*d) - (b*c^2*e*ArcTanh[c*Sqrt[x]])/d^2 - (a + b*Ar
cTanh[c*Sqrt[x]])/(2*d*x^2) + (e*(a + b*ArcTanh[c*Sqrt[x]]))/(d^2*x) + (2*e
^2*(a + b*ArcTanh[c*Sqrt[x]])*Log[2/(1 + c*Sqrt[x])])/d^3 - (e^2*(a + b*Arc
Tanh[c*Sqrt[x]])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*Sqrt[x]))/((c*Sqrt[-d] - Sqrt
[e])*(1 + c*Sqrt[x]))])/d^3 - (e^2*(a + b*ArcTanh[c*Sqrt[x]])*Log[(2*c*(Sqr
t[-d] + Sqrt[e]*Sqrt[x]))/((c*Sqrt[-d] + Sqrt[e])*(1 + c*Sqrt[x]))])/d^3 +
(a*e^2*Log[x])/d^3 - (b*e^2*PolyLog[2, 1 - 2/(1 + c*Sqrt[x])])/d^3 + (b*e^2
*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*Sqrt[x]))/((c*Sqrt[-d] - Sqrt[e])*
(1 + c*Sqrt[x]))])/(2*d^3) + (b*e^2*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]

```
*Sqrt[x]))/((c*Sqrt[-d] + Sqrt[e])*(1 + c*Sqrt[x]))])/(2*d^3) - (b*e^2*Poly
Log[2, -(c*Sqrt[x])])/d^3 + (b*e^2*PolyLog[2, c*Sqrt[x]])/d^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
```

```
x][[2]], Expon[Pq, x]]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6044

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTanh[c*
x])^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c,
```

```
d, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && (GtQ[q, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a + b\tanh^{-1}\left(c\sqrt{x}\right)}{x^3(d + ex)}\,dx = 2\,\mathrm{Subst}\left(\int \frac{a + b\tanh^{-1}(cx)}{dx^5 + ex^7}\,dx, x, \sqrt{x}\right)$$

$$= 2\,\mathrm{Subst}\left(\int \frac{a + b\tanh^{-1}(cx)}{x^5\left(d + ex^2\right)}\,dx, x, \sqrt{x}\right)$$

$$= \frac{2\,\mathrm{Subst}\left(\int \frac{a + b\tanh^{-1}(cx)}{x^5}\,dx, x, \sqrt{x}\right)}{d} - \frac{(2e)\,\mathrm{Subst}\left(\int \frac{a + b\tanh^{-1}(cx)}{x^3\left(d + ex^2\right)}\,dx, x, \sqrt{x}\right)}{d}$$

$$= -\frac{a + b\tanh^{-1}\left(c\sqrt{x}\right)}{2dx^2} + \frac{(bc)\,\mathrm{Subst}\left(\int \frac{1}{x^4\left(1 - c^2x^2\right)}\,dx, x, \sqrt{x}\right)}{2d} - \frac{(2e)\,\mathrm{Subst}\left(\int \frac{a + b\tanh^{-1}}{x^3}\right)}{d^2}$$

$$= -\frac{bc}{6dx^{3/2}} - \frac{a + b\tanh^{-1}\left(c\sqrt{x}\right)}{2dx^2} + \frac{e\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{d^2x} + \frac{\left(bc^3\right)\,\mathrm{Subst}\left(\int \frac{1}{x^2\left(1 - c^2}\right)}{2d}$$

$$= -\frac{bc}{6dx^{3/2}} - \frac{bc^3}{2d\sqrt{x}} + \frac{bce}{d^2\sqrt{x}} - \frac{a + b\tanh^{-1}\left(c\sqrt{x}\right)}{2dx^2} + \frac{e\left(a + b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{d^2x} + \frac{\left(bc^5\right.}{}$$

$$= -\frac{bc}{6dx^{3/2}} - \frac{bc^3}{2d\sqrt{x}} + \frac{bce}{d^2\sqrt{x}} + \frac{bc^4\tanh^{-1}\left(c\sqrt{x}\right)}{2d} - \frac{bc^2e\tanh^{-1}\left(c\sqrt{x}\right)}{d^2} - \frac{a + b\tanh}{2d}$$

$$= -\frac{bc}{6dx^{3/2}} - \frac{bc^3}{2d\sqrt{x}} + \frac{bce}{d^2\sqrt{x}} + \frac{bc^4\tanh^{-1}\left(c\sqrt{x}\right)}{2d} - \frac{bc^2e\tanh^{-1}\left(c\sqrt{x}\right)}{d^2} - \frac{a + b\tanh}{2d}$$

$$= -\frac{bc}{6dx^{3/2}} - \frac{bc^3}{2d\sqrt{x}} + \frac{bce}{d^2\sqrt{x}} + \frac{bc^4\tanh^{-1}\left(c\sqrt{x}\right)}{2d} - \frac{bc^2e\tanh^{-1}\left(c\sqrt{x}\right)}{d^2} - \frac{a + b\tanh}{2d}$$

$$= -\frac{bc}{6dx^{3/2}} - \frac{bc^3}{2d\sqrt{x}} + \frac{bce}{d^2\sqrt{x}} + \frac{bc^4\tanh^{-1}\left(c\sqrt{x}\right)}{2d} - \frac{bc^2e\tanh^{-1}\left(c\sqrt{x}\right)}{d^2} - \frac{a + b\tanh}{2d}$$

$$= -\frac{bc}{6dx^{3/2}} - \frac{bc^3}{2d\sqrt{x}} + \frac{bce}{d^2\sqrt{x}} + \frac{bc^4\tanh^{-1}\left(c\sqrt{x}\right)}{2d} - \frac{bc^2e\tanh^{-1}\left(c\sqrt{x}\right)}{d^2} - \frac{a + b\tanh}{2d}$$

Mathematica [A]   time = 2.74, size = 394, normalized size = 0.78

$$-\frac{3ad^2 + 6ae^2x^2\log(d+ex) - 6adex - 6ae^2x^2\log(x) + b\left(3e^2x^2\left(\mathrm{Li}_2\left(-\frac{(dc^2+e)e^{2\tanh^{-1}(c\sqrt{x})}}{dc^2-2\sqrt{-d}\sqrt{e}\,c-e}\right) + \mathrm{Li}_2\left(-\frac{(dc^2+e)e^{2\tanh^{-1}(c\sqrt{x})}}{dc^2+2\sqrt{-d}\sqrt{e}\,c-e}\right)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/(x^3*(d + e*x)),x]

[Out] -1/6*(3*a*d^2 - 6*a*d*e*x - 6*a*e^2*x^2*Log[x] + 6*a*e^2*x^2*Log[d + e*x] +
 b*(c*d*Sqrt[x]*(d + 3*c^2*d*x - 6*e*x) - 3*ArcTanh[c*Sqrt[x]]*(d*(-1 + c^2
*x)*(d + c^2*d*x - 2*e*x) + 2*e^2*x^2*ArcTanh[c*Sqrt[x]] + 4*e^2*x^2*Log[1
- E^(-2*ArcTanh[c*Sqrt[x]])]) + 6*e^2*x^2*PolyLog[2, E^(-2*ArcTanh[c*Sqrt[x
]])] + 3*e^2*x^2*(2*ArcTanh[c*Sqrt[x]]*(-ArcTanh[c*Sqrt[x]] + Log[1 + ((c^2
*d + e)*E^(2*ArcTanh[c*Sqrt[x]]))/(c^2*d - 2*c*Sqrt[-d]*Sqrt[e] - e)] + Log
[1 + ((c^2*d + e)*E^(2*ArcTanh[c*Sqrt[x]]))/(c^2*d + 2*c*Sqrt[-d]*Sqrt[e] -
 e)]) + PolyLog[2, -(((c^2*d + e)*E^(2*ArcTanh[c*Sqrt[x]]))/(c^2*d - 2*c*Sq
rt[-d]*Sqrt[e] - e))] + PolyLog[2, -(((c^2*d + e)*E^(2*ArcTanh[c*Sqrt[x]]))
/(c^2*d + 2*c*Sqrt[-d]*Sqrt[e] - e))]))))/(d^3*x^2)
```

fricas [F]   time = 0.70, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b\,\mathrm{artanh}\left(c\sqrt{x}\right)+a}{ex^4+dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^3/(e*x+d),x, algorithm="fricas")

[Out] integral((b*arctanh(c*sqrt(x)) + a)/(e*x^4 + d*x^3), x)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{artanh}\left(c\sqrt{x}\right)+a}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^3/(e*x+d),x, algorithm="giac")

[Out] integrate((b*arctanh(c*sqrt(x)) + a)/((e*x + d)*x^3), x)
```

maple [A]   time = 0.08, size = 741, normalized size = 1.46

$$-\frac{be^2\,\mathrm{dilog}\left(\frac{c\sqrt{-de}-e(1+c\sqrt{x})+e}{c\sqrt{-de}+e}\right)}{2d^3} - \frac{be^2\,\mathrm{dilog}\left(\frac{c\sqrt{-de}+e(1+c\sqrt{x})-e}{c\sqrt{-de}-e}\right)}{2d^3} - \frac{b\,\mathrm{arctanh}\left(c\sqrt{x}\right)}{2d\,x^2} + \frac{c^4b\ln\left(1+c\sqrt{x}\right)}{4d} - \frac{c^4b\ln\left(c\sqrt{x}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))/x^3/(e*x+d),x)
```

```
[Out] 1/4*c^4*b/d*ln(1+c*x^(1/2))-1/4*c^4*b/d*ln(c*x^(1/2)-1)-b/d^3*e^2*dilog(c*x
^(1/2))-b/d^3*e^2*dilog(1+c*x^(1/2))+1/2*b/d^3*e^2*dilog((c*(-d*e)^(1/2)-e*
(c*x^(1/2)-1)-e)/(c*(-d*e)^(1/2)-e))+1/2*b/d^3*e^2*dilog((c*(-d*e)^(1/2)+e*
(c*x^(1/2)-1)+e)/(c*(-d*e)^(1/2)+e))-1/2*b/d^3*e^2*dilog((c*(-d*e)^(1/2)-e*
(1+c*x^(1/2))+e)/(c*(-d*e)^(1/2)+e))-1/2*b/d^3*e^2*dilog((c*(-d*e)^(1/2)+e*
(1+c*x^(1/2))-e)/(c*(-d*e)^(1/2)-e))-1/2*b*arctanh(c*x^(1/2))/d/x^2+2*a/d^3
*e^2*ln(c*x^(1/2))-a*e^2/d^3*ln(c^2*e*x+c^2*d)+a/d^2*e/x-1/2*c^2*b/d^2*ln(1
+c*x^(1/2))*e-1/2*a/d/x^2+b*c*e/d^2/x^(1/2)-b*arctanh(c*x^(1/2))*e^2/d^3*ln
(c^2*e*x+c^2*d)-1/2*b/d^3*e^2*ln(c*x^(1/2)-1)*ln(c^2*e*x+c^2*d)+1/2*b/d^3*e
^2*ln(c*x^(1/2)-1)*ln((c*(-d*e)^(1/2)-e*(c*x^(1/2)-1)-e)/(c*(-d*e)^(1/2)-e)
)+b*arctanh(c*x^(1/2))/d^2*e/x+1/2*b/d^3*e^2*ln(c*x^(1/2)-1)*ln((c*(-d*e)^(
1/2)+e*(c*x^(1/2)-1)+e)/(c*(-d*e)^(1/2)+e))+1/2*b/d^3*e^2*ln(1+c*x^(1/2))*l
n(c^2*e*x+c^2*d)-1/2*b/d^3*e^2*ln(1+c*x^(1/2))*ln((c*(-d*e)^(1/2)-e*(1+c*x^
(1/2))+e)/(c*(-d*e)^(1/2)+e))-1/2*b/d^3*e^2*ln(1+c*x^(1/2))*ln((c*(-d*e)^(1
/2)+e*(1+c*x^(1/2))-e)/(c*(-d*e)^(1/2)-e))+1/2*c^2*b/d^2*ln(c*x^(1/2)-1)*e-
b/d^3*e^2*ln(c*x^(1/2))*ln(1+c*x^(1/2))+2*b*arctanh(c*x^(1/2))/d^3*e^2*ln(c
*x^(1/2))-1/6*b*c/d/x^(3/2)-1/2*b*c^3/d/x^(1/2)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,a\!\left(\frac{2\,e^2\log\left(ex+d\right)}{d^3}-\frac{2\,e^2\log(x)}{d^3}-\frac{2\,ex-d}{d^2x^2}\right)+b\int\frac{\log\left(c\sqrt{x}+1\right)}{2\left(ex^{\frac{7}{2}}+dx^{\frac{5}{2}}\right)\sqrt{x}}\,dx-b\int\frac{\log\left(-c\sqrt{x}+1\right)}{2\left(ex^{\frac{7}{2}}+dx^{\frac{5}{2}}\right)\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*a*(2*e^2*log(e*x + d)/d^3 - 2*e^2*log(x)/d^3 - (2*e*x - d)/(d^2*x^2))
+ b*integrate(1/2*log(c*sqrt(x) + 1)/((e*x^(7/2) + d*x^(5/2))*sqrt(x)), x)
- b*integrate(1/2*log(-c*sqrt(x) + 1)/((e*x^(7/2) + d*x^(5/2))*sqrt(x)), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{a+b\operatorname{atanh}\left(c\,\sqrt{x}\right)}{x^3\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))/(x^3*(d + e*x)),x)
```

```
[Out] int((a + b*atanh(c*x^(1/2)))/(x^3*(d + e*x)), x)
```

sympy **[F(-1)]**   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(c*x**(1/2)))/x**3/(e*x+d),x)

[Out] Timed out

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1   Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
        If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
          If[LeafCount[result]<=2*LeafCount[optimal],
            "A",
          "B"],
        "C"],
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      "C",
    "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2   Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017


#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                    see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

     leaf_count_result:=leafcount(result);
     #do NOT call ExpnType() if leaf size is too large. Recursion problem
     if leaf_count_result > 500000 then
         return "B";
     fi;

     leaf_count_optimal:=leafcount(optimal);

     ExpnType_result:=ExpnType(result);
     ExpnType_optimal:=ExpnType(optimal);

     if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
     fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
     if debug then
          print("ExpnType_result<=ExpnType_optimal");
     fi;
```

```
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                    print("both result and optimal complex");
                fi;
                #both result and optimal complex
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A";
                else
                    return "B";
                end if
            else #result contains complex but optimal is not
                if debug then
                    print("result contains complex but optimal is not");
                fi;
                return "C";
            end if
        else # result do not contain complex
                # this assumes optimal do not as well
                if debug then
                    print("result do not contain complex, this assumes optimal do not
    as well");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    if debug then
                        print("leaf_count_result<=2*leaf_count_optimal");
                    fi;
                    return "A";
                else
                    if debug then
                        print("leaf_count_result>2*leaf_count_optimal");
                    fi;
                    return "B";
                end if
        end if
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C";
    end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
        1
    else
        max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
          1
      else
          max(2,ExpnType(op(1,expn)))
      end if
    else
          max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif HypergeometricFunctionQ(op(0,expn)) then
      max(5,apply(max,map(ExpnType,[op(expn)])))
    elif AppellFunctionQ(op(0,expn)) then
      max(6,apply(max,map(ExpnType,[op(expn)])))
    elif op(0,expn)='int' then
      max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
    end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
      op(2,u)
  else
      apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3   Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#               Port of original Maple grading function by
#               Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#               added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
            fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
            gamma,loggamma,digamma,zeta,polylog,LambertW,
            elliptic_f,elliptic_e,elliptic_pi,exp_polar
        ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```python
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
ExpnType(op(1,expn)),ExpnType(op(2,expn)))
     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
 expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
 Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9


#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
 well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
            ]
    if debug:
        if m:
```

```
                print ("func ", func , " is elementary_function")
            else:
                print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e',
            'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
                  return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False


     except AttributeError as error:
        return False



def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:    #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
     elif expn.operator() == operator.pow:  #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
     elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
            return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
        elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
    func)
            return max(3,expnType(expn.operands()[0]))
        elif is_special_function(expn.operator()): #is_special_function(expn.func)
            m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
            return max(4,m1)    #max(4,m1)
        elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
            m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(
    expn.args)))
            return max(5,m1)    #max(5,m1)
        elif is_appell_function(expn.operator()):
            m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(
    expn.args)))
            return max(6,m1)    #max(6,m1)
        elif str(expn).find("Integral") != -1: #this will never happen, since it
                    #is checked before calling the grading function that is passed.
                    #but kept it here.
            m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(
    expn.args)))
            return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
        else:
            return 9


#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
 else:
     return "C"
```